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CANARY BREEDING. A PARTIAL ANALYSIS OF RECORDS FROM 1891—1909.

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I. INTRODUCTORY AND HISTORICAL.

FROM the commencement of my canary and hybrid breeding experiments in 1891, it has been my custom to make notes of the different matings and their results. These refer mostly to fancy points for show purposes, but at the same time include questions of colour and quality of plumage, of sex, of eye colour, of cinnamon inheritance, and other subjects which have much scientific importance at the present time.

Inheritance in Canaries. The appearance of an interesting set of canary and hybrid breeding experiments entitled "Inheritance in Canaries" by Charles B. Davenport, and published by the Carnegie Institute of Washington in 1908, has induced me to look over and summarize my records, in order to see how far they agree with the conclusions stated in that paper. Before giving my own results

and comparing them with those referred to, it is necessary to indicate certain points in the planning of Davenport's experiments, which may possibly account for considerable difference in our conclusions.

It would have been advisable for Davenport to have selected the original stock with much greater care. Most of the original birds used were of the Hartz mountain variety (page 8), which is bred purely for song quite regardless of colour and crest, the two points concerning which the author wished to test Mendel's theories, and also the practical rules of fanciers.

"A few of the Norwich type" (page 8) were used and one is represented (Plate I, fig. 2) which no crest breeder would have in his room, and which shows that the "Norwich type" used, so far as established crest properties are concerned, was quite as poor as the nondescript Hartz crests.

The author's sole criterion of a perfect crest (even though the requisite shape is given on page 8) seems to be absence of a bald occiput. Now this bald spot accompanies most of the perfect crests of the present day. See my figs. 3, 5. Plate IV, fig. 1.

The fancier's idea of an imperfect crest is well depicted in 118 ♂ (*loc. cit.* Plate I, fig. 1) which is called perfect (page 12). This bird has practically no front to its crest—one of the worst faults—and is deficient in radiation over the eye. It might well be called an intergrade form, neither crest nor plainhead, and such birds are well known to appear occasionally in the best bird rooms. This fact is so well known that no bird with a crest showing this imperfection in the slightest degree, is used for crest-breeding purposes. I have frequently seen, and occasionally bred, these intergrade forms, varying from short fronts to complete absence of frontal crest, the posterior half only being present, and spreading from a line on the top of the head, instead of radiating evenly from a central spot. Thus it would appear that "crest" is not strictly "alternate" in inheritance.

While the application of Mendelian principles to the inheritance of a tuft of feathers, or rudimentary crest, is interesting, it is unfortunate that the terms "perfect" and "imperfect" were used, and that the bald occiput was regarded as the sign of an imperfect crest (page 11).

Similarly the disagreement with Russ with regard to "green × yellow" mating always producing "mottled" progeny (page 15), may be explained; for Russ states that it is "important that two pure-bred birds be mated." Now, neither in the Hartz nor Norwich canaries had this been the case, for pure greens and pure "yellows" are not bred in these varieties (the only exception being green Norwich in very limited numbers).

Meaning of Terms. Before I show that results vary according to the pedigree of the birds used in spite of markings, it is necessary to point out certain misunderstandings of terms. Among fanciers, the term "yellow" is synonymous with "jonque," and "buff" with "mealy." Yellow and buff are the terms usually employed, and "yellow" does not mean merely absence of dark feathering, as in

Davenport's paper, but a particular quality of feather which may occur in a green, variegated, or clear bird. The term "clear" signifies complete absence of dark feathers (Plate III, figs. 1 and 4).

The plumage qualities "yellow" and "buff." One of the most important and interesting points in plumage is the difference between "yellow" and "buff" in the technical sense. In Davenport's paper no experiments are recorded concerning the inheritance of these primary and essential qualities, nor do they seem to be understood, for "jonque" (yellow) is not merely the "dense bright shade" of a colour, nor "mealy" (buff) the "dilute, dull tint" of a colour (*vid.* page 14). There may be numerous shades of yellow, also of buff, and the richest buff may be denser and brighter than the poorest yellow. The buff feathering is thicker, has more substance, and shows pale or buff tips. The yellow feathering is thinner, has less bulk, is more silky in quality, and shows no buff tips or mealiness (Plate III, figs. 1 and 2). In addition to having less bulk of feather, the yellow bird is usually smaller in body and more lanky than the buff*.

Davenport and Russ. No. 82 of my muling strain, a yellow variegated cock now in his seventh year, has green feathers on forehead, cheeks, on a back band $\frac{3}{4}$" broad, on both wings except three primaries on one side, and four on the other: the two outer tail feathers on each side are also green. Mated with clear hens bred from clear birds, 82's record to date is:—clears 15, even-marked 3, variegated 2, the even-marked being green on wings only (secondaries) in one case; on eyes, wings and tail (six pointed) in the second; and marked on four innermost wing quills on one side, and one on the other, in the third case. The two variegated young are very similar to 82, one, however, showing a stronger tendency to even-marking. This record of 82 would seem to support Russ in his statement that the young of such a cross resemble either parent, and that any mixture of colours shows itself in the shape of even-marking†.

* Bateson (*Mendel's Principles of Heredity*, 1909, p. 298) writes as follows: "The colours of Canaries are mostly of this class (unfixable because the result of the meeting of dissimilar gametes) and, in order to obtain the requisite shades of yellow, various crosses between *pure-coloured* varieties are made, scarcely any being bred pure for exhibition."

It is difficult for a fancier to understand the meaning of this sentence, as the requisite shade of yellow is obtained from variegated birds, which cannot be called "pure-coloured."

The only way I can catch any meaning in it, is to suppose that the technical qualities of "yellow" and "buff" are not understood by the author, and that he refers to the universal rule in the fancy to mate a "yellow" with a "buff," but this is not done for the purpose of producing the requisite shade of yellow, but to maintain the structure and quality of feather desirable, and to prevent deterioration in stamina and size.

† Bateson (*loc. cit.* p. 43) gives: "Presence of black, as in green and pied types, dominant to absence of black, as in the various yellows and cinnamons."

This cannot be universally true, for if so, my variegated yellow cock, No. 82, would have given very different results.

Moreover, this season I have two yellow variegated cocks similar in appearance to No. 82, and similarly bred, paired to cinnamon-variegated hens, and, so far, out of seven young reared, only two show any sign of dark plumage; in one of these merely a tick on the head, and in the other small wing-marks, and a small head mark.

We certainly have not in this case the equal numbers of "mottled and yellow" (variegated and clear) which the Mendelian theory, as interpreted by Davenport, demands.

II. Canary Hybrids.

Davenport and Canary Hybrids. With some statements concerning hybrid breeding, I must also express disagreement, and the use of Hartz canaries is again the probable explanation of our differences. These canaries are known to have a greater tendency to produce variegated hybrids than the general run of other sorts.

In the experience of all hybrid breeders, hybrids with the "yellow" canary do *not* "frequently show more or less of the canary yellow" (page 23), the great bulk of them being completely dark, the lightly variegated being very rare, and the clear ones in Britain being scarcely more numerous than the fingers of one hand.

The same criticism applies to the following statement on page 23: "Consequently when the yellow canary is crossed with a pigmented canary, or with a finch, the hybrids are mottled." With regard to the relative frequency of the different light hybrids, the goldfinch crosses are commonest because most are bred—the goldfinch being a favourite cage bird. Quite as light ones may be obtained from the linnet, redpoll, twite (Plate III, fig. 3), greenfinch (Plate III, fig. 4), and siskin (Plate III, fig. 1). Since 1891 I have bred a great number of all the different sorts of canary hybrids. The great bulk of these have been quite dark (Plate III, fig. 2): several almost clear of different kinds have appeared, but not until this season (1908) have I been able to say that a perfectly clear mule has fallen to my lot (Plate III, fig. 1). The following table gives the relative proportions of hybrids bred from my strain, which has been carefully built up since 1891, for the purpose of producing "light mules."

Table of Canary Hybrids bred since 1891, *arranged to show Plumage Colour.*

D=Dark plumage, with no white or clear feathers.
S. V.=Slightly variegated, a few small white or clear spots in an otherwise dark plumage.
V=Variegated, from ¼ clear to large marks (eyes, wings, and tail).
L. V.=Lightly variegated, from even marks to small ticks of dark on a clear plumage.
Cl.=Clear, complete absence of dark feathers.

	D	*S. V.*	*V*	*L. V.*	*Cl.*	Totals
Goldfinch (*C. elegans*) ...	172	74	75	19	0	340
Siskin (*C. spinus*)	35	8	4	1	1	49
Linnet (*L. cannabina*) ...	61	17	17	0	0	95
Greenfinch (*L. chloris*) ...	19	4	4	1	0	28
Redpoll (*L. rufescens*) ...	6	7	1	0	0	14
Totals	293	110	101	21	1	526

With five or six exceptions these hybrids have all been bred from male wild bird and female canary. From 1904 to 1908 the sex was noted as far as possible. Some mules are difficult to sex, and all are difficult to keep in numbers because of their pugnacity. The results recorded were: males 110, females 25, unknown 48. After making allowance for calling some females males, these results will still show that there is an excess of males in canary hybrids.

Clear Hybrids. Besides an almost clear goldfinch-canary hybrid* which was bred by myself, I have at present in my bird-room a clear yellow siskin-canary mule† also bred by myself, and which is the only clear siskin-canary hybrid in existence (Plate III, fig. 1). I also own at present a clear buff greenfinch-canary hybrid‡, the only clear greenfinch mule I have seen (Plate III, fig. 4). Also two almost clear twite-canary hybrids§. This list shows that not only the nearly clear but the wholly clear hybrid does occur, although extremely rare.

III. CINNAMON AND OTHER ALLIED WILD SPORTS.

Cinnamon Inheritance. Although many general rules have been found to apply to canary breeding, there are some interesting exceptions, and the mystery of cinnamon blood is one which has long been, and still is, the puzzle of the fancy. It may be well to state first what is already known concerning the breeding of cinnamon canaries, before adding any fresh information.

Cinnamon colour of plumage (Plate I, fig. 1) in young birds can be obtained only by using a cinnamon or cinnamon-bred male bird. If a male having no cinnamon blood be mated to a self-cinnamon female, the young have no cinnamon feathers. If a cinnamon or cinnamon-bred male be mated to a female with no cinnamon blood, all the young which show any cinnamon feathers are females.

In addition to these known facts, I have proved this year that a self-cinnamon male mated to a clear female with pink eyes—the sign of cinnamon descent—may have cinnamon-variegated male progeny.

I have also bred this season a yellow male bird—clear except four small cinnamon-ticked saddle feathers—from two clear parents that were pink-eyed, but had shown no cinnamon feathers in their pedigree for at least two generations.

Crosses of Pink-Eyed ♂s and Dark-Eyed ♀s. My records also show that if a pink-eyed male be mated to a dark-eyed female, all the clear, variegated, and green, dark-eyed progeny are males, and all the clear, variegated-cinnamon, and self-cinnamon, pink-eyed progeny are females: the sexes also occur in equal numbers.

* Clear except four small dark ticks, which gained second prize at the Crystal Place in 1907, and other leading prizes.

† First prize, City of Glasgow; first prize, Glasgow and West of Scotland; first, Aberdeen; first, Scottish National at Edinburgh (1908); first prize and championship diploma, Crystal Palace, 1909.

‡ Second Prize, Glasgow and West of Scotland, 1908; second, Aberdeen; second, Scottish National (1908).

§ One of them second prize, Glasgow and West of Scotland, 1908; first, Scottish National (Edinburgh), 1908 (Plate III, fig. 3).

The terms "pink-eye" and "dark-eye" are strictly reserved for those chicks in which these qualities were noted within two days of hatching.

These results agree with those of Doncaster with regard to sex inheritance in the moth *Abraxas grossulariata*, and its variety *lacticolor*, which appear in Report IV to the Evolution Committee of the Royal Society.

The summary of my matings give the following results (see Appendix B, p. 40):

Pink-eyed ♂ × dark-eyed ♀ : 14 matings* = 24 dark-eyed ♂ + 21 pink-eyed ♀ + 4 dark-eyed ○ + 7 pink-eyed ○.

If we add the 4 dark-eyed ○ to the 24 dark-eyed ♂, and the 7 pinked-eyed ○ to the 21 pink-eyed ♀, which would be probably a correct procedure, we should have equality of the sexes and 28 dark-eyed males against 28 pink-eyed females.

A fuller discussion of this subject will be found in Section VII (iii) below.

Wild Cinnamon Sports and Cinnamon Hybrids. For some years I have been collecting all the wild cinnamon birds which I could acquire. Any rarity of this sort is usually called a cock in order to enhance its value (a remark which applies also to many museum specimens), but I have now had a fair number in my aviaries, and have always succeeded in getting these cinnamon "cocks" to lay (e.g. Plate II, fig. 2)†. Some of these have also been examined post-mortem and the sex verified. I have also seen and examined many cinnamon wild birds belonging to other fanciers. I have further bred six cinnamon hybrids of different sorts (Plate I, fig. 1), and seen and examined several others. All these cinnamon wild birds and hybrids I have found to be females.

Therefore I feel confident in stating that (*a*) all cinnamon wild birds are females, and (*b*) that male cinnamon hybrids must be very rare, for, so far, I have seen none‡. These statements are based upon :—

1. Twenty cinnamon wild birds and cinnamon hybrids seen at bird-shows, and in bird-rooms.

2. The following living examples at present in my possession:

   (*a*) Four cinnamon and very pale cinnamon (almost white) greenfinches (*L. chloris*) (e.g. Plate II, fig. 2).

   (*b*) One cinnamon yellowhammer (*E. citrinella*).

   (*c*) Three cinnamon canary-greenfinch hybrids (Plate I, fig. 1).

3. The following formerly in my possession:

   (*a*) One self-cinnamon canary-greenfinch hybrid.

   (*b*) One self-cinnamon canary-linnet (*L. cannabina*) hybrid bred by myself.

* Sex uncertain = ○.

† The cinnamon greenfinch hen usually lays pale cinnamon eggs devoid of markings unless mating has taken place.

‡ John L. Burnett, Govan, informs me that he has bred male cinnamon greenfinch hybrids.

4. The following preserved specimens which died in my possession, the sex being verified post-mortem:

(*a*) Three self-cinnamon greenfinches (*L. chloris*)*.

(*b*) One self-cinnamon goldfinch (*Carduelis elegans*).

(*c*) One self-cinnamon sparrow (*Passer domesticus*).

(*d*) One very pale cinnamon (almost white) sparrow (*P. domesticus*).

(*e*) One self-cinnamon linnet (*L. cannabina*).

5. One very pale cinnamon (almost white) blackbird (*T. merula*) which was shot in October 1908, sent to me by Mr John Dixon, Wigton, and examined post-mortem (Plate II, fig. 3).

6. One cinnamon goldfinch (*Carduelis elegans*) which belonged to Mr John Hector, Aberdeen, was known to be a female during life, and has now been preserved and presented to me.

7. One very pale cinnamon (almost white) starling (*Sturnus vulgaris*) caught by a cat in Aberdeen recently, and now preserved in good condition, and in my possession. This bird has every appearance of being a female.

All of these cinnamon sports and hybrids are of the female sex.

Wild White Sports and White Hybrids. While I have found all self-cinnamon sports in wild birds, and also all that show the faintest shade of cinnamon colour in their plumage to be females, I have also been impressed with the fact that most that show any noticeable amount of pure white plumage are males (e.g. Plate II, fig. 1).

The following have been verified post-mortem, and most of them are in my possession (they are all males):—

1. One white corn bunting (*E. miliaria*)†. This bird has two or three wing quills ticked with dark colour, all the rest of the bird being clear.

2. Two almost clear linnets (*L. cannabina*).

* Professor Dean kindly prepared sections from corresponding parts of the eyes of one of these cinnamon greenfinch hens, and of a normal wild male greenfinch: the latter had considerably more pigment generally but especially in the posterior hemisphere, and it was much blacker even in the most sparsely distributed areas than in the cinnamon where it was of a rich brown colour.

Professor J. Arthur Thomson and his assistant Dr John Rennie kindly cut sections of the following eyes of canaries:—1. Yellow variegated ♂ bred from cinnamon father (*Fdc*). 2. Clear buff crest ♀. 3. Cinnamon-marked (wings) ♀. 4. Clear yellow pink-eyed hen—paternal aunt of the white canary (Plate I, fig. 4).

The amount and colour of the pigment differed in the order given; in 1, it formed a fairly thick black line, with brownish shades in its thinner parts: in 2 the line was thinner, and dark brown in sparsely distributed areas: in 3, there was very much less pigment forming a thin golden brown line, uniformly interrupted with clearer spaces between the pigment cells: in 4, the pigment formed a still paler and thinner golden line, with larger clear inter-spaces between the pigment cells.

† Shot in Durris, Aberdeenshire, in the autumn of 1908, and presented to me by Mr McDonald, Schoolmaster, after being stuffed by the late Mr George Sim, Naturalist.

3. One half-clear redpoll (mealy) (*L. linaria*).

4. One almost clear yellowhammer (*E. citrinella*).

5. One ¾ clear red grouse (*L. scoticus*).

6. One ¾ clear blackbird (*T. merula*).

7. One almost clear chaffinch (*F. coelebs*) shot in Aberdeenshire in Dec. 1908 (Plate II, fig. 1). This beautiful bird, shot ruthlessly by a gamekeeper during the severe snowstorm, when it might easily have been caught alive, is white all over with the following exceptions: (1) the 8th and 9th quills and 3 or 4 coverts of the right wing are of the normal dark colour; (2) the left wing has the 5th quill grizzled, the 7th, 8th and 9th dark normal colour, and 3 or 4 coverts also dark; (3) there are also 3 or 4 ticked scapulars on each side. The upper breast has a faint reddish-brown, and the dorsal region a yellow tinge, which is also seen on the rump (*vid.* coloured figure).

8. Dr Henry of Kemnay, Aberdeenshire, sent me on June 15th, a beautiful pure white male wood pigeon (*Columba palumbus*), which had been shot a day or two previously. It is "clear" (no dark feathers): the eyes are not noticeably abnormal.

I have bred hybrids this season from the following male birds:

1. One ¾ white (clear) linnet (*L. cannabina*).

2. One ⅓ white (clear) linnet (*L. cannabina*).

Also from two other males of the same description in previous seasons.

I do not say that *all* more or less white sports in wild birds are males, for I have at present two linnets with one or two white spots about 4—6 millimetres in size and these are undoubtedly females, and I have seen a pure white female pheasant (*Phasianus colchicus*).

Clear (white) and almost clear female hybrids also occur now and then. I believe, however, that the majority in this case is greatly on the male side, just as we have seen the preponderance in the case of cinnamon hybrids to be on that of the female.

It should be possible to produce male cinnamon hybrids by mating a self-cinnamon cock canary with say a self-cinnamon greenfinch. The nearest approach I have made to this is a male canary-greenfinch hybrid of a peculiar greyish colour (neither the ordinary "dark" mule, nor cinnamon) which I bred last year from a clear yellow cock canary and a self-cinnamon greenfinch—the canary being bred from a clear yellow cock and a buff green hen probably of cinnamon descent.

Origin of the Cinnamon Canary. This question of correlation of sex with cinnamon colour is a most interesting and puzzling one, and may explain the rarity of cinnamon varieties in wild birds. It also increases our interest in the origin of the cinnamon canary.

It would almost appear that we must look for and supply another sort of complementary colour—perhaps greens from cinnamon—before the cinnamon female is capable of producing a male with sufficient cinnamon blood to propagate the variety (*vid.* Greenfinch family in Summary of Conclusions). In the case of the canary it is possible that the cinnamon variety was propagated by the mating of original wild green cocks (Plate I, fig. 2) with the self-cinnamon female sports, the green males from this cross being capable, when mated with cinnamon females, of producing cinnamon males for the propagation of the variety*.

IV. THE CINNAMON CANARY AS THE FOUNDATION OF VARIETY.

The Cinnamon Canary as the Source of all our Varieties. As I believe this interesting canary—the cinnamon—to be the starting point, after the wild green bird, of all the present varieties of canaries, I shall give, at this stage, the grounds for this belief, which has been founded mainly on a study of my cinnamon wild birds, and of the cinnamon and other hybrids I have produced. I have satisfied myself that cinnamon blood, wherever found, indicates the presence of a character, essentially sporting or varying not only in respect to colour and type of plumage, but also to type generally, e.g. size and form of body. I would first draw attention to the following points:

(1) Three rich coloured self-cinnamon greenfinches (e.g. Plate II, fig. 2) acquired last year, and kept outdoors, have this year moulted pale cinnamon.

(2) One of the pale cinnamon greenfinches of last year has moulted paler still this year. It is now creamy white (cf. the blackbird, Plate II, fig. 3), but still shows traces of its original rich cinnamon colour.

We learn from this to recognise several shades and intensities of the cinnamon colour which occurs in wild birds; this fact we must remember later when we discuss the earliest varieties of the canary.

(3) Two cinnamon-variegated canary-greenfinch hybrids (e.g. Plate I, fig. 1) bred by me in 1906 and 1907, each with a small white spot on the nape, and two or three white tail feathers (the rest of the feathers being self-cinnamon), have this year largely increased the white areas of plumage—the heads, tail feathers and coverts of both showing white areas which were previously cinnamon.

(4) A self-cinnamon canary-linnet hybrid bred by me developed several white tail feathers at the second moult.

(5) One of the cinnamon-variegated canary-greenfinch hybrids (Plate I, fig. 1) after taking second prize at Bathgate in perfect plumage, has moulted in its second year into a Dutch Frill hybrid (cf. Plate V, fig. 1), the body feathers, although still cinnamon, showing the most extraordinary twists and turns, just as if it had been pulled backwards through a thick hedge, as Tegetmeier says of the Frizzle Fowl,

* C. L. W. Noorduijn (Groningen) tells me that at Ponta Delgada, the wild green canary is crossed with "yellow," and that cinnamon-variegated birds are thus obtained.

which must have originated, I believe, in a similar manner, from the cinnamon Cochin. At all events, I can say decidedly in the case of the hybrid, that there was no Dutch Frill blood in the canary parent, which belongs to a strain I have bred for many years.

(6) This same hybrid, along with the frilled feathers, has assumed a semi-upright position, thus indicating the origin of our canaries of shape and position, e.g. Lancashire, Belgian and Scotch Fancy, which are supposed to have come from the Old Dutch variety (cf. Plate V, fig. 1 and fig. 4, p. 23).

(7) I have also noted that cinnamon canaries and hybrids sometimes develop an extra number of tail feathers: one cinnamon-variegated bird of this year has thirteen. I have also two hybrids from cinnamon birds with fourteen each.

(8) I have bred and now possess alive two siskin-canary hybrids, one with a perfectly shaped golden yellow cap (most of the rest of the bird being dark), and the other with a beautiful silver-spangled back (most of the rest of the bird being clear) (Plate I, fig. 3).

The cap and spangled back are both characteristics of the Lizard canary, a specimen of which I never possessed (Plate IV, fig. 3). The strain of the canary parent is entirely free from any Lizard cross. The spangled back appeared at the first moult, just as occurs in the spangling of the Lizard canary. I have also bred several other siskin-canary hybrids with irregular or "broken" Lizard caps. The canary parent in each case was of a strain with cinnamon blood but with no Lizard cross.

In addition to the probability that the Frizzle Fowl has originated from the cinnamon Cochin, it is also likely that the different spangled varieties of poultry have had a similar origin.

We also know that Lizard and London Fancy (Plate IV, figs. 2 and 3) canaries both show the same unstable character of plumage as the cinnamon, both of these varieties being fit to exhibit only during the show season after their first moult, owing to subsequent changes in plumage.

Consequently we may safely infer that both Lizard and London Fancy canaries have been derived from cinnamon canaries.

The "Pink-eye" in Canaries. Before giving my matings of the crested variety and the results of my breeding for colour and quality of plumage, a short reference is here necessary to "pink-eye"—*the* sign of cinnamon blood—in canaries. In the adult canary, pink-eye very seldom shows sufficiently to attract attention while the bird is in the cage—in this respect differing from pink-eyed albinos, e.g. Russian rabbit, and also many wild birds. I have at present, a clear yellow yellowhammer (*E. citrinella*), with pale grey irides, and whose bright red eyes can be seen in any position of the bird in its cage: this bird can see to pick up a mealworm only from a well-illuminated part of the cage bottom (Plate II, fig. 4).

Acuteness of vision is seldom diminished in the pink-eyed canary; in it the pink-eye is best seen by holding the bird with its beak directed to the source of light (a roof-light is best) and to the examiner's face. In this position both brightly illuminated pupils (the fundus reflex) will be seen. The brightness depends mostly on the size of the pupil, and on the amount of top shadow, e.g. a crested bird will show it more clearly than a plainhead.

As an ordinary dark-eyed bird may show a small amount of redness of pupil when held in the position described, and as the degree of brightness depends on the amount of illumination—just as in using the ophthalmoscope in the case of the human eye—the only certain way to tell a pink-eyed canary is to examine the chick immediately after hatching, when the unopened eye region will be seen to be of a distinct pink (fleshy-pink) colour and quite different from that of its dark-eyed nest companions. For this reason also it is preferable to use the term "pink-eye" rather than "red" or "brown-red" eye.

V. THE EVOLUTION OF THE CANARY. EVIDENCE FROM HERVIEUX*.

One is inclined to attach much importance to this author's statements as they bear evidence of being founded on actual experiments, and are not mere repetitions from other writers.

Indeed many of the stories concerning the breeding of those mules, which we, at the present time, regard as impossible, e.g. chaffinch-canary, yellowhammer-canary, have probably arisen from the too free interpretation and misunderstanding of Hervieux's statements.

For he instances the above two hybrids, but only to illustrate his nomenclature. "A male canary being coupled with a female chaffinch, the young which come from them are named Serins mulets de Pinçon" and the others the same:

"Canary mule of the linnet."

"Canary mule of the yellowhammer (Bruant)."

"Canary mule of the goldfinch."

Having stated this he adds:

"Of all those birds of which I am about to speak, those which one pairs most commonly with our canaries, are the goldfinches, male and female, for the others are but seldom used above all at present, so that is an experiment which some new Fanciers (nouveaux Curieux) wish to make, to see what sorts of mules are produced from these different birds."

This clearly proves that Hervieux had no personal knowledge of the yellow-hammer-canary and chaffinch-canary hybrids, which subsequent writers (e.g. Buffon) state, on this inadequate evidence, to have been bred.

* *Traité des Serins de Canarie, par Hervieux* (1713). There are various editions from 1709—1785. The oldest most complete one is that of 1713 which I have used.

This prepares us to receive Hervieux's list of varieties of the canary in 1713 with a considerable amount of assurance that we are dealing with facts, which, being properly interpreted, are of the utmost importance.

But we meet in all translators, and in many subsequent writers (whether professed translators or not) with great errors even in the rendering of the variety-names.

For instance, the writer of the canary article in Rees' *Encyclopaedia* (1819) translates the term "Isabelle" as pink, and gaily proceeds to speak of the Pink Canary, and also the Pink Canary with red eyes! This translation occurs also in Buffon.

Before quoting the varieties, in order to understand the nomenclature, it is desirable to give the following extracts: "Il faut remarquer qu'il y a bien des Serins dont je viens de parler, qui ont outre la queue blanche, des plumes blanches à une aile, et souvent aux deux ailes; mais malgré cette différence particulière, les Curieux ne leur donnent pas un autre nom, que Serin à queue blanche, ou race de Panachez," which may be translated as follows:—

"It is necessary to mention that there are many Canaries of which I am about to speak, which have besides the white tail, the feathers white of one wing, and often of both wings; but in spite of this particular difference, Fanciers (Curieux) do not give them another name than Canary with white tail, or Variegated kind."

It is evident that variegation, as we now know it, was just beginning at that time, and that it arose from the sporting types of which "Gris," "Jaune" and "Blonde" are mentioned. For Hervieux says that we know canaries of these types when they are of the variegated race, as having:—

1. Several white feathers in the tail.
2. Several white claws (ergots).
3. Le duvet.

The transitional stage of the bird is also shown by Hervieux's uncertainty as to the kinds to include under the term "variegated race," for he also adds (p. 272): "I say also that there are canaries which are of the variegated race, which have not, however, any of the three marks, which I have given above, or which have not even one of them; so that it is necessary to leave it to the good faith of those who sell them to you for the variegated race."

These explanations prove that the term "race de Panachez" strictly means with tail white, with a few white body feathers, or with both these variations from self-grey, or self-cinnamon in their different shades.

In our muling experiments we have shown that the first variations to occur are these particular ones, viz. white feathers in tail, and small white spot at the back of the head, or on other parts of the body (Plate I, fig. 1). Our term variegated is denoted towards the end of the list ("which commences with the commonest and finishes with the most rare") by the single word "panaché," e.g. "Serin Panaché commun."

To understand the list, we must also recollect, as I have shown in my experiments with wild cinnamon sports, that self-cinnamon is a varying colour, and frequently changes in the same bird through various shades of pale cinnamon to a creamy white, often with a gloss of yellow on the surface (blond doré) (Plate II, fig. 3).

We must also remember that the colour grey is like cinnamon, one of the pallid variations which occur in nature, e.g. grey greenfinch, a very beautiful example of which, a female, was shown at the Scottish National Show 1909, and is the property of Mr J. W. Bruce, Coldstream. This bird is said to be three years old, and not to have changed its colour—but it is quite likely that other examples might grow paler just as the cinnamon type does—for present day grey and grizzle crests invariably moult lighter each year until ultimately they become clear*.

The mottled or spangled type of variation (Plate IV, fig. 3) (in my opinion, closely allied to cinnamon), also appears in the agate varieties.

Having considered these preliminary points, we are now in a position to interpret the list itself:—

Noms que l'on donne aux Serins, selon leurs différentes couleurs.

Je croy qu'il est apropos de marquer ici les noms que l'on donne communément aux serins, selon leurs différentes couleurs; afin que l'on sçache en quelle classe, ou plûtôt en quel degré de beauté sont les serins que l'on a, ou ceux que l'on souhaite avoir; pour cet effet je me suis proposé de les nommer par ordre, en commençant par les plus communs, et finissant par les plus rares.

1. Serin Gris commun.
   The ordinary grey canary.

2. Serin Gris aux duvets† et aux pattes blanches, qu'on appelle Race de Panachez.
   Slightly variegated frilled canary with white feet.

3. Serin Gris à queue blanche, race de Panachez.
   Slightly variegated frilled canary with white tail.

4. Serin Blond commun.
   The ordinary pale canary.

5. Serin Blond aux yeux rouges.
   The pale canary with pink eyes.

* By the kindness of Mr Alex. Cochrane, Edinburgh, I now possess a silver-grey linnet (*L. Cannabina*) —also of female sex.

† "Duvets" means the light feathers which adorn the under surface of the body of birds, and may be translated downy or frilled—for it is this part of the bird that first shows the tendency to excess of feather seen in Dutch Frills. Hervieux's explanation of "le duvet" at page 271 may be translated as follows:—"which shows itself, when taking your canary in your hand, you find on it, on blowing it under the body and stomach, a little white down (un petit duvet blanc) and in consequence of a different colour from the natural plumage."

He also adds:—"There are some canaries which have much more of this down than others. This is what one finds with the Fanciers, one they call Serins au petit duvet, that is to say, those which show a little; and the others they call Serins au grand duvet, that is to say, those which have much: this down does not appear usually till near the moult."

6. Serin Blond doré.
   The pale canary glossed with yellow.
7. Serin Blond aux duvets, race de Panachez.
   Slightly variegated pale frilled canary.
8. Serin Blond à queue blanche, race de Panachez.
   Slightly variegated pale canary with white tail.
9. Serin Jaune commun.
   The lemon-yellow canary.
10. Serin Jaune aux duvets, race de Panachez.
    Slightly variegated frilled lemon-yellow canary.
11. Serin Jaune à queue blanche, race de Panachez.
    Slightly variegated lemon-yellow canary with white tail.
12. Serin Agate commun.
    The original Lizard canary.
13. Serin Agate aux yeux rouges.
    The Lizard with pink eyes (showing cinnamon origin).
14. Serin Agate à queue blanche, race de Panachez.
    Slightly variegated Lizard canary with white tail.
15. Serin Agate aux duvets, race de Panachez.
    Slightly variegated frilled Lizard.
16. Serin Isabelle commun.
    The original cinnamon canary.
17. Serin Isabelle aux yeux rouges.
    The cinnamon canary with pink eyes.
18. Serin Isabelle doré.
    The cinnamon canary glossed with yellow.
19. Serin Isabelle aux duvets, race de Panachez.
    Slightly variegated frilled cinnamon.
20. Serin Isabelle à queue blanche, race de Panachez.
    Slightly variegated cinnamon with white tail.
21. Serin Blanc, aux yeux rouges (Plate I, fig. 4).
    The white canary with pink eyes.
22. Serin Panaché commun.
    The original variegated canary.
23. Serin Panaché aux yeux rouges.
    Grey-variegated canary with pink eyes.
24. Serin Panaché de blond.
    Pale cinnamon-variegated canary.
25. Serin Panaché de blond aux yeux rouges.
    Pale cinnamon-variegated canary with pink eyes.

26. Serin Panaché de noir*.
Green-variegated canary (e.g. figs. 2 and 3).

27. Serin Panaché de noir-jonquille, aux yeux rouges.
Cinnamon-green variegated canary with pink eyes.

28. Serin Panaché de noir-jonquille et regulier.
The London Fancy canary (Plate IV, fig. 2).

29. Serin Plein, qui sont à present les plus rares.
Clear orange-yellow canary, which is, at present, the rarest.

[30†. The crest canary (or rather the crowned) which is one of the most beautiful (Buffon).]

The study of this most instructive list which begins with the commonest and ends with the rarest, combined with a knowledge of the nature and behaviour of sports in wild birds generally, proves distinctly the "sport" origin of all the varieties of the canary.

In classes 1—3 we have the grey canary varying in the direction of frilled and white feathers, and white feet.

In 4—8, the pale type (either of grey or cinnamon) shows the same variations, but in addition a more marked tendency to albinism (pink eye), and towards the differentiation between "yellow" and "buff."

In 9—11, the uniformly lemon-yellow canary shows similar plumage variations.

In 12—15, the original Lizard proclaims its cinnamon descent by having pink eyes, besides the plumage changes like the others.

In *Canary and Cage-Bird Life* for April 16, 1909, Mr L. Butterworth's lecture to the Rochdale Ornithological Club on "The Lizard Canary Fancy Past and Present" is given (Plate IV, fig. 3). In it, this lemon-yellow variation with its tendency to become paler is described in connection with the Lizard canary of forty years ago. At the same time the "duvet" or frilled variety appeared. As these statements from an experienced and observant fancier are important historically, I give them in full, premising that this lemon-yellow colour in mules is well known to be due to cinnamon inheritance.

"When I first started to breed the Lizard canary there was a strain of Lizards which were very plentiful in and around Rochdale known as the lemon Lizard, or lemon jonque, on account of the cap being a pale yellow colour somewhat the colour of a lemon. In its nest feathers it had a back full of straight, narrow rowing, but after its first moult the colour of its cap and the tips of the small feathers were of the same pale yellow colour, the spangle being not nearly so

* Noir—Such quills and tail feathers are mostly black or smoky when spread out; but when in position show their yellowish-green edging mainly.

† In the 1793 London edition of Buffon's *Natural History*, Class 30 is included in Hervieux's 1713 list of varieties, where I have been unable to find it.

All the varieties except 29 and 30 are mentioned in the 1709 and 1711 editions of Hervieux.

distinct as that of the orange-coloured variety. Breeders, seeing that it stood no chance on the show bench, refused to breed with it, and consequently, in a few years, the strain died out.

"About the same time, there was another strain known as the flat or hollow-backed Lizard. This was a class of bird with a back full of large, distinct spangling, or moons as we called them. The moons were distributed all over the back, and not in straight, regular rows as you see them in the Lizards of to-day. This class of bird had very often a *split* or *parting* down the centre of the back, and as it very rarely got into the money at any show, gradually became scarce, until it has almost met with the same fate as the lemon jonque. I should never pair two golds or two silvers together without a special reason*.....

"I remember experimenting in this direction many years ago. I paired a gold cock with a gold hen, and succeeded in breeding some decent young from the pair. Then I inbred with two of the young ones, also both golds. The result was the feathers on the young birds bred from the inbred pair, instead of lying close to the body, grew the wrong way about; they turned up over the back just like those on a *Frizzle* Fowl, which convinced me that you can go too far in that direction" (cf. fig. 1).

In Classes 16—20, the original cinnamon displays similar variations to those in previous classes.

Class 21 is specially interesting to us, as the only white canary ever seen by British fanciers was exhibited at the last Crystal Palace show, and is undoubtedly of cinnamon descent (Plate I, fig. 4).

In 22—26, we have the start of our present day variegated varieties (cf. figs. 2 and 3).

Class 27 is interesting as indicating what I call a cinnamon-green variegation, for these birds, the produce of a cinnamon cock with a dark-eyed hen, are all males and undoubtedly show more pinkness of the eye than other green-variegated birds. They also frequently show a tendency to the dark-green—almost black—London Fancy markings.

In Class 28, we have the start of the London Fancy—now almost extinct—and its occurrence immediately after Class 27 may be of some assistance in re-establishing this beautiful variety (Plate IV, fig. 2).

In Class 29, we have the appearance, of which I have had experience, of a rich orange-yellow bird. The special quality of rich colour which characterizes the Norwich canary has probably its origin here.

Class 30, mentioned by Buffon as being in Hervieux's list, whether there or not, at all events shows that crest was known about 1750.

In our cinnamon muling experiments we have shown that position is correlated with cinnamon sporting and frills.

* "Gold" is the yellow variety of Lizard, and "silver" the buff.

In this manner our birds of position—Lancashire (fig. 4, p. 23), Yorkshire (Plate I, fig. 4), Dutch Frill (Plate V, fig. 1), Belgian, etc.—have arisen.

We have thus been able to trace the origin of all our present day varieties.

It only remains for some enterprising fancier to follow out the experiments farther and introduce some new varieties, e.g. fantail, trumpeter, black, tumbler, silkie, and many others.

In the English (1718) translation of Hervieux the term "duvet" is taken to mean *rough-footed*, and "panaché" and "race de Panachez" are both translated *copple-crowned*. The one interpretation is as nonsensical as the other, and quite as bad as that of the writer already mentioned who describes "Isabelle" as pink. But the reference to copple-crowns is interesting, as probably indicating the existence of a crested canary in England before 1718. One feels, however, that in the case of this translator one has to deal with a poultry or pigeon, and not a canary fancier.

Note. I may mention that Temminck in his *Histoire Naturelle Générale des Gallinacés* describes the silk fowl under the name of the "Coq à Duvet," and gives it the scientific title of *Gallus Lanatus* (Tegetmeier's *Poultry Book*, 1867). Also in support of my interpretation of "panaché," let me quote the following:—

"Description des Couleurs d'un Canari Panaché, observé avec M. de Montbeillard:"

"The shades and arrangement of the colours of the variegated canaries differ exceedingly; some are black on the head, others not; some are spotted irregularly, and others with great regularity. The differences of colour are commonly perceived only on the upper part of the bird; they consist of two large black spots on each wing, the one before and the other behind, in a large crescent of the same colour placed on the back, pointing its concavity towards the head, and joining by its horns to the two anterior black spots of the wings. Lastly, the tail is surrounded behind by an half-collar of grey, which seems to be a compound colour resulting from the intimate mixture of black and yellow." (Buffon's *Nat. Hist. of Birds*, London, 1793.)

Albin's Song-Birds. At this stage also, let me refer shortly to another old book on song-birds that agrees in every detail with my interpretation of Hervieux's list of varieties of the canary, and also adds some additional information. The title is:

A Natural History of English Song-Birds, and such of the Foreign as are usually brought over, and esteem'd for their Singing, etc. By Mr Eleazer Albin. London 1759 (3rd Edit.).

Albin's knowledge of the song-birds he mentions, and their proper treatment in confinement is so complete and excellent (e.g. his treatment of the goldfinch with regard to hemp seed) that this little book would be an up-to-date guide at the present day. On this account I attach much importance to his list of varieties of the canary given on page 86.

Albin's List of Varieties of Canary in 1759.

(1) "Bright lovely yellow, with jet-black spots." This undoubtedly describes the London Fancy canary, which like the Lizard ("agate" of Hervieux) we believe to be derived from the cinnamon, and which is now almost extinct (Plate IV, fig. 2).

(2) "The Mealy-Bird, so named from the mealy kind of Colour which seems to cover his Feathers." This is the buff bird of the present day.

(3) "Mottled-Birds: their chief colour is white mottled with black or brownish spots." These are our green-variegated and cinnamon-variegated varieties.

(4) "All yellow." Our clear yellow.

(5) "All white" (Plate I, fig. 4).

(6) "Grey." This is the original grey, the "Serin gris" of Hervieux, which is closely allied to the cinnamon canary of to-day.

(7) Other varieties not named.

The Epitome of the Art of Husbandry. London, 1675. By J. B., Gent. At this date in England canaries were green, and variegation had evidently not occurred, for the author—Joseph Blagrove, who is particularly well informed with regard to singing-birds—says (p. 107) "Many Country-People cannot distinguish a Canary from one of our common Green-Birds, etc."

He also writes (p. 106): "The first I shall begin withal is, the Bird called the Canary-Bird, because the original of that Bird came from thence (I hold this to be the best Song-Bird): But now with industry they breed them very plentifully in Germany, and in Italy also; and they have bred *some few here in England*, though as yet not anything to the purpose as they do in other Countries."

VI. Comparative Evolution of other Domesticated Species.

The Evolution of the Domestic Fowl. It will probably be found that in other domesticated animals, a similar line of development has been followed, and I have mentioned one or two points of similarity in the domestic fowl. In it, the game varieties seem to form a more or less direct line from *Gallus bankiva*, while those which show greatest diversity in type generally, and in plumage, owe this variability to the original Cochin, which in 1867 not only included a definite cinnamon variety and a white variety in its family, but also had as its prevailing colour Buff* of various shades, e.g. Lemon Cochins, Silver Buff Cochins, etc. (*vid.* Tegetmeier's *Poultry Book*, 1867).

* "Buff" denotes colour in poultry, and is not used in the technical canary sense.

The Cochin shows the variability which we have seen to occur in cinnamon canaries and hybrids.

(1) According to Tegetmeier (*Poultry Book*, 1867) this variety of fowl has the defect of "twisted primary quill feathers much more frequently than any other."

It is probably because this "defect" was made a disqualifying point in poultry shows, that the Frizzled Cochin has not become an established variety.

(2) The Silk Cochin or Emu Fowl is known.

(3) Grouse and Partridge Cochins represent the spangled varieties.

(4) "There is a tendency in Cochins to produce an extraordinary number of cocks in nearly every brood" (Tegetmeier). We have thus a sexual peculiarity in Cochins just as in cinnamon Canaries (cf. *p. e.* ♂ × *d. e.* ♀ matings).

(5) Cochins are also peculiarly subject to visual defects like albino birds. "The eye should be red...in all cases of blindness pearl-eyed birds" (a further stage of albinism) "have been the sufferers" (Tegetmeier). This pearl eye is said to be "very hereditary" in Cochins (Wright's *Poultry Book*, 1902). Davenport in *Inheritance in Poultry*, quoting McGrew (1904, p. 526), mentions the Buff Cochin as probably the oldest Chinese variety, and cites records of the oldest monastery—Hoangho—to the effect that this fowl was cultivated by the brotherhood 1500 years ago.

From the same author, a very important confirmation of our theory is obtained. Referring to the indigenous Buff Cochin of China, a traveller says that "no two can be found of exactly the same color; some are a chestnut-color, others darker, and some quite light" (McGrew, 1901, p. 527). With regard to the Buff Cochins first imported into England, Wright agrees with Tegetmeier in saying that the colour varied from lightest silver buff and silver cinnamon, through lemons and buffs, to the deepest coloured cinnamons.

Thus we have in the Cochin the same variability that we have seen to occur in our cinnamon sports and hybrids, and we can understand how *Gallus bankiva*, through a cinnamon sport, might be the ancestor of all our present varieties without the aid of a separate ancestor for the Aseel-Malay group. We have, moreover, evidence of the sporting tendency in *G. bankiva*, for Darwin, quoting Mr Blyth, says that the species varies considerably in the wild state, some from near the Himalayas being *paler* coloured than those from other parts of India (Darwin, *The Variation of Animals and Plants under domestication*, 1875, Vol. I. p. 247).

An interesting point also, in connection with this cinnamon-sport origin of all our domestic varieties (which sport occurs, as we have shown, from the female side), is the following statement by Blumenbach, 1831 (given by Tegetmeier): "What we have observed above concerning the aberrations of the formative nisus, namely, that it occurs less frequently in animals of the male sex than in

females, is confirmed by the examples of this variety of poultry distinguished by the protuberance on the head: for of this deformity very slight traces indeed are found in the cocks, and those but seldom."

Darwin supports Blumenbach in his statement that this protuberance, with its accompanying crest, was originally confined to the female sex (*loc. cit.* p. 270).

Corroborative Evidence.

(A) Since writing the above, Mr Lewer has sent me a most interesting article on the "Origin of our Breeds of Poultry," by Henry Scherren, F.Z.S., M.B.O.U., which appeared in *Feathered World* for Oct. 11, 1907 (with coloured plate).

This will be found to corroborate my theory. Aldrovandus' classification in 1599 is given:

(1) Common farm poultry—with game characteristics—the female slightly crested.

(2) Paduans—a crested variety with pale coloured (yellow) beak and legs and the wild plumage broken up with white, green, red, and yellow.

(3) A "buskined" or feather-legged race with similar characteristics. Evidently the original Cochin.

(4) A dwarf race—the original bantam.

(5) Turkish fowls—in which Lewis Wright saw a fairly strong resemblance to the Pencilled Hamburghs. Compare my origin of the Lizard canary.

(6) Persians—tailless or rumpless fowls.

Aldrovandus also mentions:—

(7) Frizzled Fowl.

(8) Woolly Fowl—the Silk Fowl of the present day.

(B) *Buff Poultry and Cinnamon Canaries.* The term "buff" in poultry, indicates colour, not quality of feather. The buff colour has been grafted on to the different varieties of poultry, exactly in the same manner as cinnamon in canaries, e.g. cinnamon Norwich, cinnamon Crests, etc.

Davenport, after showing that the buff colour of the Cochin is of high antiquity, and stating that it has been transferred to many other breeds by crossing, e.g. Buff Wyandotte, quotes McGrew (1901, p. 24):

"Two distinct lines were produced under different methods. One was formed from Wyandotte—Buff Cochin cross; the other came through the Rhode Island Red—Wyandotte cross. The Rhode Island Red is, however, as is well-known, a direct descendant of the Buff Cochin. The Buff Plymouth Rocks were derived *directly* or *indirectly* from the Buff Cochin. The history of the Buff Leghorn is the same—the offspring of a yellow Danish Leghorn cock and Buff Cochin pullets,

mated with a yellow Leghorn hen. The produce, three-fourths yellow Leghorn and one-fourth Buff Cochin, gave (Wyckoff, 1904, p. 527) the first Buff Leghorns ever shown."

The Buff "Orpingtons"—a highly modern and mongrel breed—have a similar history, being chiefly Buff Cochin and Dorking (Wright, 1902, p. 296).

The behaviour of the buff colour in these crosses is exactly the same as we have shown to occur in the cinnaman colour of canaries—for the Buff Leghorns, for instance, were not obtained directly from the buff mother but from her sons, in the same manner as we have demonstrated cinnamon feathers in canaries, to be inherited from the male side. We may safely assume that the buff varieties of poultry correspond with the cinnamon varieties of the canary, that both are due to an original early cinnamon sport, and that this sport or mutation is, in all probability, the cause of the great diversity of all the varieties of the species.

(C) *Early "Sport" in the Pigeon.* In the case of the pigeon also, evidence is adducible which proves that the first change from the Blue Rock pigeon (*Columba livia*) was one in the direction of albinism.

In *The Dovecot and Aviary* (Rev. E. S. Dixon, 1851) the author gives a passage in full from Varro, who lived from 116—27 B.C. He translates it thus:

"If ever you should establish a Dovery, you would consider the birds your own, although they were wild. For two sorts of pigeons are usually kept in a Dovery: the one belonging to rural districts, and as others call it, a Rock Pigeon, which is kept in towers, and among the beams and rafters (*columinibus*) of a farmhouse, and which is on that account named Columba, since from natural timidity it seeks the highest of roofs; whence it happens that the rustic pigeons especially seek for towers, to which they may at their own pleasure fly from the fields, and return thither. The second kind of pigeons is more quiet; and contented with the food given at home, it accustomes itself to feed within the limits of the gate. This kind is of a *white* colour principally, but the country sort is without white or variegated colours. From these two original stocks a third mixed or mongrel kind is bred for the sake of the produce."

VII. CANARY BREEDING.

(i) *Matings of Crested Variety** (cf. figs. 2 and 3, also Plate IV, figs. 1 and 4). In Appendix I. (p. 33) is given the actual data of my matings with explanation of the symbols used. They may be summarised as follows:

38 crest ♂ × crest-bred plainhead ♀ = 66 crest + 64 plainhead,

or, in symbols,	$38\,C\,♂ \times c\,♀ = 66C + 64c.$
Again,	$44\,c\,♂ \times C\,♀ = 60C + 63c.$
Together,	253 offspring, $126C + 127c.$
Further,	$15\,C\,♂ \times C\,♀ = 34C + 17c.$

* By the comparatively downless state of its head the newly hatched crest may always be distinguished from the crest-bred plainhead.

From these results it follows that:

(1) Reciprocal matings of crest and crest-bred plainhead ($C \times c$) result in practically an equal number of crested and crest-bred plainhead young*.

(2) Double cresting ($C \times C$) gives twice the number of crests as compared with crest-bred plainheads.

(3) Double crest-bred plainhead breeding ($c \times c$) is so well-known never to produce a single crest that I have not included any matings of that sort in my paper. In this mating crestlessness is recessive (fig. 2).

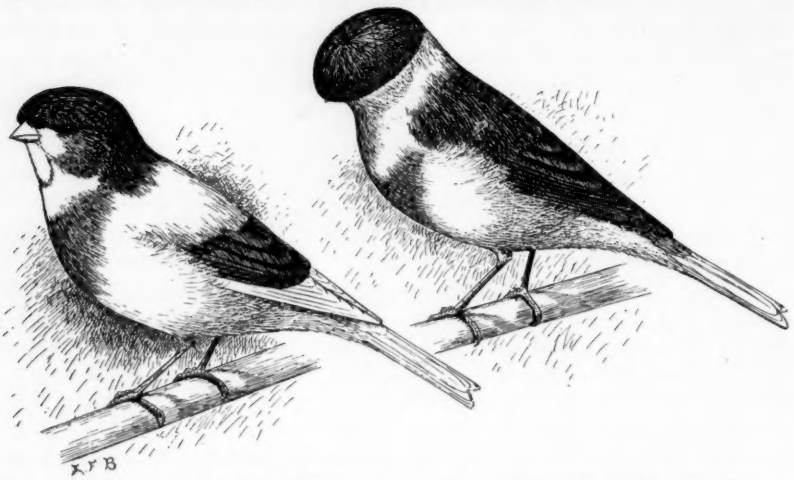


FIG. 2. Variegated crest-bred Norwich Plainhead. First Prize, St Helens, December, 1908.

FIG. 3. Heavily variegated Norwich Crest. First Prize, St Helens, December, 1908.

From an original sketch by E. F. Bailey. By permission of *Canary and Cage Bird Life*.

(4) No crested bird (fig. 3) in my reciprocal matings of crest and crest-bred plainhead has produced crests only, or in sufficient number to be worthy of note: those producing most are:

2. '91 *C. d. y.* ♂ = *C* 7, *c* 1.

5. '98 *C. m. y.* ♀ (*c. gr. b.* × *C. t. y. Lan.* {3. '97}) = *C* 6, *c* 1.

2. '99 *C. gr.* ♂ (*C. m.* × *c. v.* 1. '98) = *C* 6, *c* 1.

(5) Those birds used in double-crest ($C \times C$) matings which produced most crested young were:

5. '97 *C. m.* (*C. m.* × *c. v.*) = *C* 5, *c* 1.

5. '98 *C. t.* = *C* 4.

1. '06 *C. cl.* (*c. m.* × *C. cl. Lan.* 5. '03) = *C* 2, *c* 3.

5. '07 „ = *C* 3.

3. '08 „ = *C* 3.

* Skulls from this mating were preserved and none showed the condition of *hernia cerebri*.

(6) Thus, reciprocal $C \times c$ matings, resulting in practically 50 % crests and 50 % crest-bred plainheads, agree with the Mendelian result of a heterozygote × homozygote union; it is difficult to get a corresponding agreement in the $C \times C$ matings, which give twice the number of crests as compared with crest-bred plainheads.

(7) Of 160 crests bred by me, 14 are noted as having bad fronts. The great bulk, however, were high class crests and held principal honours at all Scottish shows. The same is true of the crest-bred plainheads, one of which, "Cock o' the North"—*c. gr.* (*c. v.* × *C. gr.* 8. '93)—defeated "Devastation," the Crystal Palace Champion of two years' standing, under Mr Bexon, one of the best known English judges. Such highly bred birds are less prolific and more delicate than the crests

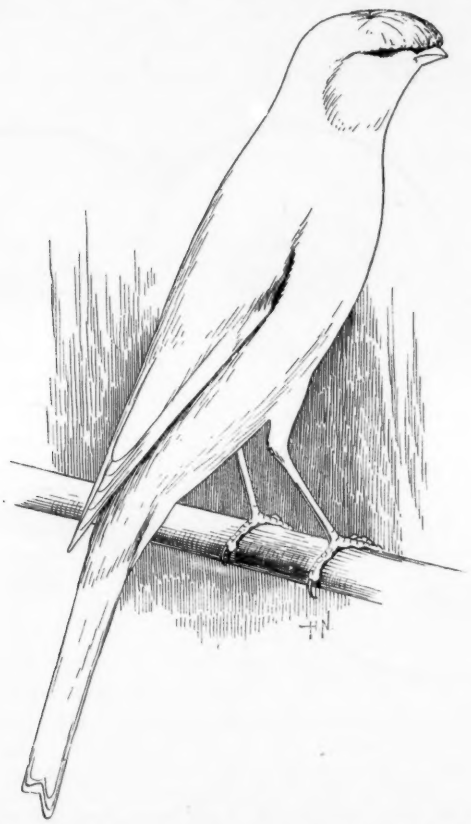


FIG. 4. Typical Lancashire Coppy. To illustrate frontal with absence of back crest, and upright position. From a sketch by H. Norman. By permission of *Cage Birds*.

used by Davenport in his experiments: they are also very subject to lenticular degeneration, many becoming blind from cataract shortly after the first or second moult. This is, no doubt, connected with the excessive development of head feather, which is an essential point in the variety (Plate IV, fig. 1).

(8) I have mentioned that almost $\frac{1}{12}$ of the crests had deficient frontal crest development: it may be well also to state that in the Lancashire variety of crested canary (the "Coppy") we have a crest with a front and no back—this being the characteristic of the Coppy crest (fig. 4). In it also there is no bald occiput, the feather lying close and smooth at the nape like an ordinary crest-bred plainhead.

It is also necessary in breeding the ordinary or Norwich crest (the variety of my experiments) to occasionally introduce Lancashire blood in order to get the best results. Hence we have a complex set of conditions existing, which results in probably $\frac{3}{4}$ of the highest class crests of the day having bald occiputs.

(9) It may be stated definitely that "perfect crest" and "bald occiput" may co-exist (fig. 5).

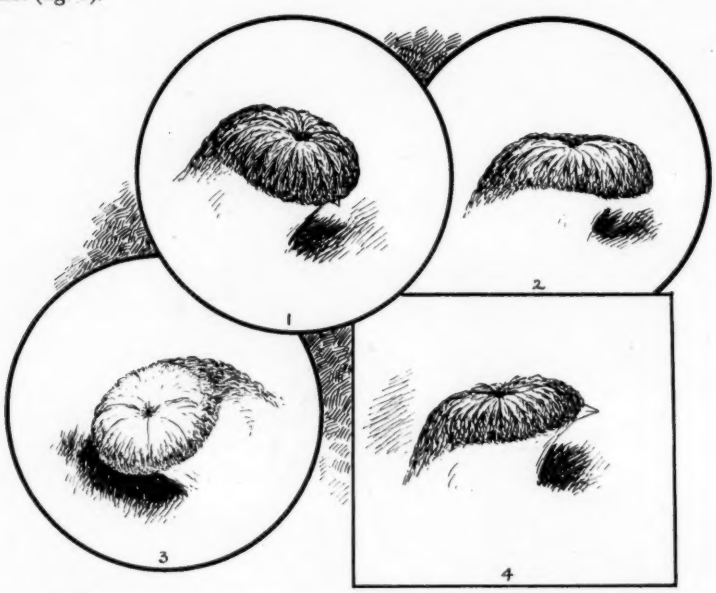


Fig. 5. A Study in "Crests." From sketches taken by H. Norman at Camberwell (L. and P. O. S.) Show, October, 1906. Class 6. Any variety Crested Canary.

No. 1. First Prize. Perfect shape, good density, droop, and radiation.
No. 2. Second Prize. Heavy frontal, weaker back crest, centre too far back.
No. 3. Third Prize. Smaller crest of good shape, droop, and radiation.
No. 4. Fourth Prize. Deficient in size, density, and radiation.

By permission of *Cage Birds*.

It is also a fact that no partly bald plainheads occur in the crested variety—a proportion of $\frac{1}{16}$ is necessary according to Davenport's system of allelomorphs: see his p. 13.

Double-Cresting and Baldness. With regard to double-cresting, of which I have had considerable experience, we have seen that the proportion of crests was doubled. None of these 34 crests and 17 crest-bred plainheads showed any sign of general baldness. I have been, however, careful to select perfectly shaped crests with closely lying backs. I have often heard of, and seen, partially bald-headed crests, which were said to be the result of double-cresting. In these there was always some skin affection of an eczematous or parasitic nature, which would sufficiently account for the condition apart from simple baldness*.

Results of Double-Cresting. In my experience double-cresting may be continued for several generations with the best results as far as shape, droop and radiation (for the present-day crest is not "flat" as Davenport quotes from Blakston in

* Bateson (*Mendel's Principles of Heredity*, 1909) writes (quoting Evolution Committee Reports and Davenport): "The crest of the Crested Canaries is *always* bred for Shows by mating cresteds with plain-headed birds." (p. 298.)

This is, by no means, universally the case, as double-cresting is a well-known method of pairing to produce winners.

"This neatly-laid appearance (of the Crest) is *only* produced when the bird is heterozygous for the crest factor." (p. 298.)

The following were crests of perfect shape, and prize-winners:

C. cl. ♀ (*C. cl.* × *C. v.* 2 : 2. 06).

C. t. ♀ (*C. cl.* × *C. v.* 2 : 2. '06).

C. m. ♂ and *C. m.* ♀ : 3. '02 had crest-bred father, but both father and mother were bred from double-crests.

In 1894, Mr John Hector, Aberdeen, bred a green crested hen from double-crests, which gained first prizes at Dundee and Aberdeen, and was then sold to Mr Fred. Weinberg, Dundee, who gained many leading prizes with it, and believed it to be the best crest he had ever owned.

Mr Jas. Wallace, Aberdeen, gained 3rd prize, Class 9, Crystal Palace, 1909, with a clear yellow crest which he bred from double-crests.

Mr F. W. Barnett, Fakenham, Norfolk, who is now, and has been for many years, the much envied chief of crest breeders, has favoured me with his experience of double-cresting, which will be found to agree with my own.

Mr Barnett has not exhibited any birds for some years (most winners, however, have been bred by him), but he tells me he won at the Crystal Palace ten or twelve years ago with a crested bird bred from two crests; he has bred several others in a similar way which could have won in good company, had they been shown by him. His practice, when in need of fresh blood, is to mate a Lancashire Coppy (Fig. 4) with a Crest (Fig. 3) which "*hit very well*"; he also agrees with me in stating that indiscriminate double-cresting often throws mop crests, the reverse of bald heads. These mop crests are not the bald heads of the fancier as supposed by Bateson (p. 37).

"The mating of two crested parents is by several authors said to give rise to some bald birds." (p. 37.)

Bateson here, like Darwin, repeats a belief which undoubtedly does obtain in the fancy, but which my experience does not corroborate—for I have never seen baldness which could be attributed to double-cresting, and I have seen much of it that could be easily explained by simpler means, e.g. *lice*, to the attacks of which the *heads* and *necks* of these heavily feathered birds are specially prone, and which are, as a rule, unrecognised by fanciers.

Darwin also states erroneously that there is a feather-footed breed of canary (1875, *The Variation of Animals and Plants under domestication*, Vol. I. p. 311) (cf. p. 17 above and note on p. 13).

1880, *vid.* p. 11) are concerned; but there is certainly, unless care be exercised, a tendency for the bald occiput, not to become greater in area, but to be surrounded by aberrant feathers, which cause a dishevelled appearance at the back of the head—the "horned" crest in fact.

It is possible that this condition might develop into a structure like the shell crest of the Trumpeter pigeon, whose rose crest exactly resembles the crest of a modern crested canary, covering eyes and beak in even, more or less, circular radiation. In this pigeon double-cresting is the rule: there are no crest-bred plainheads, and no baldness occurs (Plate V, fig. 6).

Plumage-colour in the Crested Variety. We have already seen that my results with regard to "mottled × yellow," or more correctly, variegated × clear matings in my non-crested or muling strain did not agree with Davenport's conclusions.

The same is true of plumage-colour in my crested strain, the results of which I now give, with the additional explanation that Davenport's "mottled × green" corresponds with my variegated × green, and his "mottled × mottled" with my variegated × variegated matings.

Matings		Progeny		
		variegated	green	clear
Variegated × variegated, 36 matings	=	103	6	12
Variegated × clear, 24 matings	=	68	0	17
Variegated × green, 24 matings	=	58	13	1

The variegated × variegated proportion is roughly 51 : 3 : 6, which does not correspond, even approximately, to the expected 9 : 3 : 4.

In neither of the other two sets of matings is the expected 50 % of variegateds and of clears, of variegateds and of greens obtained.

(ii) *Inheritance of Yellow and Buff Qualities.* The quality of colour being comparatively unimportant in the crested variety, I regret that sufficient attention was not given to noting the relative proportions of yellow and buff progeny. This point was carefully attended to in the notes of my non-crested or muling strain, where colour is of great importance.

There is a wide-spread belief that yellow or jonque feather is inherited much more strongly from the male than from the female parent. The figures from my crested strain are not sufficiently full to be of great importance in definitely deciding this question.

Proportion of Yellow to Buff Progeny in the Crested Variety.

Matings		Progeny				
		Yellows		Buffs		Unknown
18 yellow ♂ × buff ♀	=	22	+	10	+	22
14 buff ♂ × yellow ♀	=	23	+	18	+	16

Thus yellow ♂ × buff ♀ notes 22 yellows in a total of 54, and buff ♂ × yellow ♀ notes 23 yellows in a total of 57.

Such as they are these figures indicate that there is no great difference in the number of yellow progeny obtained, between using a buff cock and a yellow cock. They also show a tendency to partial dominance of yellow, the proportions in those noted being 22 : 10 in the first set, and 23 : 18 in the second. The "unknowns," however, form too large an element in the offspring to allow of any great stress being laid on these results.

"Buff" recessive. It may be stated definitely that no yellow progeny can be obtained from two buff parents. Any reported cases of this sort are probably due to one of the parents being really a poor yellow and not a good or "high" buff.

Some care has to be exercised in distinguishing these birds, but in every case the buff tips of the feathers about the head, neck and saddle—the frost or meal—can be made out in the buff bird, and is absent in the yellow (Plate III, figs. 1 and 2).

Double-buff matings in my Crested Strain.

Number of Matings	Progeny	
65	193 buffs	0 yellows

Reciprocal Yellow and Buff Matings, Muling Strain. The following are the results in detail of reciprocal yellow and buff matings, and double yellow matings in my muling strain since 1891:

Yellow ♂ × Buff ♀. Number of Matings, 26.

	Progeny.	
Yellow	Buff	Unknown (whether yellow or buff)
20 male	19 male	3 male
33 female	15 female	1 female
4 sex unknown	4 sex unknown	6 sex unknown
57	38	10

Proportion of yellows to buffs = 57 : 38, or 3 : 2.
„ males to females = 42 : 49.

Buff ♂ × yellow ♀. Number of Matings, 20.

	Progeny.	
Yellow	Buff	Unknown (whether yellow or buff)
20 male	12 male	1 sex unknown
22 female	12 female	
2 sex unknown	4 sex unknown	
44	28	1

Proportion of yellows to buffs = 11 : 7, nearly 3 : 2.
„ males to females = 32 : 34.

Putting the reciprocal matings together we have

Yellows = 101 Buffs = 66

Proportion of yellows to buffs = almost exactly 3 : 2.

„ males to females = 74 : 83.

Double-yellow (*yellow* × *yellow*) *Matings. Number of Matings,* 18.

	Progeny.	
Yellow	Buff	Unknown (whether yellow or buff)
22 male	8 male	4 female
26 female	8 female	
2 sex unknown		
50	16	4

Proportion of yellows to buffs = 3 : 1 nearly.

„ males to females = 30 : 38.

From these tables it will be seen that the belief that more yellow progeny came from yellow male and buff female than from buff male and yellow female matings, is not substantiated. The proportion in both cases is almost exactly 3 : 2.

It is further evident that double-yellowing practically doubles the number of yellows obtained, the proportion then being almost exactly 3 : 1, instead of 3 : 2.

Females are rather more numerous than males in all the matings.

The following birds are those in my muling strain which have produced most yellow progeny :

(1) Variegated yellow ♀ (bred from double-yellows) = 6 yellows—males and females.

(2) Clear yellow ♂ (bred from yellow ♂ × buff ♀) = 3 *cl. y.* ♂ + 3 *cl. y.* ♀.

(3) Cinnamon ticked yellow ♀ = 5 *y.* ♀ + 1 *y.* ♂ + 3 ?

(4) Ticked yellow ♀ (bred from a treble-yellowed ♂ × *m. b.* ♀) = 5 *y.* ♀ + 1 *y.* ♂.

(5) Variegated yellow ♂ (bred from double yellows) = 4 *y.* ♂ + 6 *y.* ♀ + 2 *b.* ♂.

The Effect of Double-yellowing. While double-yellow mating is thus shown to improve the colour of a strain, it must be done cautiously, as feather becomes thin and scanty, and the general health of the stock suffers if this method be used even for three or four generations in succession. It seems to be true that reciprocal yellow and buff matings should be the general rule, if the health of any strain is to be kept good. I have not been able, so far, to prove that any yellow, or any crested bird, was truly homozygous. The yellow quality of feather, like the crest, seems to be of a compound nature ; it is associated especially with thinness and scantiness of plumage, and results usually, if double-yellowing be continued to any extent, in a featherless condition of the body, especially on its under aspect.

(iii) *Inheritance of Eye Colour.* I now pass to my matings bearing on pink and dark eyes. The matings are all tabled in Appendix B, and the table below gives a summary of the results reached.

The possible matings are as follows:

I. Pure Pink-eyed ♂ × pure Dark-eyed ♀.

II. Pink-eyed ♂ × Pink-eyed ♀.

III. Pure Dark-eyed ♂ × Pink-eyed ♀.

III. 1. Dark-eyed ♂ × Pink-eyed ♀ from Pink-eyed Father and Dark-eyed Mother.

III. 2. Dark-eyed ♂ from Pink-eyed Father × Pink-eyed ♀.

III. 3. Dark-eyed ♂ from Pink-eyed Mother × Pink-eyed ♀.

III. 4. Dark-eyed ♂ from Pink-eyed Father × Pink-eyed ♀ from Pink-eyed Father and Dark-eyed Mother.

III. 5. Dark-eyed ♂ from Pink-eyed Mother × Pink-eyed ♀ from Pink-eyed Father and Dark-eyed Mother.

IV. Dark-eyed ♂ × Dark-eyed ♀, both pure.

IV. 1. Dark-eyed ♂ × Dark-eyed ♀ from Pink-eyed Mother.

IV. 2. Dark-eyed ♂ from Pink-eyed Mother × Dark-eyed ♀ from Pink-eyed Mother.

IV. 3. Dark-eyed ♂ from Pink-eyed Father × Dark-eyed ♀ (pure).

IV. 4. Dark-eyed ♂ from Pink-eyed Mother × Dark-eyed ♀ (pure).

Now according to the view of Bateson and Punnet every pink-eyed female is alike in gametic character, and every dark-eyed female is also alike. The dark-eyed male may, however, be homozygous or heterozygous in eye colour. Thus there is no distinction between a dark-eyed female of dark-eyed descent, and one which has arisen from a pink-eyed ancestry by one dark-eyed cross. Unfortunately the data are not sufficiently numerous, and the knowledge of sex is so incomplete in the case of the offspring that it is not possible to test effectively some of the fundamental points of this theory which depends on the hypotheses that (*a*) sex is heterozygous in the female and (*b*) that there is a repulsion between femaleness and dark-eyedness.

Thus whether a dark-eyed female comes from a dark-eyed line or not, her gametic constitution is represented by *p. d.* ♂♀ and her ova are *p.* ♀ and *d.* ♂, but a dark-eyed male may be either *d. d.* ♂♂, the homozygous type, or *p. d.* ♂♂, the heterozygous type, the gametes being *d.* ♂, *d.* ♂ and *p.* ♂, *d.* ♂ respectively. It is perfectly easy to deduce from this the expected proportions of offspring both as to sex and eye colour. But the numbers, especially those sexed, in the subclasses III. 1—5, and IV. 1—4, are quite insufficient to draw conclusions as to the fundamental hypotheses. Such conclusions, however, as can be drawn are at no point opposed to the theory above developed.

Let us look at the categories in order:

I. This is *p.p.* ♂♂ × *p.d.* ♂♀ for there is no "pure" dark-eyed female. The result should be 50 °/₀ pink-eyed females and 50 °/₀ (*p.d.* ♂♂) dark-eyed heterozygous males. Actually we have ♂ 24 + ○ 4 dark-eyed and ♀ 21 + ○ 7 pink-eyed offspring.

II. This is *p.p.* ♂♂ × *p.p.* ♂♀ and should give 50 °/₀ of pink-eyed ♂'s and 50 °/₀ of pink-eyed ♀'s. The result ♂ 9 and ♀ 13 is not opposed to this as there were ○ 13.

III. This is *d.d.* ♂♂ × *p.p.* ♂♀. The result should be all offspring dark-eyed and the sexes in equal numbers. There were 110 dark-eyed and no pink-eyed. Of the canaries alone ♂ 20, ♀ 21, and ○ 9. The results accord with the theory.

III. 1. This is *d.d.* ♂♂ × *p.p.* ♀♂, the same as III. according to the theory. There were 22 offspring all dark-eyed, sexes are not available.

III. 2—5. It would be interesting to consider these separately, but the material is not adequate. According to the theory they are all *p.d.* ♂♂ × *p.p.* ♂♀, and should give rise to 25 °/₀ pink-eyed females, 25 °/₀ pink-eyed males, 25 °/₀ dark-eyed females and 25 °/₀ dark-eyed heterozygous males. All together there were in fact 10 pink-eyed and 10 dark-eyed offspring, and 3 doubtful. The sexes were not ascertained.

IV. This is on the theory *d.d.* ♂♂ × *d.p.* ♂♀ and should give all dark-eyed, there were 283 offspring all dark-eyed, and equality of sexes. There were ♂ 96 and ♀ 93 and many unsexed.

IV. 1. This is *d.d.* ♂♂ and *d.p.* ♂♀. This again should give 50 °/₀ heterozygous dark-eyed males and 50 °/₀ dark-eyed females. There were 16 dark-eyed offspring in all, sex not recorded.

IV. 2 is *d.p.* ♂♂ × *d.p.* ♀♂ and should give 25 °/₀ pink-eyed females, 25 °/₀ dark-eyed females and 50 °/₀ dark-eyed males. There were 15 dark-eyed, and 6 pink-eyed progeny.

IV. 3—4 should also give the same result. We have altogether (IV. 2—4) 43 offspring, 11 pink-eyed and 32 dark-eyed; a result exceedingly close to the expected. There are too few sexed to allow of any definite conclusion. The whole subject deserves to be treated from far greater numbers. Thus far there is nothing in my observations—like the four dark-eyed hens of Durham and Marryat from *d.p.* ♂♀ × *p.p.* ♂♂—at variance with the theory.

VIII. Summary of Conclusions.

I. All canary varieties have arisen from a grey or cinnamon sport occurring in the female—the pallid type of variation which occurs at present among many wild birds.

This theory is advanced from a study of:

1. Wild sports generally, in nature and in confinement.

2. Cinnamon, and cinnamon-bred hybrids, which frequently show characteristics of canary varieties arising *de novo*.

3. The earliest canary literature.

4. Collateral evidence of a similar nature in poultry and pigeons.

II. "Dark-eye" and "Pink-eye" are found to behave generally in Mendelian fashion, for from Group IV. (*d. e.* × *d. e.*) it is evident that there is a homozygous type of dark-eyed canary—also from

Group IV. 2, 3, 4, it appears that there is also a heterozygous or impure form occurring in the male as well as the female. The pink-eyed birds being homozygous, if we arrange our groups of matings according to Mendelian principles we find:

Group III. *d. e.* × *p. e.*

Matings 33 Progeny 110 Dark-eyed 110

In this group dark-eye is dominant, and pink-eye recessive.

The following heterozygous matings:

IV.	2.	*d. e. F.* × *d. e. F.*	Matings	5	Progeny	21	*d. e.*	15	*p. e.*	6
„	3.	*F. d. e.* × *d. e.*	„	3	„	12	„	9	„	3
„	4.	*d. e. F.* × *d. e.*	„	4	„	10	„	8	„	2
						43		32		11

The result closely approximates to 3 : 1.

The following heterozygous × homozygous matings:

III.	2.	*F. d. e.* × *p. e.*	Matings	1	Progeny	3	*d. e.*	1	*p. e.*	2	
„	3.	*d. e. F.* × *p. e.*	„	3	„	12	„	4	„	5.	Unknown 3.
„	4.	*F. d. e.* × *F. p. e.*	„	1	„	2	„	0	„	2	
„	5.	*d. e. F.* × *F. p. e.*	„	2	„	6	„	5	„	1	
						23		10		10	

This result gives the required 50 % of each.

One would expect the female of the homozygous type of dark-eyed canary to be homozygous as well as the male, and I have evidence of this in several of my females giving very large percentages of dark-eyed males when mated to pink-eyed males—the proportion of six to one occurring several times. It is probable that a homozygous dark-eyed female would be completely dominant over the cinnamon male, and that male dark-eyed progeny only would result.

I can prove the occurrence of wild heterozygous males by the following most interesting result which I have just obtained, viz. a family of young greenfinches which are leaving their nest to-day (June 8th). The father is a wild caught bird which I selected as being of the colour which I think indicates a heterozygous nature not only in wild birds, but also in canaries, viz. a colour I call cinnamon-green. To any casual observer, however, the bird would pass as a normally coloured greenfinch. The mother is one of my pale cinnamon—almost creamy—white greenfinches already mentioned. The family of five consists of four of a cinnamon

and one of a greyish type all much paler than the normally coloured young of greenfinches. It is too early to sex the young, but I hope to have, at last, seen a *male* cinnamon greenfinch*! The cinnamons appear to be males, and the grey a female.

III. Other characters which are seen to behave as recessives are (*a*) buffness, (*b*) crest-bred plainheadedness, and their corresponding qualities (*a*) yellowness, (*b*) crestedness, exhibit more or less imperfect dominance.

There can be no doubt that the majority of crests at present are heterozygous with regard to crest, although this cannot be essential to crest formation as stated by Bateson (*Principles of Heredity*, 1909), for the Trumpeter pigeon, which is homozygous, has a similar arrangement in its rose crest (Plate V, fig. 6). Bateson also, following Davenport, is in error in assuming that the fancier's bald crest is an upstanding crest which shows the bald occiput. Such a crest is the very reverse of bald, and is the well-known "mop" crest of the fancy.

IV. My points of disagreement with C. B. Davenport centre mainly in the material used in his experiments, and in his interpretation of fancy points with regard to crest and plumage colour.

His "green × yellow" matings do not correspond with my "green × clear" partly because his "green" is not green but variegated (*vide* Plate I. fig. 1, where the pied throat shows the bird to be variegated). Similarly nearly all "clears" have dark underflue, or dark spots on bill or feet, which strictly constitutes them variegated. Hence Davenport's "green × yellow" is really variegated × variegated, and naturally from this mating one would expect variegated or "mottled" young.

Moreover, greens often become variegated, variegateds often become ticked, and ticked birds often become clear during their life-time.

Davenport's perfect crest, as has been shown, is far short of what is considered perfect at the present day, and the perfect crest does not depend on the absence of a bald occiput.

The technical qualities "yellow" and "buff" are important in any study of the plumage of the canary, and are not considered.

V. It is essential in studying Mendelian phenomena as occurring in fancy varieties that the most strict definition of the characters under examination be made, and that their nomenclature, and behaviour under varying conditions, be thoroughly understood. A fancier is trained to detect differences which others are quite unable to see, and his success depends on the careful balancing of factors which to the uninitiated are unobservable. It would therefore save much confusion if sharply defined facts only were taken into consideration meantime, and if no assumptions were made such as those referring to "bald occiput," "baldness," "perfect crest," etc.

An expression of my deep indebtedness is due to Professor Karl Pearson, for much assistance and advice in the preparation of this paper.

* Three of the cinnamons died, and on examination were found to be males.

APPENDIX.

CANARY MATINGS.

A. *Matings of the Crested Variety.*

Meaning of Symbols. Males are mentioned first, females second, e.g. ♂ × ♀. *C.*=crest. *c.*=crest-bred plainhead (from crested stock). *gr.*=self-green. *v.*=variegated to any extent between green and *m*=marked on head, wings and tail, or head and one wing with or without tail marking. *C. d.*=clear body, dark crest. *C. t.*=ticked crest. *c. t.*=ticked crest-bred. Body usually almost clear, but there may be ticking, or grizzling, i.e. faint marking on the head, body, wings, or tail. *cl.*=clear. *Lan.*=Lancashire variety [crested—(coppy)—or plain-head]. *Pl.*=plainhead—not crest-bred. *Cin.*=self-cinnamon. *b.*=buff. *y.*=yellow. Sex of progeny is mentioned when known. The rule in crest breeding is to double-buff, i.e. both parents are generally buff birds in order to increase feather and size. Hence all birds are *b.* (=buff), when *y.* (=yellow) is not mentioned in any mating. If *y.* be mentioned then *b.* is used for those buffs which have been noted, no symbol being given for those unknown.

Example. *C. t. b.* (bad front) (*C. t.* × *c. cl.*: 3. '04) × *Pl. t. y.*=1 *C. t. y.* ♂ (bad front)+1 *C. t. b.* ♂ (bad front)+1 *c. t. y.* ♂ +1 *c. m. y.* ♀.

The above formula means:

A ticked buff crested cock with bad front to his crest (from mating no. 3. 1904 of ticked crested cock with clear crest-bred hen) mated with ticked yellow plainhead, bred the following young: one ticked yellow crested cock with bad front, one ticked buff crested cock with bad front, one ticked yellow crest-bred cock, one marked yellow crest-bred hen.

Matings and Progeny.

1891. 1. *c. v.* × *C. cl.* (*c. t.* × *C. v.*)=1 *c. v.* ♂.

2. *C. d. y.* × *c. v. b.*=3 *C. v.*+2 *C. t. b.* × 1 *c. v. b.*

1892. 1. *C. m.* (*c.* × *C.*) × *c. v.* (*c* × *C.*)=2 *C. m.* ♂ +1 *C. gr.* ♂ +1 *c. gr.* ♂ +1 *c. v.* ♂.

2. *C.* × *c. v.*=1 *c. v.* ♂ +1 *c. cl.* ♀.

3. *C. m.* (*c.* × *C.*) × *c. v.*=1 *C. t.* ♂ +1 *c. v.* ♂.

4. *c. v.* (*c. v.* × *C. cl.*: 1. '91) × *C. v.*=*C. v.* ♀.

5. *c. v.* × *C. v.*=1 *C. cl.*+1 *C.*

6. *c. v. y.* × *C. gr. b.* (*c. y.* × *C. v. b.*)=1 *C. d. y.* ♀ +1 *C. gr.* ♀ +2 *c. gr.* ♀ +1 *c. y. gr.* ♂ +1 *c. v.* ♀

7. *c. v.* × *C. cl.* (*c. t.* × *C. v.*)=1 *C. v.* ♀ +1. *c. v.* ♂.

8. *C. m.* (*c.* × *C.*) × *c. cl. Lan.*=1 *C. d.* ♀ × 1 *c. cl.* ♀.

9. *C.* × *c. cl. Lan.*=2 *c. cl.* (one ♀).

10. *C. d. y.* × *c. cl. b. Lan.*=2 *C. d. y.*

1893. 1. *C. gr.* (*C. m.* × *c. v.*: 1. '92) × *c. cl.* (*C. m.* × *c. cl. Lan.*: 8. '92) = 1 *C. m.* ♂ + 1 *C. m.* ♀ + 1 *c. v.* ♂ + 1 *c. v.* ♀ + 1 *c. m.* ♂.

2. *C. m.* (*c.* × *C.*) × *c. cl.* (*C. m.* × *c. cl. Lan.*: 8. '92) = *C. m.* ♂.

3. *c. v.* × *C. v.* (*c. v.* × *C. v.*: 4. '92) = 1 *C. gr.* ♂ + 1 *c. v.* ♂ + 1 *c. cl.* ♀.

4. *c. v.* (*c. v.* × *C. cl.*: 1. '91) × *C. cl.* (*c. t.* × *C. v.*) = 3 *C. m.* ♂ + 1 *c. t.* ♂.

5. *C. t.* (*C. m.* × *c. v.*: 3. '92) × *c. gr.* (*c. v. y.* × *C. gr. b.*: 6. '92) = 1 *C. m.* ♂ + 1 *C. m.* + 2 *c. v.* ♀.

6. *c. gr. y.* (*c. v. y.* × *C. gr. b.*: 6. '92) × *C. d. b.* (*C. m.* × *c. cl. Lan.*: 8. '92) = *c. v.*

7. *C. m.* (*c* × *C*) × *C. gr.* (*c. v. y.* × *C. gr. b.*: 6. '92) = 1 *C. gr.* ♂ + 1 *C. v.* ♂ + 1 *c. v.* ♂ + 1 *c. v.* ♀ + 1 *c. v.*

8. *c. v.* (*c. v.* × *C. cl.*: 7. '92) × *C. gr.* (*c. v. y.* × *C. gr. b.*: 6. '92) = *c. gr.* ♂.

9. *c. v.* × *C. gr.* (*c. y.* × *C. v.*) = 1 *C. gr.* ♂ + 1 *C. gr.* ♀ + 1 *C. v.* ♀ + 2 *c. v.* ♂.

10. *C. m.* (*c.* × *C.*) × *C. gr.* (*c. y.* × *C. v. b.*) = 2 *C. v.* ♂.

1894. 1. *c. gr.* (*c. v.* × *C. gr.*: 9. '93) × *C. v.* (*c. v.* × *C. gr.*: 9. '93) = 1 *c. v.* ♀ + 1 *c. cl.* ♀.

2. *c. gr.* (*c. v.* × *C. gr.*: 9. '93) × *C. gr.* (*c. v. y.* × *C. gr. b.*: 6. '92) = 1 *C. gr.* ♂ + 1 *c. gr.* ♂.

3. *C. v.* (*C. m.* × *C. gr.*: 7. '93) × *c. cl.* (*C.* × *c. cl. Lan.*: 9. '92) = 2 *C. v.* ♀ + 1 *C. v.* ♂.

4. *c. gr. y.* (*c. v. y.* × *C. gr. b.*: 6. '92) × *C. m. b.* (*C. gr.* × *c. cl.*: 1. '93) = *c. v.* ♂.

1895. 1. *C. t. y.* (*Lan.*) × *c. cl. b. Lan.* = *C. t.* + *c. m.* ♂.

2. *C. v.* (*C. m.* × *C. gr.*: 7. '93) × *c. v.* (*c. gr.* × *C. v.*: 1. '94) = *C. v.* ♀.

3. *c. v.* (*c. v.* × *C. cl.*: 1. '91) × *C. m.* = 1 *C. m.* × 1 *c. v.* ♂ + 1 *c. v.* + 2 *c. m.*

4. *c. gr. y.* (*c. v. y.* × *C. gr. b.*: 6. '92) × *C. b. v.* 1 (*C. v.* × *c. cl.*: 3. '94) = *c. m. y.*

1896. 1. *C. gr.* × *C. v.* 2 (*C. v.* × *c. cl.*: 3. '94) = 2 *C. v.* + 1 *c. v.*

2. *C. t.* × *C. v.* (*C. v.* × *c. v.*: 2. '95) = 2 *c. v.*

3. *c. v.* (*c. v.* × *C. m.*: 3. '95) × *C. t.* = 1 *c. cl.* ♂.

4. *c. v.* (*c. v.* × *C. cl.*: 7. '92) × *C. gr.* = 1 *C. v.* ♂ + 1 *C. v.* ♀ + 1 *c. v.* + 1 *c. v.* ♀ + 1 *c. gr.* ♂ + 1 *c. gr.*

5. *C. t.* × *c. gr.* = 1 *c. gr.* + 1 *c. t.*

1897. 1. *C. y.* (*Lan.*) × *C. v. b.* (*C. v.* × *c. v.*: 2. '95) = 1 *C. m.* + 1 *c. v. b.*

2. *C. m.* (*C. m.* × *c. v.*) × *c. v.* (*c. v.* × *C. gr.*: 4. '96) = 1 *C. t.* + 1 *c. v.* ♂ + 1 *c. v.* + 1 *c. t.*

3. *c. gr. b.* (*c. v.* × *C. gr.*: 4. '96) × *C. t. y.* (*Lan.*) = 1 *C. d. y* ♀ + 1 *C. m. y.* ♀ + 2 *C. m. b.* (bad fronts) + 2 *c. v. y.* + 1 *c. v. y.* ♀ + 2 *c. v. b.* + 1. *c. v. y.* ♂.

4. *c. m.* (*C. t. y. Lan.* × *c. cl. Lan.*) × *C. v.* (*c. v.* × *C. gr.*: 4. '96) = 1 *c. gr.* ♀.

5. *C. m.* (*C. m.* × *c. v.*) × *C. v.* (*c. v.* × *C. gr.*: 4. '96) = 2 *C. v.* ♂ + 1 *C. m.* + 1 *C. t.* ♀.

1898. 1. *C. m.* (*C. m.* × *c. v.*) × *c. v.* (*c. v.* × *C. gr.*: 4. '96) = 1 *C. gr.* ♂ + 1 *C. m.* ♀.

2. *c. gr.* (*c. v.* × *C. gr.*: 4. '96) × *C. t.* (*C. m.* × *C. v.*: 5. '97) = 1 *C. v.* ♂ + 1 *C. v.* ♀ + 1 *c. v.* ♀ + 1 *c. v.*

3. *c. cl.* (*c. v.* × *C. t.*: 3. '96) × *C. v.* (*c. v.* × *C. gr.*: 4. '96) = 1 *C. m.* ♂ × 1 *c. v.* ♂ + 1 *c. v.* ♀.

4. *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. b. v.* 2 (*C. v.* × *c. cl.*: 3. '94) = 1 *C. m. y.*

5. *C. t. b.* × *C. m. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) = 1 *C. t.* + 2 *C. v. b.* ♂ + 1 *C. v.*

6. *C. t. b.* × *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) = 1 *C. d.* ♀ + 1 *C. t. y.* ♀ + 1 *C. t.* ♀ + 1 *c. m. y.* ♀ + 1 *c. cl.* ♀ + 1 *c. t.* ♀

1899. 1. *C. gr. b.* (*C. m.* × *c. v.*: 1. '98) × *c. m. y.* (*C. t. b.* × *c. v. y.*: 6. '98) = 1 *C.* + 2 *C. v. y.* ♀ + 1 *c. v. y.* ♂.

2. *C. gr.* (*C. m.* × *c. v.*: 1. '98) × *c. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = *C. v.* ♂.

3. *C. t. b.* × *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) = 1 *C. v. b.* ♀ + 2 *C. t. y.* ♀ + 2 *c. v.* ♂ + 1 *c. v. b.* ♀.

4. *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. t. b.* (*C. m.* × *C. v.*: 5. '97) = *c. v. b.* ♂.

5. *C. m.* (*C. m.* × *c. v.*) × *c. v.* (*c. gr.* × *C. t.*: 2. '98) = *c. v.* ♀.

6. *C. v.* (*C. m.* × *C. v.*: 5. '97) × *c. gr.* (*c. m.* × *C. v.*: 4. '97) = *C. gr.* ♂ (bad front).

7. *c. gr.* (*c. v.* × *C. gr.*: 4. '96) × *C. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = 2 *C. m.* (bad fronts) + 1 *c. m.*

8. *c. cl. b.* (*c. v.* × *C. t.*: 3. '96) × *C. m. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) = *C. m. y.* ♀ + *C. t. b.* ♀ + *c. cl. b.* ♀.

1900. 1. *C. m.* (*C. m.* × *c. v.*) × *C. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = *C. t.* ♀ + *c. cl.* ♀.

2. *c. gr. b.* (*c. v.* × *C. gr.*: 4. '96) × *C. v. y.* (*C. gr. b.* × *c. m. y.*: 1. '99) = *C. gr. b.* (rough).

3. *C. v. b.* (*C. gr.* × *c. t.*: 2. '99) × *C. t. y.* (*C. t. b.* × *c. v. y.*: 3. '99) = *C. t. y.*

4. *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. v. b.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *C. v. b.* ♂ + 1 *C. m.* (bad front) + 2 *C. t.* (one bad front).

5. *c. v.* (*C. t. b.* × *c. v. y.*: 3. '99) × *C. t.* (*c. cl. b.* × *C. m. y.*: 8. '99) = 1 *C. m.* ♂ + 1 *C. v.* ♀ + 1 *c. m.*

1901. 1. *c. cl.* (*c. v.* × *C. t.*: 3. '96) × *C. t.* (*C. m.* × *C. t.*: 1. 1900) = *c.*

2. *C. gr.* (*C. m.* × *c. v.*: 1. '98) × *C. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = 1 *C. m.* ♂ + 1 *C. m.* ♀ + 1 *c. m.* ♂.

3. *C. v.* (*c. v. y.* × *C. v. b.*: 4. 1900) × *c. cl.* (*C. m.* × *C. t.*: 1. 1900) = *C. m.*

4 *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. v. b.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *C. v. b.* ♂ + 1 *C. gr. y.* ♂ (bad front) + 1 *C. m. b.* ♀ + 1 *C. v. y* ♀ + 1 *c. v. y* ♂ + 1 *c. v. y.* ♀ + 1 *c. v. y.* ♀ + 1 *c. v. b.* ♂ + 1 *c. m. b.* ♂ + 1 *c. cl. y.* ♀.

1902. 1. *C. v. b.* (*c. v. y.* × *C. v. b.*: 4. 1900) × *c. cl. y.* (*c. v. y.* × *C. v. b.*: 4. '01) = 2 *c. v.*

2. *c. cl.* (*c. v.* × *C. t.*: 3. '96) × *C. v.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *C. m.* + 1 *C. t.* ♀ + 1 *c. v.*

3. *c. m.* (*C. gr.* × *C. t.*: 2. '01) × *C. t.* (*C. m.* × *C. t.*: 1. 1900) = 1 *C. m.* ♂ + 1 *C. m.* ♀ (both champions).

1903. 1. *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. v. b.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *c. m. y.* + 1 *c. v. y.*

2. *c. v.* × *C. t.* (*c. cl.* × *C. v.*: 2. '02) = 2 *C. m.* (both bad fronts) + *c. v.*

3. *c. m.* (*C. gr.* × *C. t.*: 2. '01) × *C. gr.* = 1 *c. gr.* ♂ + 1 *c. m.*

4. *c. cl.* (*c. v.* × *C. t.*: 3. '96) × *C. m.* = 1 *C. t.* ♂ + 1 *c. m.* ♀.

5. *c. m.* (*C. gr.* × *C. t.*: 2. '01) × *C. cl. Lan.* = 1 *C. cl.* ♂ + 1 *C. t.* + 1 *c. cl.*

1904. 1. *c. gr.* (*c. m.* × *C. gr.*: 3. '03) × *C. t.* = *c. v.* ♂.

2. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. m.* = *c. v.* ♂ + *c. cl.* ♀.

3. *C. t.* (*c. cl.* × *C. m.*: 4. '03) × *c. cl.* = 1 *C. t.* ♂ (bad front) × 2 *cl.* ♀.

1905. 1. *C. m.* × *c. cl.* (*C. t.* × *c. cl.*: 3. '04) = 1 *C. v.* ♂ + 1 *C. t.* ♀ + 1 *c. v.* ♂ + 1 *c. m.* ♀.

2. *C. cl. b.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. cl. y.* = 3 *C. cl.*

3. *C. t. Lan.* × *c. cl.* (*C. cl.* × *c. m.*: 2. '04) = *C. t* + *c. cl.*

4. *C. m.* × *c. m.* (*c. cl.* × *C. m.*: 4. '03) = 3 *C. v.* ♀ + 1 *C. t.* ♀ + 2 *c. m.* ♂ + 1 *c. m.* ♀ + 1 *c. cl.* ♂ + 3 *c. v.*

5. *C. t. b.* (bad front) (*C. t.* × *c. cl.*: 3. '04) × *Pl. t. y.* = 1 *C. t. y.* ♂ (bad front) + 1 *C. t. b.* ♂ (bad front) + 1 *c. t. y.* ♂ + 1 *c. m. y.* ♀.

1906. 1. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 1 (*C. m.* × *c. m.*: 4. '05) = 1 *C. t.* ♂ + 1 *C. t.* + 2 *c. v.* ♀ + 1 *c. m.* ♀.

2. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 2 (*C. m.* × *c. m.*: 4. '05) = 1 *C. t.* ♂ + 1 *C. t.* ♀ + 1 *C. cl.* ♀ + 1 *C. t.* (bad front) + 1 *C. t.* + 1 *c. m.* + 1 *c. m.* ♀ + 1 *c. cl.* ♂.

3. *c. gr.* (*c. m.* × *C. gr.*: 3. '03) × *C. t.* (*C. m.* × *c. m.*: 4. '05) = *C. v.*

4. *C. t. Lan.* × *c. m.* (*C. m.* × *c. cl.*: 1. '05) = 1 *C. m.* ♂ + 1 *c. v.* ♀ + 1 *c. v.* + 1 *c. v.* + 1 *c. cl.* ♀ + 1 *c. v.* + 1 *c. cl.*

5. *C. m.* × *c. m.* (*c. cl.* × *C. m.*: 4. '03) = 1 *C. v.* ♂ + 3 *C. v.* + 1 *c. cl.* ♀ + 2 *c.*

1907. 1. *C. cl. b.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *self-cin. y.* = 1 *C. v. b.* ♂ (bad front) + 1 *C. v. y.* ♀ + 1 *c. v. b.* ♂.

2. *c. v. y.* (*c. cl. y.* × *self-cin. b.*) × *C. cl. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. v. y.* ♂ (bad front) + 1 *C. cl. y.* ♀ + 1 *c. cl. y.* ♀.

3. *c. v. y.* (*c. cl. y.* × *self-cin. b.*) × *C. t. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. v. y.* ♂ + 1 *C. v. y.* ♂.

4. *c. cl. y.* (*self-cin. y.* × *c. m. y.*) × *C. v. b.* 2 (*C. m.* × *c. m.*: 4. '05) = *C. m. y.* ♂ + *C. cin. m. b.* ♀ + *c. cl. y.* ♀.

5. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 1 (*C. m.* × *c. m.*: 4. '05) = 1 *C. cl.* + 1 *C. v.* + 1 *C. cl.* + 1 *c. m.* + 1 *c. v.*

6. *C. t. b.* (*C. cl.* × *C. v.* 2: 2. '06) × *c. v. y.* (*c. cl. y.* × *self-cin. b.*) = 1 *C. t.* + 1 *C. v. y.* ♀ + 1 *C. t. y.* ♀ + 1 *c. v. b.* ♀ + 1 *c. v. b.* ♂ + 1 *c. m.* ♂ + 1 *c. m. y.* ♀ + 1 *c. cl.*

7. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. cl.* (*C. m.* × *c. m.*: 5. '06) = 1 *C. t.* + 2 *C. cl.* + 1 *c. cl.*

8. *C. t.* (*C. cl.* × *C. v.* 1: 1. '06) × *c. v.* (*C. cl.* × *C. v.* 1: 1. '06) = 1 *C. v.*

9. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. v.* (*C. cl.* × *C. v.* 1: 1. '06) = 1 *C. t.* + 1 *C. t.* + 1 *c. v.* ♀ + 1 *c. v.* ♀.

10. *c. cl. y.* (*Pl. cl. y.* × *c. m. b.*) × *C. t. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. t.* + 1 *C. t.* + 1 *c. cl. y.* ♀ + 3 *c. v.*

1908. 1. *C. m. y.* (*c. cl. y.* × *C. v. b.* 2: 4. '07) × *C. cl. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. m.* + 1 *C. cl.*

2. *c. v. b.* (*C. t. b.* × *c. v. y.*: 6. '07) × *C. v. y.* (*C. cl. b.* × *self-cin. y.*: 1. '07) = 1 *C. m. b.* ♀ + 1 *C. m. y.* ♀.

3. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 1 (*C. m.* × *c. m.*: 4. '05) = 2 *C. t.* ♂ + 1 *C. v.* ♀.

B. *Cinnamon Matings, Dark-eyed Matings and Cross or Hybrid Matings.*

Terms. *p. e.* = pink-eyed on hatching and pure pink-eyed bred so far as known. *d. e.* = dark-eyed on hatching and pure dark-eyed bred so far as known. *F.* = hybrid nature of inheritance. Other abbreviations as in other tables. *Cin.* = Cinnamon and pink-eyed on hatching. *t.* (ticked) *m.* (marked), and *v.* (variegated) birds are dark-eyed on hatching.

I. *p. e.* × *d. e.* *Pink-eyed Male mated with Dark-eyed Female.*

1898. 1. *cl. y. p. e.* × *t. y.* = 1 *m. y.* ♂ + 1 *cin. v.* ♀ + 1 *cl. y. p. e.* ♀.

2. *cl. y. p. e.* × *cl. b. d. e.* = 2 *cl. y. d. e.* ♂ + 1 *cl. y. p. e.* + 1 *cl. b. p. e.*

3. *cl. y. p. e.* × *cl. b. d. e.* = 3 *cl. y. p. e.* ♀.

1904. 1. *cl. b. p. e.* × Linnet (*L. cannabina*) = 2 *dark* ♂ + 1 *self-cin.* ♀ (linnet mules).

2. *self-cin. b.* × *cl. y. d. e.* = 1 *v. b.* ♂ + 1 *m. y.* ♂ + 1 *v. b. cin.* ♀.

1906. 1. *cl. b. p. e.*×*cl. y. d. e.*=2 *t. y.* ♂ +1 *cl. b. d. e.* ♂.
2. *cl. b. p. e.*×*cl. y. d. e.*=1 *yn. b.* ♂ +1 *cl. y. p. e.* ♀ +1 *t. cin. y.* ♀.
3. *cl. b. p. e.*×*m. y.*=1 *t. b. d. e.* ♂ +1 *cl. y. d. e.* ♂ +1 *cl. b. d. e.* ♂ +1 *cl. y. p. e.* ♀.
4. *self-cin. y.*×*v. y.*=1 *gr. y.* ♂ +1 *self-cin.* ♀ +1 *self-cin. y.* ♀ +1 *v. cin. y.* ♀.

1907. 1. *cl. b. p. e.*×*cl. y. d. e.*=1 *d. e.*+1 *cl. y. d. e.* ♂ +3 *cl. b. p. e.* ♀.
2. *self-cin. y.*×*t. b.*=1 *v. y.* ♂ +1 *v. b.* ♂ +1 *v. cin. y.* ♀ +1 *v. cin. b.* ♀.
3. *cl. y. p. e.*×*v. b.*=1 *v. y.* ♂ +1 *v. cin. b.* ♀ +1 *cl. y. p. e.* ♀.
4. *cl. b. p. e.*×*v. y.*=2 *v. y.* ♂ +1 *v. cin. y.* ♀ +1 *d. e.*+4 *p. e.*
5. *self-cin. b.*×*cl. y. d. e.*=3 *v. y.* ♂ +2 *d. e.*+1 *p. e.*

II. *p. e.* × *p. e.* *Pink-eyed Male mated with Pink-eyed Female.*

1898. *p. e.*×*cl. y. p. e.*=1 *cl. y. p. e.* ♂ +2 *cin. t. y.* ♀ +1 *cin. t. y.*

1904. 1. *self-cin. b.*×*cl. y. p. e.*=2 *cin. v. y.* ♀ +1 *cin. v. b.* ♀ +1 *cin. t. b.* ♀.
2. *self-cin. b.*×*cl. b. p. e.*=*cl. b. p. e.* ♂ +*cin. v. b.* ♀.

1905. 1. *cl. b. p. e.*×*cl. y. p. e.*=1 *cl. y. p. e.*+2 *cl. b. p. e.*+3 *cl. p. e.*
2. *self-cin. y.*×*self-cin. b.*=*self-cin. y.* ♂ +*self-cin. y.* ♀ +*self-cin. b.* ♂ +*self-cin b.* ♀ +1 *p. e.*

1908. *self-cin. b.*×*cl. y. p. e.*=2 *cin. v. y.* ♂ +3 *cin. v. b.* ♀ +1 *cin. v. y.* ♀.

1909. 1. *cin. t. b.*×*cl. y. p. e.*=3. *p. e.*
2. *self-cin. b.*×*self-cin. y.*=3 *p. e.*
3. *cin. t. y.*×*cin. t. b.*=2 *cl. p. e.*

III. *d. e.* × *p. e.* *Dark-eyed Male mated with Pink-eyed Female.*

1895. *cl. b. d. e.*×*cin. t.*=*cl. b. d. e.* ♂ +*cl. b. d, e.* ♀ +2 *cl. b. d. e.*+*t. b.*

1899. *d. e.*×*cin. t. y.*=1 *d. e.* (linnet mule).

1901. *t. y.*×*cin. t. y.*=*m. b.* ♂ +*m. y.* ♀ +2 *m.* ♀ +1 *cl. y. d. e.* ♀.

1902. *t. y.*×*cin. t. y.*=*v. y.* ♂ +*t. y.* ♀ +2 *cl. y.* ♀.

1904. 1. *d. e.*×*cl. y. p. e.*=*d. e.* ♂ +*d. e.* (goldfinch mules).
2. *d. e.*×*cin. v. y.*=8 *d. e.* (goldfinch mules).
3. *cl. y. d. e.*×*self-cin. y.*=*m. y.* ♂.
4. *cl. y. d. e.*×*self-cin. y.*=*v. y.* ♂ +*gr.* ♀ +*v.* ♀ +*v. y.* ♀.
5. *cl. y. d. e.*×*self-cin. b.*=*m. b.* ♂ +*v. b.* ♀ +*m.*

1905. 1. *cl. y. d. e.*×*cin. v. y.*=*m. y.* ♂ +*cl. y. d. e.* ♀ +*t. b.* ♀.
2. *d. e.*×*cin. v. y.*=3 *d. e.* (goldfinch mules).
3. *d. e.*×*cin. m. b.*=7 *d. e.* (goldfinch mules).
4. *d. e.*×*cin. v. y.* (*self-cin. b.*×*cl. y. p. e.*)=3 *d. e.* (siskin mules).
5. *d. e.*×*cin. v. y.* (*self-cin. b.*×*cl. y. p. e.*)=2 *d. e.* (goldfinch mules).
6. *cl. y. d. e.*×*self-cin. b.*=*v. y.* ♂ +2 *v. b.* ♂ +*v. y.* ♀.

1906. 1. *cl. y. d. e.* × *cin. m. b.* = 2 *v. y.* ♂ + *v. b.* ♂ + *t. y.* ♂ + *v. y.* ♀.
2. *d. e.* × *cin. m. b.* = 3 *d. e.* (siskin mules).
3. *d. e.* × *cin. m. b.* = 1 *d. e.* (goldfinch mule).
4. *cl. y. d. e.* × *self-cin. y.* = *m. y.* ♂ + *m. y.* + *v. b.* ♂ + 4 *v. y.* ♀.

1907. 1. *C. cl. b. d. e* × *self-cin. y.* = *C. v. b.* ♂ + *C. v. y.* ♀ + *c. v. b.* ♂.
2. *d. e.* × *cl. y. p. e.* = 6 *d. e.* (linnet mules).
3. *d. e.* × *cl. y. p. e.* = 1 *d. e.* (redpoll mule).
4. *t. y.* × *self-cin. b.* = 2 *v. b.* ♂ + 2 *v. y.*
5. *c. cl. d. e.* × *self-cin. y.* = *cl. d. e.*
6. *cl. b. d. e.* × *self-cin. y.* = *v.*

1908. 1. *d. e.* × *cin. m. y.* = 1 *d. e.* (siskin mule).
2. *d. e.* × *cl. y. p. e.* = 1 *d. e.* (siskin mule).
3. *d. e.* × *cl. y. p. e.* = 4 *d. e.* (linnet mules).
4. *d. e.* × *cin. v. y.* = 3 *d. e.* (siskin mules).
5. *d. e.* × *cin. v. b.* = 9 *d. e.* (linnet mules).

1909. 1. *d. e.* × *cl. b. p. e.* = 2 *d. e.* (siskin mules).
2. *d. e.* × *cl. b. p. e.* = 1 *d. e.* (redpoll mule).
3. *d. e.* × *cin. v. b.* = 2 *d. e.* (linnet mules).

III 1. *d. e.* × *p. e. F.* *Dark-eyed Male mated with Pink-eyed Female from Pink-eyed Father.*

1905. *d. e.* × *cin. v. b.* (*self-cin. b.* × *cl. y. d. e*) = 4 *d. e.* (siskin mules).

1907. 1. *d. e.* × *cin. v. y.* (*p. e.* × *d. e.*) = 4 *d. e.* (goldfinch mules).
2. *d. e.* × *cin. v. y.* (*p. e.* × *d. e.*) = 1 *d. e.* (siskin mule).

1908. 1. *d. e.* × *C. cl. y. p. e.* (*p. e.* × *d. e.*) = 3 *d. e.* (linnet mules).
2. *d. e.* × *cin. v. y.* (*p. e.* × *d. e. F.*) = 8 *d. e.* (linnet mules).
3. *d. e.* × *cin. v. b.* (*self-cin. y.* × *d. e.*) = 2 *d. e.* (siskin mules).

III 2. *d. e. F.* × *p. e.* *Dark-eyed Male from Pink-eyed Father mated with Pink-eyed Female.*

1908. *v. y.* (*self-cin. b.* × *d. e.*) × *cin. v. b.* (*self-cin. b.* × *cl. y. p. e.* = 1 *cin. v.* + 1 *cl. p. e.* + 1 *v.*

III 3. *d. e. F.* × *p. e.* *Dark-eyed Male from Pink-eyed Mother mated with Pink-eyed Female.*

1905. 1. *v. y.* (*d. e.* × *p. e.*) × *cl. b. p. e.* = 1 *cl. b. p. e* ♂. + 3 *cl.*
2. *m. y.* (*d. e.* × *p. e.*) × *self-cin. y.* = 3 *d. e.* + 2 *p. e.*

1909. *v. y.* (*d. e. F.* × *p. e.*) × *cin. v. b.* (*self-cin. b.* × *cl. y. p. e.*) = 1 *cin. v.* + 1 *cl. p. e.* + 1 *cl. d. e.*

III 4. *d. e. F. × p. e. F. Dark-eyed Male from Pink-eyed Father mated with Pink-eyed Female from Pink-eyed Father.*

1908. *v. y.* (*p. e.* × *d. e. F.*) × *cl. b. p. e.* (*p. e.* × *d. e.*) = *cin. t. b.* ♂ + *cl. b. p. e.* ♀.

III 5. *d. e. F. × p. e. F. Dark-eyed Male from Pink-eyed Mother mated with Pink-eyed Female from Pink-eyed Father.*

1905. *v. y.* (*d. e.* × *self-cin. b.*) × *cin. v. b.* (*self-cin. y.* × *d. e.*) = *cl. p. e.*

1908. *v. b.* (*d. e.* × *p. e.*) × *cin. v. y.* (*self-cin. y.* × *d. e.*) = 5 *d. e.*

IV. *d. e. × d. e. Dark-eyed Male mated with Dark-eyed Female.*

Vid. Matings of Crested Variety. Appendix A.

IV 1. *d. e. × d. e. F. Dark-eyed Male mated with Dark-eyed Female from Pink-eyed Mother.*

1905. *d. e.* × *cl. y. d. e.* (*d. e.* × *p. e.*) = 3 *d. e.* (goldfinch mules).

1906. *d. e.* × *cl. y. d. e.* (*d. e. F.* × *p. e.*) = 4 *d. e.* + 1 *d. e.* (goldfinch mules).

1908. *d. e.* × *m. y.* (*d. e.* × *self-cin. y.*) = 6 *d. e.* (goldfinch mules).

1909. *d. e.* × *m. y.* (*d. e.* × *self-cin. y*) = 2 *d. e.* (greenfinch mules).

IV 2. *d. e. F. × d. e. F. Dark-eyed Male from Pink-eyed Mother mated with Dark-eyed Female from Pink-eyed Mother.*

1905. *v. y.* (*d. e.* × *p. e.*) × *cl. y. d. e.* (*d. e.* × *p. e.*) = 2 *t. y.* ♂ + 1 *cl. y. p. e.* ♀.

1906. *v. y.* (*d. e.* × *p. e.*) × *cl. y. d. e.* (*d. e.* × *p. e.*) = 1 *v. y.* ♂ + 1 *cl. b. d. e.* ♂ + 1 *cl. y. d. e.* ♀ + 1 *cl. y. p. e.* ♀.

1907. 1. *v. y.* (*d. e.* × *self-cin.*) × *cl. b. d. e.* (*d. e.* × *p. e.*) = 1 *v. y.* ♂ + 1 *cl. y. p. e.* ♀ + 1 *cl. y. d. e.* ♀.

2. *v. y.* (*d. e.* × *self-cin.*) × *cl. b. d. e.* (*d. e.* × *p. e.*) = 2 *v. y.* ♂ + 5 *d. e.* + 2 *p. e.*

3. *m. b.* (*d. e.* × *self-cin.*) × *m. y.* (*d. e.* × *self-cin. y*) = 1 *d. e.* + 1 *p. e.*

IV 3. *d. e. F. × d. e. Dark-eyed Male from Pink-eyed Father mated with Dark-eyed Female.*

1905. *m. y.* (*self-cin. b.* × *cl. y. d. e.*) × *d. e.* (greenfinch ♀) = 4 *d. e.* ♂ + 1 *cin. v.* ♀ + 2 *d. e.* (greenfinch mules).

1908. *C. m. y.* (*p. e.* × *d. e.*) × *C. cl. b. d. e.* = *C. m.* + *C. cl. d. e.*

1909. *C. m. y.* (*p. e.* × *d. e.*) × *C. d. e.* = *C. gr.* + *c. cin. m.* + *c. cl. y. p. e.*

IV 4. *d. e. F. × d. e. Dark-eyed Male from Pink-eyed Mother mated with Dark-eyed Female.*

1904–07. *v. y.* (*d. e.* × *p. e.*) × *d. e.* (greenfinch ♀) = 4 *d. e.* (greenfinch mules).

1906. *m. y.* (*cl. y. d. e.* × *self-cin. y.*) × *d. e.* (greenfinch ♀) = *cin. v.* ♀ (greenfinch mule).

1908. 1. *m. y.* (*d. e.* × *p. e.*) × *d. e.* (greenfinch ♀) = 3 *d. e.* (greenfinch mules).

2. *m. y.* (*d. e.* × *p. e.*) × *d. e.* (siskin ♀) = 1 *d. e.* + 1 *p. e.* (siskin mules).

SUMMARY OF MATINGS*.

I. *p. e.* × *d. e.*†

Matings 14. Progeny 56. Dark-Eyed ♂ 24, ○ 4=28. Pink-Eyed ♀ 21, ○ 7‡=28.

II. *p. e.* × *p. e.*

Matings 9. Progeny 35. Pink-Eyed ♂ 9, ♀ 13, ○ 13=35. Dark-Eyed 0.

III. *d. e.* × *p. e.*

Matings 33. Progeny 110. Dark-Eyed 110. Pink-Eyed 0. Of the canaries there were ♂ 20, ♀ 21, ○ 9.

III 1. *d. e.* × *F. p. e.*

Matings 6. Progeny 22. Dark-Eyed 22. Pink-Eyed 0.

III 2. *F. d. e.* × *p. e.*

Mating 1. Progeny 3. Dark-Eyed 1. Pink-Eyed 2.

III. 3. *d. e. F.* × *p. e.*

Matings 3. Progeny 12. Dark-Eyed 4. Pink-Eyed 5. Doubtful 3.

III. 4. *F. d. e.* × *F. p. e.*

Mating 1. Progeny 2. Pink-Eyed 2.

III. 5. *d. e. F.* × *F. p. e.*

Matings 2. Progeny 6. Dark-Eyed 5. Pink-Eyed 1.

IV. *d. e.* × *d. e.*

Matings 91. Progeny 283. Dark-Eyed 283. Pink-Eyed 0.

IV. 1. *d. e.* × *d. e. F.*

Matings 4. Progeny 16. Dark-Eyed 16.

IV. 2. *d. e. F.* × *d. e. F.*

Matings 5. Progeny 21. Dark-Eyed ♂ 7, ♀ 2, ○ 6=15. Pink-Eyed ♀ 3, ○ 3=6.

IV. 3. *F. d. e.* × *d. e.*

Matings 3. Progeny 12. Dark-Eyed 9. Pink-Eyed 3.

IV. 4. *d. e. F.* × *d. e.*

Matings 4. Progeny 10. Dark-Eyed 8. Pink-Eyed ♀ 1, ○ 1=2.

* ♂ = male, ♀ = female, ○ = sex unknown.

† These matings may also be considered from the standpoint of colour:

2.	*y.* × *y.* matings = *y.* ♂ 2	+	*y.* ♀ 3
4.	*y.* × *b.* matings = *y.* ♂ 4		*b.* ♂ 1
	y. ♀ 5		*b.* ♀ 2
	Yellows 9		Buffs 3

Sex unascertained in 1 *y. p. e.* and 1 *b. p. e.* both probably ♀

7*b.* × *y.* =	*y.* ♂ 10	+	*b.* ♂ 5
	y. ♀ 4		*b.* ♀ 4
	Yellows 14		Buffs 9

Colour unascertained in 4 *d. e.* and 5 *p. e.*

‡ This mating gave no certain black-eyed ♀s as in the case of Durham and Marryat, *Report to R. S. Evolution Co.* IV. p. 58, where also it is inaccurately stated that "pink-eyed hen and black-eyed cock gives all young of both sexes black-eyed."

DESCRIPTION OF PLATES.

PLATE I*. *Canaries and Canary Hybrids.*

1. Cinnamon-variegated Buff Canary ♂ × Greenfinch ♀ Hybrid showing Dutch Frill characteristics. A female.

2. The original Wild Canary (*Serinus canaria*).

3. Variegated Buff Siskin ♂ × Canary ♀ Male Hybrid, showing Lizard cap and silver-spangled back. Bred 1908. V.H.C. Crystal Palace, Feb. 1909, in Class 103 (Mixed Hybrids).

4. The White Canary. Bred by W. Kiesel. First prize, Class 66, Crystal Palace, 1909. The first white canary ever seen by living fanciers in this country. By kind permission of the owner, I examined the parents and relations of this bird, and found distinct traces of cinnamon inheritance.

PLATE II*. *Wild Sports.*

1. Almost clear Chaffinch (*Fringilla coelebs*) ♂.

2. Rich Self-cinnamon Greenfinch (*Ligurinus chloris*). One of several living examples in my possession—all females.

3. Very pale Cinnamon (creamy-white) Blackbird (*Turdus merula*), female (examined post-mortem). Other two specimens of a similar nature were shown in Class 123, Crystal Palace, 1909—both females. Also in same class a *pure* white one, with orange bill and red eyes, had third prize—undoubtedly a male specimen.

4. Clear Yellow Yellowhammer (*Emberiza citrinella*)—bright red eyes. At present alive and in my possession. Probably a female.

PLATE III*. *Canary Hybrids.*

1. Clear Yellow Siskin-Canary Hybrid—the only one known. Male. 1st prize, City of Glasgow, 1908; 1st prize, Class 64, Glasgow and West of Scotland, 1908; 1st prize, Class 34, Aberdeen, 1908; 1st prize, Class 87, Scottish National (Edinburgh), 1908–9; 1st prize, Class 95, Crystal Palace, 1909. Also championship diploma and two special prizes for best hybrid, Crystal Palace, 1909.

2. Dark Buff Siskin-Canary Hybrid. Male. I reared twenty-two of these—yellows and buffs—in 1908.

3. Ticked Buff Twite-Canary Hybrid. Male. 2nd prize, Class 65, Glasgow and West of Scotland, 1908; 1st prize, Class 89, Scottish National (Edinburgh), 1908–09; 3rd prize, Class 95, Crystal Palace, 1909.

4. Clear Buff Greenfinch-Canary Hybrid. Male. 2nd prize, Class 64, Glasgow and West of Scotland, 1908; 2nd prize, Class 34, Aberdeen, 1908; 2nd prize, Class 87, Scottish National (Edinburgh), 1908–09; 4th prize, Class 95, Crystal Palace, 1909.

* From original water-colour drawings by H. Norman.

PLATE IV*. *Canary Varieties.*

1. Buff, wing-marked Dark Crested Canary ♂.
2. Yellow (*Jonque*) London Fancy Canary ♂, in show plumage.
3. Gold Lizard Canary ♂, in show plumage and colour-fed.
4. Clear Yellow Crest-Bred Plainhead Canary ♂.

PLATE V.

Fig. 1. Variegated Buff Dutch Frill Canary.

Fig. 6. Trumpeter Pigeon.

ILLUSTRATIONS IN THE TEXT.

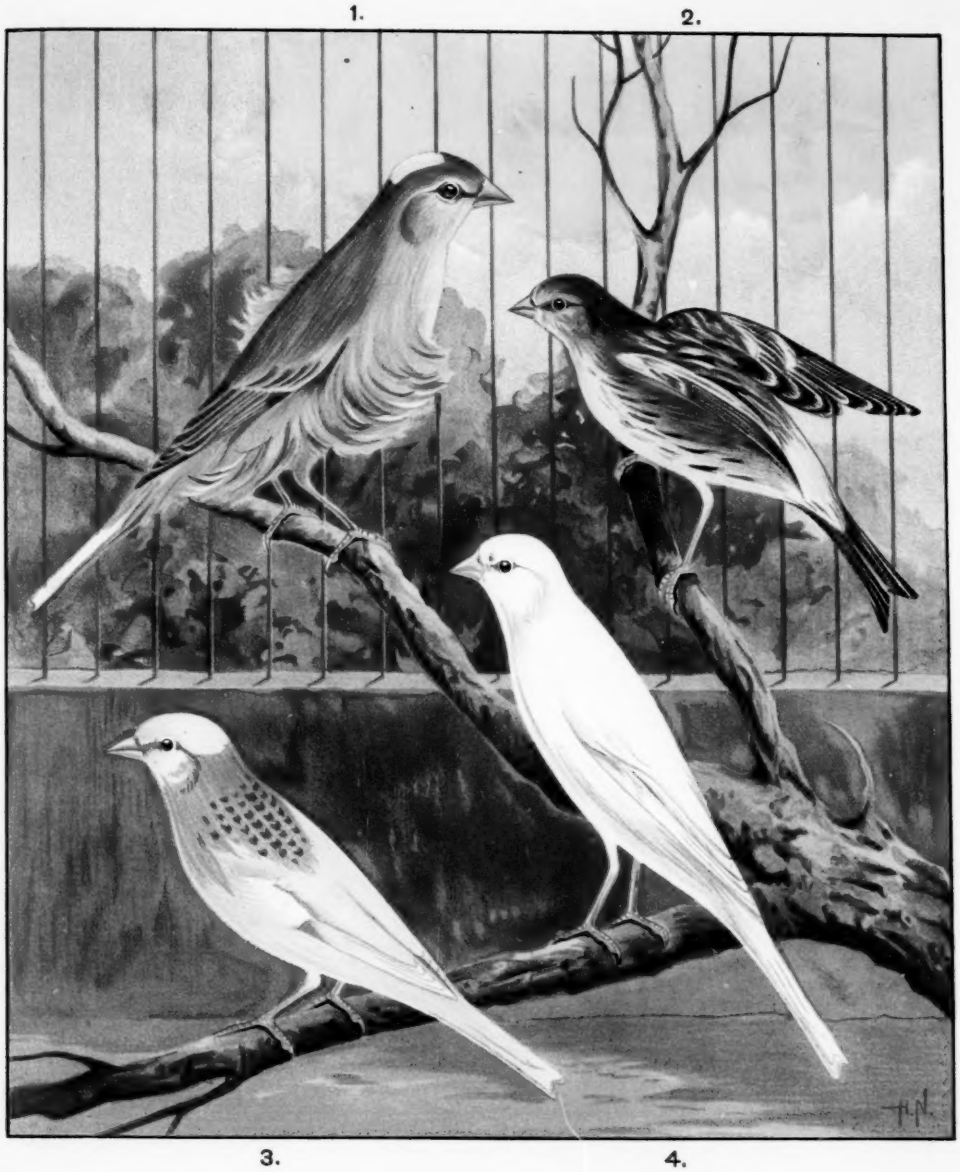
Fig. 2. Variegated Crest-Bred Norwich Plainhead. See p. 22.

Fig. 3. Heavily Variegated Norwich Crest. See p. 22.

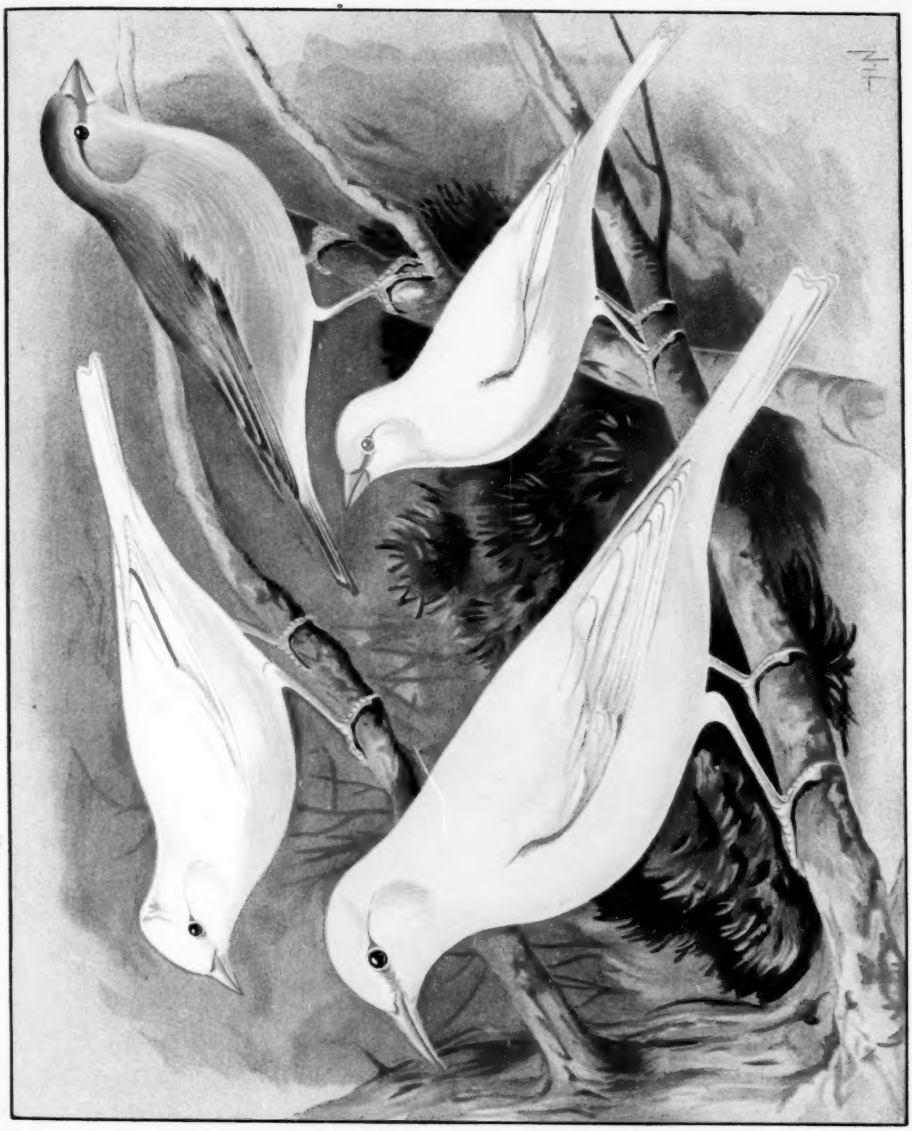
Fig. 4. Typical Lancashire Coppy. See p. 23.

Fig. 5. A Study in Crests. See p. 24.

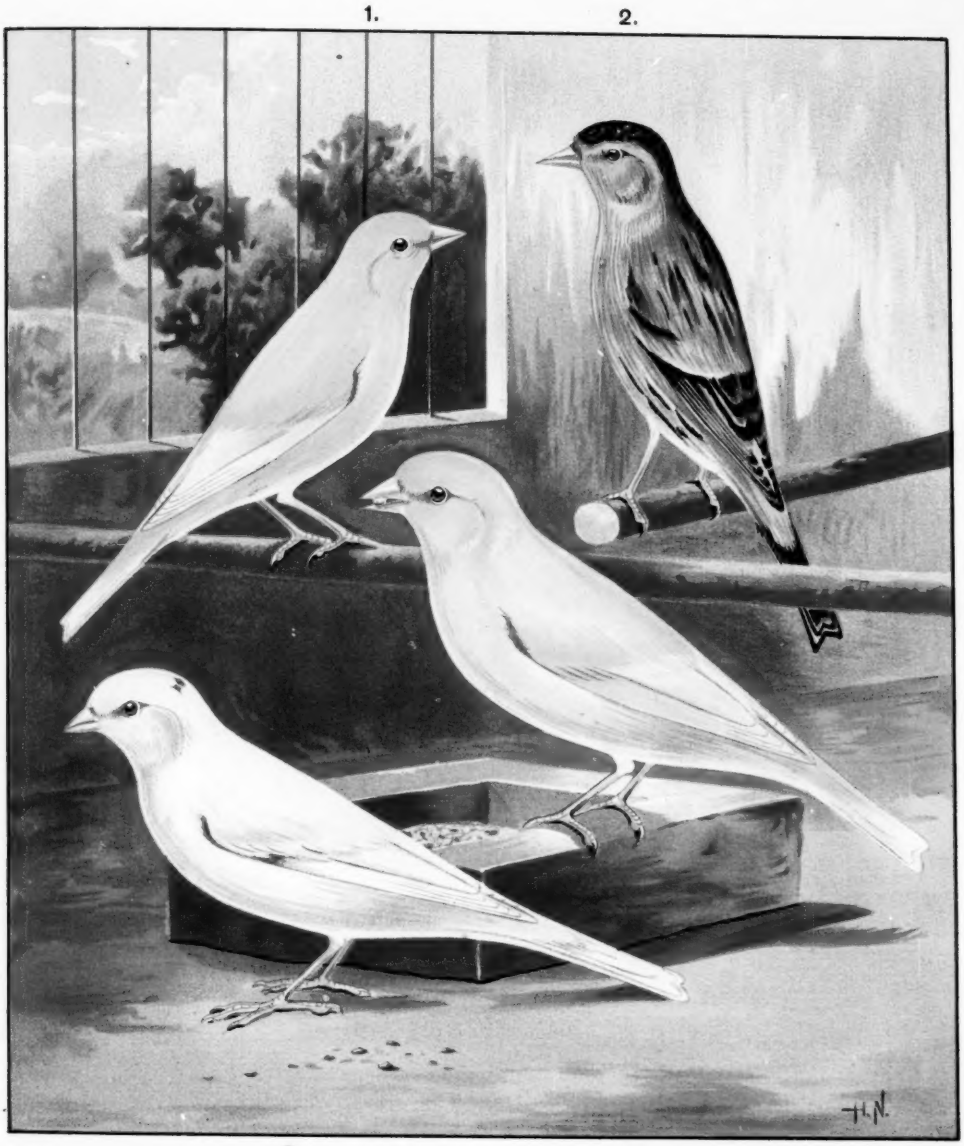
* By permission of *Canary and Cage Bird Life.*



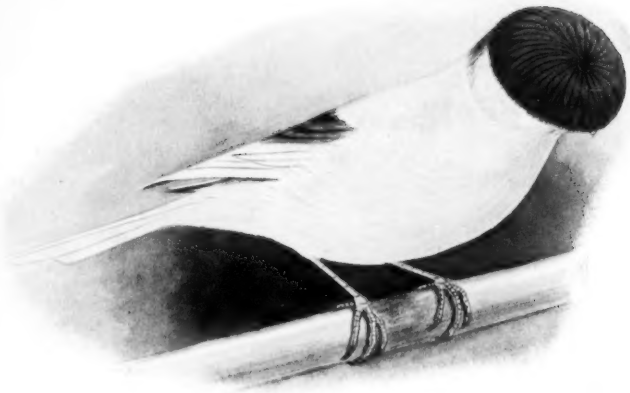
CANARY AND CANARY HYBRIDS.



WILD SPORTS.



CANARY HYBRIDS.



1

3

2

4

CANARY VARIETIES.

1. CRESTED. 2. LONDON FANCY. 3. LIZARD. 4. CREST-BRED.

*(Reprinted by kind permission from "*CANARY AND CAGE-BIRD LIFE.*")*

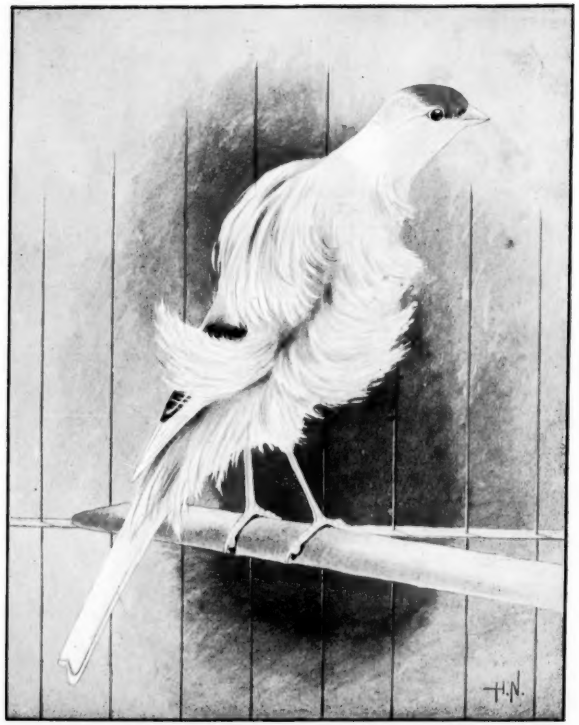


FIG. 1. Variegated buff Dutch Frill Canary. First Prize, Crystal Palace, February 1909. From drawing by H. Norman.

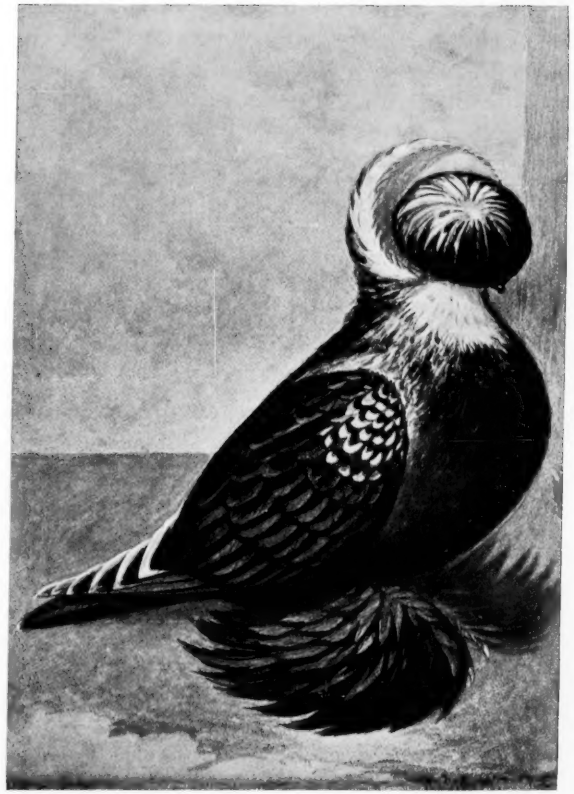


FIG. 6. Trumpeter Pigeon. To illustrate Rose Crest and Shell Crest. By permission of *Feathered World*.

TABLES OF THE Γ-FUNCTION.

THE following tables have been prepared by Mr J. H. Duffell, Associate of the Institute of Actuaries, from the standard tables in Legendre's *Traité des Fonctions Elliptiques*, Paris, 1825—8. Those tables give the values of the log Γ-functions to 12 places of decimals. The present tables provide the values of $\log \Gamma(p)$ from $p = 1{\cdot}000$ to $2{\cdot}000$ proceeding by differences of ·001, and recording *seven* figures. The tabled first differences are the differences of the recorded values, and not the nearest value to 7 figures of the differences of Legendre's tabled values*. It is very usual for biometricians to work with tables of 7 figure logarithms, and the need of a table of $\log \Gamma(p)$ to 7 figures has been long experienced. The original Legendre's table is in a scarce and expensive work and goes to more figures than are required in biometric practice. A number of six-figure $\Gamma(p)$ function tables have been issued, notably the clear table of Mr W. Palin Elderton (*Frequency Curves and Correlation*, C. and E. Layton, 1906). But in view of the forthcoming issue by *Biometrika* of "Tables for Statisticians and Biometricians," it seemed desirable to have stereotyped plates of the log Γ-function to 7 figures, as this number is the working standard adopted in this Journal's numerical tables.

For every value of p greater than 5, and for almost all practical purposes for any value of p greater than 2, we can readily calculate $\log \Gamma(p)$ from the formula†:

$$\log \frac{\Gamma(x+1)}{x^x e^{-x}} = 0{\cdot}399{,}0899 + \tfrac{1}{2} \log x + {\cdot}080{,}929 \sin \frac{25°{\cdot}623}{x},$$

which has the advantage of giving $\Gamma(x+1)/(x^x e^{-x})$, the quantity usually required in biometric work.

For values between 2 and 5 of p, or for values of p less than unity, the formula of reduction $\Gamma(p+1) = p\,\Gamma(p)$ and the present table must still be used.

* W. Palin Elderton has kindly tested this point theoretically, and finds that the differences of the recorded values are the best to use in such cases. ED.

† Pearson, *Biometrika*, Vol VI. p. 119. In the example line 19 for "$\log x^x e^{-x} = {\cdot}848{,}4081$" read "$\log x^x e^{-x} = 3{\cdot}526{,}3058$."

p	Log $\Gamma(p)$, Negative Characteristic, $\bar{1}$									
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·00	·999,9999	7497	5001	2512	0030	·998,7555	5087	2627	0173	$-\bar{7}727$
1·01	·997,5287	2855	0430	$-\bar{8}011$	$-\bar{5}600$	·996,3196	0798	$-\bar{8}408$	$-\bar{6}025$	3648
1·02	·995,1279	$-\bar{8}916$	$-\bar{6}561$	$-\bar{4}212$	$-\bar{1}870$	·993,9535	7207	4886	2572	0265
1·03	·992,7964	5671	3384	1104	$-\bar{8}831$	·991,6564	4305	2052	$-\bar{9}806$	$-\bar{7}567$
1·04	·990,5334	3108	0889	$-\bar{8}677$	$-\bar{6}471$	·989,4273	2080	$-\bar{9}895$	$-\bar{7}716$	$-\bar{5}544$
1·05	·988,3379	1220	$-\bar{9}068$	$-\bar{6}922$	$-\bar{4}783$	·987,2651	0525	$-\bar{8}406$	$-\bar{6}294$	$-\bar{4}188$
1·06	·986,2089	$-\bar{9}996$	$-\bar{7}910$	$-\bar{5}830$	$-\bar{3}757$	·985,1690	$-\bar{9}630$	$-\bar{7}577$	$-\bar{5}530$	$-\bar{3}489$
1·07	·984,1455	$-\bar{9}428$	$-\bar{7}407$	$-\bar{5}392$	$-\bar{3}384$	·983,1382	$-\bar{9}387$	$-\bar{7}398$	$-\bar{5}415$	$-\bar{3}439$
1·08	·982,1469	$-\bar{9}506$	$-\bar{7}549$	$-\bar{5}599$	$-\bar{3}655$	·981,1717	$-\bar{9}785$	$-\bar{7}860$	$-\bar{5}941$	$-\bar{4}029$
1·09	·980,2123	0223	$-\bar{8}329$	$-\bar{6}442$	$-\bar{4}561$	·979,2686	0818	$-\bar{8}956$	$-\bar{7}100$	$-\bar{5}250$
1·10	·978,3407	1570	$-\bar{9}738$	$-\bar{7}914$	$-\bar{6}095$	·977,4283	2476	0676	$-\bar{8}882$	$-\bar{7}095$
1·11	·976,5313	3538	1768	0005	$-\bar{8}248$	·975,6497	4753	3014	1281	$-\bar{9}555$
1·12	·974,7834	6120	4411	2709	1013	·973,9323	7638	5960	4288	2622
1·13	·973,0962	$-\bar{9}308$	$-\bar{7}659$	$-\bar{6}017$	$-\bar{4}381$	·972,2751	1126	$-\bar{9}508$	$-\bar{7}896$	$-\bar{6}289$
1·14	·971,4689	3094	1505	$-\bar{9}922$	$-\bar{8}345$	·970,6774	5209	3650	2096	0549
1·15	·969,9007	7471	5941	4417	2898	·969,1386	$-\bar{9}879$	$-\bar{8}378$	$-\bar{6}883$	$-\bar{5}393$
1·16	·968,3910	2432	0960	$-\bar{9}493$	$-\bar{8}033$	·967,6578	5129	3686	2248	0816
1·17	·966,9390	7969	6554	5145	3742	·966,2344	0952	$-\bar{9}566$	$-\bar{8}185$	$-\bar{6}810$
1·18	·965,5440	4076	2718	1366	0019	·964,8677	7341	6011	4687	3368
1·19	·964,2054	0746	$-\bar{9}444$	$-\bar{8}147$	$-\bar{6}856$	·963,5570	4290	3016	1747	0483
1·20	·962,9225	7973	6725	5484	4248	·962,3017	1792	0573	$-\bar{9}358$	$-\bar{8}150$
1·21	·961,6946	5748	4556	3369	2188	·961,1011	$-\bar{9}841$	$-\bar{8}675$	$-\bar{7}515$	$-\bar{6}361$
1·22	·960,5212	4068	2930	1796	0669	·959,9546	8430	7318	6212	5111
1·23	·959,4015	2925	1840	0760	$-\bar{9}685$	·958,8616	7553	6494	5441	4393
1·24	·958,3350	2313	1280	0253	$-\bar{9}232$	·957,8215	7204	6198	5197	4201
1·25	·957,3211	2226	1246	0271	$-\bar{9}301$	·956,8337	7377	6423	5474	4530
1·26	·956,3592	2658	1730	0806	$-\bar{9}888$	·955,8975	8067	7165	6267	5374
1·27	·955,4487	3604	2727	1855	0988	·955,0126	$-\bar{9}268$	$-\bar{8}416$	$-\bar{7}570$	$-\bar{6}728$
1·28	·954,5891	5059	4232	3410	2593	·954,1782	0975	0173	$-\bar{9}376$	$-\bar{8}585$
1·29	·953,7798	7016	6239	5467	4700	·953,3938	3181	2429	1682	0940
1·30	·953,0203	$-\bar{9}470$	$-\bar{8}743$	$-\bar{8}021$	$-\bar{7}303$	·952,6590	5883	5180	4482	3789
1·31	·952,3100	2417	1739	1065	0396	·951,9732	9073	8419	7770	7125
1·32	·951,6485	5850	5220	4595	3975	·951,3359	2748	2142	1541	0944
1·33	·951,0353	$-\bar{9}766$	$-\bar{9}184$	$-\bar{8}606$	$-\bar{8}034$	·950,7466	6903	6344	5791	5242
1·34	·950,4698	4158	3624	3094	2568	·950,2048	1532	1021	0514	0012
1·35	·949,9515	9023	8535	8052	7573	·949,7100	6630	6166	5706	5251
1·36	·949,4800	4355	3913	3477	3044	·949,2617	2194	1776	1362	0953
1·37	·949,0549	0149	$-\bar{9}754$	$-\bar{9}363$	$-\bar{8}977$	·948,8595	8218	7846	7478	7115
1·38	·948,6756	6402	6052	5707	5366	·948,5030	4698	4371	4049	3731
1·39	·948,3417	3108	2803	2503	2208	·948,1916	1630	1348	1070	0797
1·40	·948,0528	0263	0003	$-\bar{9}748$	$-\bar{9}497$	·947,9250	9008	8770	8537	8308
1·41	·947,8084	7864	7648	7437	7230	·947,7027	6829	6636	6446	6261
1·42	·947,6081	5905	5733	5565	5402	·947,5243	5089	4939	4793	4652
1·43	·947,4515	4382	4254	4130	4010	·947,3894	3783	3676	3574	3476
1·44	·947,3382	3292	3207	3125	3049	·947,2976	2908	2844	2784	2728
1·45	·947,2677	2630	2587	2549	2514	·947,2484	2459	2437	2419	2406
1·46	·947,2397	2393	2392	2396	2404	·947,2416	2432	2452	2477	2506
1·47	·947,2539	2576	2617	2662	2712	·947,2766	2824	2886	2952	3022
1·48	·947,3097	3175	3258	3345	3436	·947,3531	3630	3734	3841	3953
1.49	·947,4068	4188	4312	4440	4572	947,4708	4848	4992	5141	5293
1.50	·947,5449	5610	5774	5943	6116	·947,6292	6473	6658	6847	7040

A horizontal bar means that the third figure of the mantissa has changed, a negative sign that it must be *lowered* one unit.

Differences :—Negative down to rule										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
2503	2496	2489	2482	2475	2468	2460	2454	2446	2440	*1·00*
2432	2425	2419	2411	2404	2398	2390	2383	2377	2369	*1·01*
2363	2355	2349	2342	2335	2328	2321	2314	2307	2301	*1·02*
2293	2287	2280	2273	2267	2259	2253	2246	2239	2233	*1·03*
2226	2219	2212	2206	2198	2193	2185	2179	2172	2165	*1·04*
2159	2152	2146	2139	2132	2126	2119	2112	2106	2099	*1·05*
2093	2086	2080	2073	2067	2060	2053	2047	2041	2034	*1·06*
2027	2021	2015	2008	2002	1995	1989	1983	1976	1970	*1·07*
1963	1957	1950	1944	1938	1932	1925	1919	1912	1906	*1·08*
1900	1894	1887	1881	1875	1868	1862	1856	1850	1843	*1·09*
1837	1832	1824	1819	1812	1807	1800	1794	1787	1782	*1·10*
1775	1770	1763	1757	1751	1744	1739	1733	1726	1721	*1·11*
1714	1709	1702	1696	1690	1685	1678	1672	1666	1660	*1·12*
1654	1649	1642	1636	1630	1625	1618	1612	1607	1600	*1·13*
1595	1589	1583	1577	1571	1565	1559	1554	1547	1542	*1·14*
1536	1530	1524	1519	1512	1507	1501	1495	1490	1483	*1·15*
1478	1472	1467	1460	1455	1449	1443	1438	1432	1426	*1·16*
1421	1415	1409	1403	1398	1392	1386	1381	1375	1370	*1·17*
1364	1358	1352	1347	1342	1336	1330	1324	1319	1314	*1·18*
1308	1302	1297	1291	1286	1280	1274	1269	1264	1258	*1·19*
1252	1248	1241	1236	1231	1225	1219	1215	1208	1204	*1·20*
1198	1192	1187	1181	1177	1170	1166	1160	1154	1149	*1·21*
1144	1138	1134	1127	1123	1116	1112	1106	1101	1096	*1·22*
1090	1085	1080	1075	1069	1063	1059	1053	1048	1043	*1·23*
1037	1033	1027	1021	1017	1011	1006	1001	996	990	*1·24*
985	980	975	970	964	960	954	949	944	938	*1·25*
934	928	924	918	913	908	902	898	893	887	*1·26*
883	877	872	867	862	858	852	846	842	837	*1·27*
832	827	822	817	811	807	802	797	791	787	*1·28*
782	777	772	767	762	757	752	747	742	737	*1·29*
733	727	722	718	713	707	703	698	693	689	*1·30*
683	678	674	669	664	659	654	649	645	640	*1·31*
635	630	625	620	616	611	606	601	597	591	*1·32*
587	582	578	572	568	563	559	553	549	544	*1·33*
540	534	530	526	520	516	511	507	502	497	*1·34*
492	488	483	479	473	470	464	460	455	451	*1·35*
445	442	436	433	427	423	418	414	409	404	*1·36*
400	395	391	386	382	377	372	368	363	359	*1·37*
354	350	345	341	336	332	327	322	318	314	*1·38*
309	305	300	295	292	286	282	278	273	269	*1·39*
265	260	255	251	247	242	238	233	229	224	*1·40*
220	216	211	207	203	198	193	190	185	180	*1·41*
176	172	168	163	159	154	150	146	141	137	*1·42*
133	128	124	120	116	111	107	102	98	94	*1·43*
90	85	82	76	73	68	64	60	56	51	*1·44*
47	43	−38	−35	−30	−25	−22	−18	−13	− 9	*1·45*
− 4	− 1	+ 4	+ 8	+12	+16	+20	+25	+29	+33	*1·46*
+37	+41	45	50	54	58	62	66	70	75	*1·47*
78	83	87	91	95	99	104	107	112	115	*1·48*
120	124	128	132	136	140	144	149	152	156	*1·49*
161	164	169	173	176	181	185	189	193	197	*1·50*

* Differences change sign at horizontal rule.

p	Log $\Gamma(p)$, Negative Characteristic, $\bar{1}$									
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·51	·947,7237	7437	7642	7851	8064	·947,8281	8502	8727	8956	9189
1·52	·947,9426	9667	9912	+0161	+0414	·948,0671	0932	1196	1465	1738
1·53	·948,2015	2295	2580	2868	3161	·948,3457	3758	4062	4370	4682
1·54	·948,4998	5318	5642	5970	6302	·948,6638	6977	7321	7668	8019
1·55	·948,8374	8733	9096	9463	9834	·949,0208	0587	0969	1355	1745
1·56	·949,2139	2537	2938	3344	3753	·949,4166	4583	5004	5429	5857
1·57	·949,6289	6725	7165	7609	8056	·949,8508	8963	9422	9885	+0351
1·58	·950,0822	1296	1774	2255	2741	·950,3230	3723	4220	4720	5225
1·59	·950,5733	6245	6760	7280	7803	·950,8330	8860	9395	9933	+0475
1·60	·951,1020	1569	2122	2679	3240	·951,3804	4372	4943	5519	6098
1·61	·951,6680	7267	7857	8451	9048	·951,9649	+0254	+0862	+1475	+2091
1·62	·952,2710	3333	3960	4591	5225	·952,5863	6504	7149	7798	8451
1·63	·952,9107	9766	+0430	+1097	+1767	·953,2442	3120	3801	4486	5175
1·64	·953,5867	6563	7263	7966	8673	·953,9383	+0097	+0815	+1536	+2260
1·65	·954,2989	3721	4456	5195	5938	·954,6684	7434	8187	8944	9704
1·66	·955,0468	1236	2007	2782	3560	·955,4342	5127	5916	6708	7504
1·67	·955,8303	9106	9913	+0723	+1536	·956,2353	3174	3998	4825	5656
1·68	·956,6491	7329	8170	9015	9864	·957,0716	1571	2430	3293	4159
1·69	·957,5028	5901	6777	7657	8540	·957,9427	+0317	+1211	+2108	+3008
1·70	·958,3912	4820	5731	6645	7563	·958,8484	9409	+0337	+1268	+2203
1·71	·959,3141	4083	5028	5977	6929	·959,7884	8843	9805	+0771	+1740
1·72	·960,2712	3688	4667	5650	6636	·960,7625	8618	9614	+0613	+1616
1·73	·961,2622	3632	4645	5661	6681	·961,7704	8730	9760	+0793	+1830
1·74	·962,2869	3912	4959	6009	7062	·962,8118	9178	+0241	+1308	+2378
1·75	·963,3451	4527	5607	6690	7776	·963,8866	9959	+1055	+2155	+3258
1·76	·964,4364	5473	6586	7702	8821	·964,9944	+1070	+2199	+3331	+4467
1·77	·965,5606	6749	7894	9043	+0195	·966,1350	2509	3671	4836	6004
1·78	·966,7176	8351	9529	+0710	+1895	·967,3082	4274	5468	6665	7866
1·79	·967,9070	+0277	+1488	+2701	+3918	·968,5138	6361	7588	8818	+0051
1·80	·969,1287	2526	3768	5014	6263	·969,7515	8770	+0029	+1291	+2555
1·81	·970,3823	5095	6369	7646	8927	·971,0211	1498	2788	4082	5378
1·82	·971,6678	7981	9287	+0596	+1908	·972,3224	4542	5864	7189	8517
1·83	·972,9848	+1182	+2520	+3860	+5204	·973,6551	7900	9254	+0610	+1969
1·84	·974,3331	4697	6065	7437	8812	·975,0190	1571	2955	4342	5733
1·85	·975,7126	8522	9922	+1325	+2730	·976,4139	5551	6966	8384	9805
1·86	·977,1230	2657	4087	5521	6957	·977,8397	9839	+1285	+2734	+4186
1·87	·978,5640	7098	8559	+0023	+1490	·979,2960	4433	5909	7389	8871
1·88	·980,0356	1844	3335	4830	6327	·980,7827	9331	+0837	+2346	+3859
1·89	·981,5374	6893	8414	9939	+1466	·982,2996	4530	6066	7606	9148
1·90	·983,0693	2242	3793	5348	6905	·983,8465	+0028	+1595	+3164	+4736
1·91	·984,6311	7890	9471	+1055	+2642	·985,4232	5825	7421	9020	+0621
1·92	·986,2226	3834	5445	7058	8675	·987,0294	1917	3542	5170	6802
1·93	·987,8436	+0073	+1713	+3356	+5002	·988,6651	8302	9957	+1614	+3275
1.94	·989,4938	6605	8274	9946	+1621	·990,3299	4980	6663	8350	+0039
1·95	·991,1732	3427	5125	6826	8530	·992,0237	1947	3659	5375	7093
1·96	·992,8815	+0539	+2266	+3995	+5728	·993,7464	9202	+0943	+2688	+4435
1·97	·994,6185	7937	9693	+1451	+3213	·995,4977	6744	8513	+0286	+2062
1·98	·996,3840	5621	7405	9192	+0982	·997,2774	4569	6368	8169	9972
1·99	·998,1779	3588	5401	7216	9034	·999,0854	2678	4504	6333	8165

A horizontal bar means that the third figure of the mantissa has changed, a positive sign that it must be *raised* one unit.

DIFFERENCES:—on this page, POSITIVE										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
200	205	209	213	217	221	225	229	233	237	*1·51*
241	245	249	253	257	261	264	269	273	277	*1·52*
280	285	288	293	296	301	304	308	312	316	*1·53*
320	324	328	332	336	339	344	347	351	355	*1·54*
359	363	367	371	374	379	382	386	390	394	*1·55*
398	401	406	409	413	417	421	425	428	432	*1·56*
436	440	444	447	452	455	459	463	466	471	*1·57*
474	478	481	486	489	493	497	500	505	508	*1·58*
512	515	520	523	527	530	535	538	542	545	*1·59*
549	553	557	561	564	568	571	576	579	582	*1·60*
587	590	594	597	601	605	608	613	616	619	*1·61*
623	627	631	634	638	641	645	649	653	656	*1·62*
659	664	667	670	675	678	681	685	689	692	*1·63*
696	700	703	707	710	714	718	721	724	729	*1·64*
732	735	739	743	746	750	753	757	760	764	*1·65*
768	771	775	778	782	785	789	792	796	799	*1·66*
803	807	810	813	817	821	824	827	831	835	*1·67*
838	841	845	849	852	855	859	863	866	869	*1·68*
873	876	880	883	887	890	894	897	900	904	*1·69*
908	911	914	918	921	925	928	931	935	938	*1·70*
942	945	949	952	955	959	962	966	969	972	*1·71*
976	979	983	986	989	993	996	999	1003	1006	*1·72*
1010	1013	1016	1020	1023	1026	1030	1033	1037	1039	*1·73*
1043	1047	1050	1053	1056	1060	1063	1067	1070	1073	*1·74*
1076	1080	1083	1086	1090	1093	1096	1100	1103	1106	*1·75*
1109	1113	1116	1119	1123	1126	1129	1132	1136	1139	*1·76*
1143	1145	1149	1152	1155	1159	1162	1165	1168	1172	*1·77*
1175	1178	1181	1185	1187	1192	1194	1197	1201	1204	*1·78*
1207	1211	1213	1217	1220	1223	1227	1230	1233	1236	*1·79*
1239	1242	1246	1249	1252	1255	1259	1262	1264	1268	*1·80*
1272	1274	1277	1281	1284	1287	1290	1294	1296	1300	*1·81*
1303	1306	1309	1312	1316	1318	1322	1325	1328	1331	*1·82*
1334	1338	1340	1344	1347	1349	1354	1356	1359	1362	*1·83*
1366	1368	1372	1375	1378	1381	1384	1387	1391	1393	*1·84*
1396	1400	1403	1405	1409	1412	1415	1418	1421	1425	*1·85*
1427	1430	1434	1436	1440	1442	1446	1449	1452	1454	*1·86*
1458	1461	1464	1467	1470	1473	1476	1480	1482	1485	*1·87*
1488	1491	1495	1497	1500	1504	1506	1509	1513	1515	*1·88*
1519	1521	1525	1527	1530	1534	1536	1540	1542	1545	*1·89*
1549	1551	1555	1557	1560	1563	1567	1569	1572	1575	*1·90*
1579	1581	1584	1587	1590	1593	1596	1599	1601	1605	*1·91*
1608	1611	1613	1617	1619	1623	1625	1628	1632	1634	*1·92*
1637	1640	1643	1646	1649	1651	1655	1657	1661	1663	*1·93*
1667	1669	1672	1675	1678	1681	1683	1687	1689	1693	*1·94*
1695	1698	1701	1704	1707	1710	1712	1716	1718	1722	*1·95*
1724	1727	1729	1733	1736	1738	1741	1745	1747	1750	*1·96*
1752	1756	1758	1762	1764	1767	1769	1773	1776	1778	*1·97*
1781	1784	1787	1790	1792	1795	1799	1801	1803	1807	*1·98*
1809	1813	1815	1818	1820	1824	1826	1829	1832	1835	*1·99*

A SECOND COOPERATIVE STUDY OF *VESPA VULGARIS*. COMPARISON OF QUEENS OF A SINGLE NEST AND QUEENS OF A GENERAL POPULATION.

By E. Y. THOMSON, JULIA BELL, M.A., and KARL PEARSON, F.R.S.

(1) *Scope and Material.*

In a first biometric study of *Vespa vulgaris* made two years ago* its authors compared the size, variability and correlation of wing parts of queens, drones and workers from a single nest, which contained 129 queens, 150 drones and several hundred workers. In the present study the same measurements have been repeated on 188 queens, presumably belonging to many nests and collected in a different district. The nest of the first paper was taken in the autumn of 1905 in the neighbourhood of Godalming. The queens for this second investigation were obtained in the spring of 1908 in the neighbourhood of Gerard's Cross. As soon as the first queen wasp appeared, a reward of 1*d.* was offered for each queen wasp brought, and considerably over 200 were thus rapidly collected. Of this number certain individuals had to be rejected because their wings were damaged; a few others were rejected because their colour raised suspicion that they had been thrown in from an old nest. The specimens came in small numbers, sometimes one at a time, often alive, and there is no reason to doubt that they represent a genuine sample of the queen wasps of the autumn of 1907, which survived the winter and were starting in the spring of 1908 to establish their nests.

Now this difference of season is a very important factor to be kept constantly in view. For perfect comparison of the mean characters of a nest and of the general population, we ought to have collected our queens from a couple of hundred Surrey nests, one or two from each, in the autumn of the same year as the single nest was taken. But the difficulty of procuring the number of nests required stood largely in the way, and we determined, at any rate in a second study, to investigate the general population queen in the spring, when collecting

* A Cooperative Study of Queens, Drones and Workers in *Vespa vulgaris*. By Alexandra Wright, Alice Lee and Karl Pearson, *Biometrika*, Vol. v. p. 407.

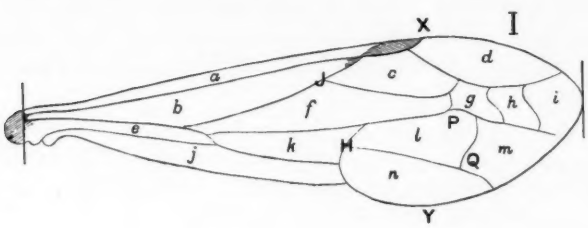
was easier. The change of district was due to the fact that we found at Gerard's Cross in Dr Alice Lee a ready helper, who organised in association with the local schoolmaster a comprehensive search party. Mr O. H. Latter of Charterhouse most kindly went through the material to exclude members of other races.

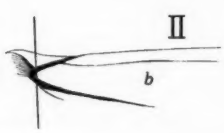
The microscopic measurements made by Miss E. Y. Thomson followed very closely the lines indicated in the earlier paper, an ocular micrometer and a Leitz 1*b* objective being used. The magnification in her case was one ocular unit = 1·46 mm. As this differs slightly from the value used by Miss Wright*, the measurements of both series when compared together in this paper have been reduced to absolute values.

(2) *Characters measured.*

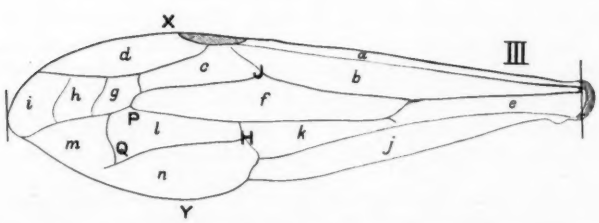
These are precisely the same as those of the first study, and the cuts of that figure reproduced below indicate the lengths measured.

RIGHT WING. *Vespa Vulgaris.*





LEFT WING. *Vespa Vulgaris.*



(A) Total length of wing reckoned from the tip of wing outside of cell *i* to the inside of the vein at the end of cell *b*. Cf. Figs. I. and II.

* The setting of the microscope differed slightly.

(B) Greatest length of cell b taken inside thickened end of veins.

(C) Length of cell f from point to further inside point on the fg vein.

(D) Greatest breadth of wing from X to wherever found, i.e. XY.

(E) Distance apart of J and H, measured inside veins of cell f.

(F) Greatest length of cell d, measured inside cell.

(G) Distance apart of points P and Q measured inside cell l.

The same indices were also determined by Miss Thomson, namely:

$$H = 100\ D/A, \qquad K = 100\ B/A,$$
$$I = 100\ E/D, \qquad L = 100\ C/A,$$
$$J = 100\ G/D, \qquad M = 100\ F/A.$$

The correlations selected for comparison were those of the previous memoir, i.e.

A and D,	G and F,	I and K,	I and L.
A and C,	G and D,	M and J,	
A and E,	B and E,	L and J,	

A few additional correlations were ascertained, as it was thought desirable to put on record the degree of resemblance between right and left wings in at least one species of wasp.

The analytical determination of the biometric constants was carried out by Miss J. Bell, and as there were 188 individuals it seemed sufficient in this case to form grouped correlation tables and calculate the correlations in the usual product moment manner, adjusting with Sheppard's corrections.

(3) *On the relative Sizes of the mean General Population and of the mean Nest Queens.*

Table I gives the general results.

Now, judged by the probable errors, we should conclude that the mean population queen was in all absolute sizes of the wing less than the mean queen of our special nest*. But a word ought to be said as to the possibility of a persistent difference between the two measurers. We fully recognised the difficulty of changing the measurer in the middle of the work, but circumstances did not permit of other action. Accordingly, to diminish personal equation, Miss Thomson practised for some time on Miss Wright's slides, and after a time adjusted her tube length until she got ocular readings in absolute agreement for all practical purposes with Miss Wright's†. But on determination of the magnifying power,

* E and G are the only characters about which any doubt could arise, and as the means of the other characters are in all cases significantly less, these doubts may be discarded.

† The degree of accordance may be illustrated by the following measurements on Queen 100 of Miss Wright's series:

Miss E. Y. Thomson:	9·20	5·34	4·35	2·68	·99	2·22	·97	for A to G respectively.
Miss A. Wright:	9·20	5·35	4·36	2·68	·99	2·23	·97	,,

1 scale unit was now found to be 1·46 mm., as against Miss Wright's 1·48 mm. The solution of this difference may lie in (*a*) a constant relative personal equation of the two measurers, (*b*) a change in Miss Wright's material during the two years which have elapsed since the measurements were made. In the case of both series the slides were dried for about a month before measurements were made; it is possible, however, that an interval of two years would produce a still further effect.

TABLE I.

Mean Values of Characters in Nest and Population Queens.

	Character	Population Queen		Nest Queen		Ratio *P. Q./N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A*	13·223 ± ·015	13·219 ± ·016	13·623 ± ·008	13·617 ± ·009	·9706	·9708
	B	7·770 ± ·010	7·818 ± ·010	8·042 ± ·006	8·058 ± ·007	·9662	·9702
	C	6·185 ± ·008	6·177 ± ·008	6·388 ± ·004	6·390 ± ·004	·9682	·9667
	D	3·933 ± ·006	3·950 ± ·007	3·989 ± ·003	3·983 ± ·003	·9860	·9917
	E	1·446 ± ·003	1·452 ± ·003	1·457 ± ·002	1·467 ± ·001	·9925	·9898
	F	3·031 ± ·005	3·045 ± ·005	3·175 ± ·003	3·191 ± ·003	·9546	·9542
	G	1·414 ± ·002	1·405 ± ·003	1·420 ± ·002	1·415 ± ·002	·9958	·9929
					Mean	·9763	·9766
Indices	*H*	·2975 ± ·0004	·2992 ± ·0005	·2928 ± ·0002	·2925 ± ·0002	1·0161	1·0229
	I	·3672 ± ·0005	·3675 ± ·0005	·3651 ± ·0004	·3687 ± ·0003	1·0057	·9967
	J	·3605 ± ·0005	·3565 ± ·0004	·3560 ± ·0004	·3552 ± ·0004	1·0126	1·0037
	K	·5878 ± ·0003	·5917 ± ·0003	·5904 ± ·0003	·5918 ± ·0003	·9956	·9998
	L	·4680 ± ·0003	·4680 ± ·0003	·4690 ± ·0002	·4694 ± ·0002	·9979	·9970
	M	·2293 ± ·0002	·2303 ± ·0002	·2331 ± ·0002	·2344 ± ·0002	·9837	·9825
					Mean	1·0019	1·0004

The difficulty about a personal equation lies in the fact that it would not extend in either case to the determination of the magnifying powers, for these were found in agreement when determined in the two cases by third observers. If the difference be due to the complete dryness of the material after two years' storage, this difference is of no importance for our present purposes, as both series were measured at practically the same interval after the preparation of the slides.

Even if we could attribute the divergence to personal equation, i.e. if Miss Thomson would have found two years ago, as she does now, that on an average she must reduce Miss Wright's measurements by the factor ·986, i.e. by about 1·4 p.c., this will not account for the average ratio of ·976, i.e. a reduction of 2·4 p.c., found between the population of queens and the queens of a single nest. That we are dealing with a real differentiation is furthermore indicated by the fact that the ratios of the absolute values differ so widely from one character to a second, the

range being from ·954 to ·996. There is, we think, no doubt that the absolute dimensions of the wings differ in the case of any single nest by as much as 1 to 4 p.c. from those of the general population. We have however still to bear in mind the possibility that this differentiation is not that of a nest against a general population, but may be that of a Surrey against a Buckinghamshire local race. Finally a third possibility is to be remembered, our spring queens may be severely selected as against the young autumn queens*. If this were so, the conclusion must be that the smaller queens have somewhat the better chance of surviving. The reader will observe that our results for left and right wings were closely in accordance, the maximum difference in the ratio of the means of the two series being in *D*, where it amounts to ·006; whereas between different characters of the same wing it can be as large as ·041. This is sufficient evidence that the differences are not due to the same source.

If we turn to the population queen, the right wing is larger for the characters *A*, *C* and *G*, and the left wing for the characters *B*, *D*, *E* and *F*. In the nest queen the right wing is larger for the characters *A*, *D* and *G*, and the left in *B*, *C*, *E* and *F*. The characters which have changed places are *C* and *D*. Actually *E* and *F* were the only characters in the nest queen and *B* and *G* in the population queen which showed significant deviations in right and left wings, owing to the size of the probable errors. But the general accordance between the two series seems to indicate that a real differentiation between right and left wing might be demonstrable with larger material.

The last column of Table I shows that when we deal with the relative proportions of the wings, there is a far closer resemblance between population queen and nest queen than for absolute dimensions. The average ratio of the indices for the right wing for the two series differs from unity by only about 2 in 1000, and in the left wing by still less. Considering the probable errors of the differences of the indices of the two series, most of these differences are undoubtedly significant, but, I think, we may assert that the degree of resemblance in pattern is greater than it is in size.

The general conclusion to be drawn from examination of our 26 characters is that there is slight but quite definite differentiation between our single nest and our population of queens; but that until further measurements are made we cannot on the basis of this table alone attribute the results without hesitation to: (*a*) local race differences, (*b*) individuality of the nest arising from heredity or nurture, or (*c*) the selective action of the winter in differentiating autumn from spring queens†.

* Mr Latter suggests as a possibility that the "nest-queens" had fluid in their wings, while the "population queens" had theirs thoroughly dried by hibernation. It is not easy to determine how far this differentiation would be removed by pickling and again drying with the slide. But further observations will be made in this direction.

† We might even suppose that without a selective death-rate the wing might to some extent be worn in six months; the fact that the internal cells like *f* and *b* are also smaller, appears to negative, however, such a hypothesis.

(4) *On the relative Variability of the General Population and of the Nest Queens. Absolute Values.*

Table II gives the absolute variabilities of the 26 characters determined, and we at once observe most marked differences between the two series.

TABLE II.

Standard Deviations of Characters for Nest and Population Queens.

	Character	Population Queen		Nest Queen		Ratio *P. Q./N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A*	·3124 ± ·0109	·3165 ± ·0110	·1428 ± ·0060	·1464 ± ·0061	2·1877	2·1619
	B	·2099 ± ·0073	·2026 ± ·0070	·1089 ± ·0046	·1174 ± ·0049	1·9275	1·7257
	C	·1644 ± ·0057	·1678 ± ·0058	·0666 ± ·0028	·0704 ± ·0030	2·4685	2·3835
	D	·1343 ± ·0047	·1438 ± ·0050	·0525 ± ·0022	·0528 ± ·0022	2·5581	2·7235
	E	·0527 ± ·0018	·0552 ± ·0019	·0329 ± ·0014	·0246 ± ·0010	1·6018	2·2439
	F	·0991 ± ·0034	·1044 ± ·0036	·0462 ± ·0019	·0468 ± ·0020	2·1450	2·2308
	G	·0495 ± ·0017	·0524 ± ·0018	·0376 ± ·0016	·0385 ± ·0016	1·3165	1·3610
					Mean	2·029	2·119
Indices	*H*	·00914 ± ·00032	·00925 ± ·00032	·00329 ± ·00014	·00314 ± ·00013	2·7781	2·9459
	I	·01022 ± ·00036	·01058 ± ·00027	·00628 ± ·00026	·00526 ± ·00022	1·6274	2·0114
	J	·01022 ± ·00036	·00881 ± ·00031	·00717 ± ·00030	·00733 ± ·00031	1·4254	1·2019
	K	·00614 ± ·00021	·00616 ± ·00021	·00422 ± ·00018	·00504 ± ·00021	1·4550	1·2222
	L	·00531 ± ·00018	·00540 ± ·00019	·00281 ± ·00012	·00285 ± ·00012	1·8897	1·8947
	M	·00487 ± ·00017	·00483 ± ·00017	·00360 ± ·00015	·00345 ± ·00014	1·3528	1·4000
					Mean	1·755	1·779

Confining our attention to the absolute variabilities, we see that the variability of the queens in a single nest is not quite half the variability of the queens of the general population. Dr Warren has recently compared the absolute variability for the head index of Natal Termites for the caste of small soldiers* in the two cases of the general population and the population of a single nest. He found for the general population small soldiers a standard deviation of 17·08, and for the average nest one of 7·37, giving a ratio of 2·32. This is of the same order as our results, 2·02 to 2·12. But Dr Warren noticed that between November and March the nest variability could alter considerably (9·00 to 6·56). Further, he was dealing with a head index, and we have compared it with absolute measurements on the wing. If we consider our index measurements on the wing they give (lower part of Table II) 1·76 to 1·78, as against his 2·32. When we consider, however, that he has only taken a single character against our averages of 14 absolute and 12 index characters, and that we get ratios above and below his value, it is difficult

* *Biometrika*, Vol. VI. pp. 336 and 339.

to lay much stress on the differences between us. In round figures we may say that the population of a nest is about half as variable as the general population.

Now it might well be supposed that the queens who survive the winter would be a somewhat highly selected group. Such selection would usually tend to reduce variability, or we might expect that the variability of the general population of queens in the autumn would be, if anything, still larger relatively to the nest population. Is then the large variation of the general population due to a great variability of the nest means resulting from local environmental influence, as Dr Warren suggests in the case of termites? Unfortunately we have in our case only one nest worked out, and we have no data on local variability or the effect of season. But first we ought to consider whether this difference is really incompatible with the effect of heredity on the single nest. Let us suppose the ancestry inbred for n generations, and the ancestral coefficients to diminish in a geometrical series:

$$r_n = \alpha\beta^{n-1},$$

thus obeying the ancestral law of heredity. Suppose the coefficient of assortative mating to be ϵ, for each generation, the mating being that of a brother and sister. Then the correlation of the nth and sth midparent will be obtained by considering the correlation of the means of two individuals only. Let these be H_n and H_s, then, following the lines and notation of a memoir on the ancestral law*, we have, if N be total population of pairs,

$$H_n = \tfrac{1}{2}(h_n + mh'_n), \qquad H_s = \tfrac{1}{2}(h_s + mh'_s),$$

$$S(H_n H_s) = N\rho_{ns}\Sigma_n\Sigma_s = \tfrac{1}{4}\{S(h_n h_s) + mS(h_n h'_s) + mS(h_s h'_n) + m^2 S(h'_n h'_s)\},$$

or
$$\rho_{ns}\Sigma_n\Sigma_s = \tfrac{1}{4}(\sigma_n\sigma_s r_{ns} + m\sigma_n\sigma'_s r_{ns'} + m\sigma_s\sigma'_n r_{n's} + m^2\sigma'_n\sigma'_s r_{n's'}).$$

Now, as in that memoir, take $m = \sigma_n/\sigma'_s = \sigma_s/\sigma'_n$, supposing no secular change. Further, note that we have

$$\Sigma_n^2 = \tfrac{1}{4}(\sigma_n^2 + m^2\sigma'^2_n + 2m\sigma_n\sigma'_n r_{nn'})$$

and $r_{nn'} = \epsilon$; thus

$$\Sigma_n^2 = \tfrac{1}{4}\,2\,(1+\epsilon)\,\sigma_n^2 = \frac{1+\epsilon}{2}\,\sigma_n^2 = \Sigma_s^2;$$

whence
$$\rho_{ns} = \frac{2}{1+\epsilon}\,\frac{r_{ns} + r_{ns'} + r_{n's} + r_{n's'}}{4}.$$

If now we suppose the ancestral influence of drone and queen to be equal, or not being equal we take r_{n-s} to represent the mean of the four values, we have for the correlation between an nth and sth midparent, which may be represented by ρ_{n-s}, for it depends only on the difference,

$$\rho_{n-s} = \frac{2}{1+\epsilon}\,r_{n-s} = \frac{2\alpha}{1+\epsilon}\,\beta^{n-s-1}$$
$$= 1/\gamma\,.\,\beta^{n-s-1},$$

if we take
$$\gamma = (1+\epsilon)/2\alpha.$$

* *R. S. Proc.* Vol. 62, p. 388.

We have next to consider the correlation between an nth midparent and the actual offspring. Let us represent it by ρ_{0n}. Then

$$N\Sigma_n\sigma_0\rho_{0n} = \tfrac{1}{2}S(h_0h_n) + m\tfrac{1}{2}\Sigma(h_0h'_n),$$

or,

$$\Sigma_n\sigma_0\rho_{0n} = \tfrac{1}{2}(\sigma_0\sigma_n r_{0n} + m\sigma_0\sigma_{n'}r_{0n'}),$$

$$\rho_{0n} = \frac{\sigma_n}{\Sigma_n}\tfrac{1}{2}(r_{0n} + r_{0n'})$$

$$= \sqrt{\frac{2}{1+\epsilon}}\, r_n = \sqrt{\frac{2}{1+\epsilon}}\,\alpha\beta^{n-1} = \sqrt{\frac{\alpha}{\gamma}}\,\beta^{n-1},$$

if r_n be again the mean correlation value of the ancestors of the two sexes in the nth degree and the offspring.

We have now to evaluate the determinant Δ and the minor Δ_{11}, obtained by cutting out its first row and column. Taking n generations of inbreeding:

$$\Delta = \begin{vmatrix} 1, & \rho_{01}, & \rho_{02}, & \rho_{03}, & \dots & \rho_{0n} \\ \rho_{01}, & 1, & \rho_{12}, & \rho_{13}, & \dots & \rho_{1n} \\ \rho_{02}, & \rho_{12}, & 1, & \rho_{23}, & \dots & \rho_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \rho_{0n}, & \rho_{1n}, & \rho_{n2}, & \rho_{n3}, & \dots & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1, & \sqrt{\alpha/\gamma}, & \sqrt{\alpha/\gamma}\,\beta, & \sqrt{\alpha/\gamma}\,\beta^2, & \dots & \sqrt{\alpha/\gamma}\,\beta^{n-1} \\ \sqrt{\alpha/\gamma}, & 1, & \frac{1}{\gamma}, & \frac{1}{\gamma}\beta, & \dots & \frac{1}{\gamma}\beta^{n-2} \\ \sqrt{\alpha/\gamma}\,\beta, & \frac{1}{\gamma}, & 1, & \frac{1}{\gamma}, & \dots & \frac{1}{\gamma}\beta^{n-3} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \sqrt{\alpha/\gamma}\,\beta^{n-1}, & \frac{1}{\gamma}\beta^{n-2}, & \frac{1}{\gamma}\beta^{n-3}, & \frac{1}{\gamma}\beta^{n-4}, & \dots & 1 \end{vmatrix}$$

$$= \alpha\gamma \begin{vmatrix} \frac{1}{\alpha\gamma}, & \frac{1}{\gamma}, & \frac{\beta}{\gamma}, & \frac{\beta^2}{\gamma}, & \dots & \frac{\beta^{n-1}}{\gamma} \\ \frac{1}{\gamma}, & 1, & \frac{1}{\gamma}, & \frac{\beta}{\gamma}, & \dots & \frac{\beta^{n-2}}{\gamma} \\ \frac{\beta}{\gamma}, & \frac{1}{\gamma}, & 1, & \frac{1}{\gamma}, & \dots & \frac{\beta^{n-3}}{\gamma} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{\beta^{n-1}}{\gamma}, & \frac{\beta^{n-2}}{\gamma}, & \frac{\beta^{n-3}}{\gamma}, & \frac{\beta^{n-4}}{\gamma}, & \dots & 1 \end{vmatrix}$$

$$= \frac{\alpha\gamma}{\gamma^{n+1}} \begin{vmatrix} \frac{1}{\alpha}, & 1, & \beta, & \beta^2, & \dots & \beta^{n-1} \\ 1, & \gamma, & 1, & \beta, & \dots & \beta^{n-2} \\ \beta, & 1, & \gamma, & 1, & \dots & \beta^{n-3} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \beta^{n-1}, & \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \dots & \gamma \end{vmatrix}.$$

Subtract β times the second row from the first and β times the second column from the first.

$$\Delta = \frac{\alpha}{\gamma^n}\begin{vmatrix} \frac{1}{\alpha}+\gamma\beta^2-2\beta, & 1-\gamma\beta, & 0, & 0, & \dots & 0 \\ 1-\gamma\beta, & \gamma, & 1, & \beta, & \dots & \beta^{n-2} \\ 0, & 1, & \gamma, & 1, & \dots & \beta^{n-3} \\ 0, & \beta, & 1, & \gamma, & \dots & \beta^{n-4} \\ \dots & \dots & \dots & & & \\ 0, & \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \dots & \gamma \end{vmatrix}$$

$$= \frac{\alpha}{\gamma^n}\left\{\left(\frac{1}{\alpha}+\gamma\beta^2-2\beta\right)D_{n-2}-(1-\gamma\beta)^2 D_{n-3}\right\},$$

where

$$D_{n-2} = \begin{vmatrix} \gamma, & 1, & \beta, & \dots & \beta^{n-2} \\ 1, & \gamma, & 1, & \dots & \beta^{n-3} \\ \beta, & 1, & \gamma, & \dots & \beta^{n-4} \\ \dots & & & & \\ \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \dots & \gamma \end{vmatrix}.$$

We now turn to the first minor Δ_{11}, for which we have:

$$\Delta_{11} = \begin{vmatrix} 1, & \frac{1}{\gamma}, & \frac{1}{\gamma}\beta, & \dots & \frac{1}{\gamma}\beta^{n-2} \\ \frac{1}{\gamma}, & 1, & \frac{1}{\gamma}, & \dots & \frac{1}{\gamma}\beta^{n-3} \\ \frac{1}{\gamma}\beta, & \frac{1}{\gamma}, & 1, & \dots & \frac{1}{\gamma}\beta^{n-4} \\ \dots & & & & \\ \frac{1}{\gamma}\beta^{n-2}, & \frac{1}{\gamma}\beta^{n-3}, & \frac{1}{\gamma}\beta^{n-4}, & \dots & 1 \end{vmatrix}$$

$$= \frac{1}{\gamma^n}\begin{vmatrix} \gamma, & 1, & \beta, & \dots & \beta^{n-2} \\ 1, & \gamma, & 1, & \dots & \beta^{n-3} \\ \beta, & 1, & \gamma, & \dots & \beta^{n-4} \\ \dots & & & & \\ \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \dots & \gamma \end{vmatrix}$$

$$= \frac{1}{\gamma^n} D_{n-2}.$$

Now if s_n be the standard-deviation of a single nest:

$$s_n = \sigma_n \sqrt{\Delta/\Delta_{11}} = \sigma_n \sqrt{\alpha\left\{\left(\frac{1}{\alpha}+\gamma\beta^2-2\beta\right)-(1-\gamma\beta)^2\frac{D_{n-3}}{D_{n-2}}\right\}}.$$

We have accordingly to find D_{n-3}/D_{n-2} when n is large. Returning to D_{n-2}, subtract β times the second row from the first, and β times the second column from the first. There results:

$$D_{n-2} = \begin{vmatrix} \gamma(1+\beta^2)-2\beta, & 1-\gamma\beta, & 0, & 0, & \dots & 0 \\ 1-\gamma\beta, & \gamma, & 1, & \beta, & \dots & \beta^{n-3} \\ 0, & 1, & \gamma, & 1, & \dots & \beta^{n-4} \\ 0, & \beta, & 1, & \gamma, & \dots & \beta^{n-5} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0, & \beta^{n-3}, & \beta^{n-4}, & \beta^{n-5}, & \dots & \gamma \end{vmatrix}$$

$$= \{\gamma(1+\beta^2)-2\beta\}\, D_{n-3} - (1-\gamma\beta)^2\, D_{n-4}.$$

Assume $D_{n-2} = C\xi^{n-2}$, and we have

$$\xi^2 - \{\gamma(1+\beta^2)-2\beta\}\,\xi + (1-\gamma\beta)^2 = 0.$$

Hence if ξ_1 and ξ_2 be the roots

$$D_{n-2} = C_1\xi_1^{n-2} + C_2\xi_2^{n-2},$$

and

$$\frac{D_{n-3}}{D_{n-2}} = \frac{C_1\xi_1^{n-3} + C_2\xi_2^{n-3}}{C_1\xi_1^{n-2} + C_2\xi_2^{n-2}}.$$

Now if ξ_1 be $> \xi_2$, this when n is large rapidly becomes equal to $\dfrac{1}{\xi_1}$.

Thus we conclude that for a great number of generations of inbreeding of brother and sister, as will usually be the case in wasps,

$$\frac{\sigma_n}{s_n} = \frac{1}{\sqrt{\alpha\left\{\left(\dfrac{1}{\alpha}+\gamma\beta^2-2\beta\right)-\dfrac{(1-\gamma\beta)^2}{\xi_1}\right\}}},$$

where ξ_1 is the greater root of the quadratic

$$\xi^2 - \{\gamma(1+\beta^2)-2\beta\}\,\xi + (1-\gamma\beta)^2 = 0.$$

The result may be written

$$\sigma_n/s_n = 1/\sqrt{1+\alpha(\xi_1-\gamma)}.$$

We have worked out this result numerically for a few cases, assuming ϵ the coefficient of assortative mating to be $= \cdot 5$ the resemblance of brother and sister in most mammals. (This step, of course, from mammals to insects yet needs verification. Cf. Warren for Aphis and Daphnia, *Biometrika*, Vol. I. p. 147.)

Case (i). The average values of biometric work are $\alpha = \frac{1}{2}$, $\beta = \frac{2}{3}$. Hence $\gamma = 1{\cdot}5$, $1-\gamma\beta$ will be nearly zero and $\xi_1 = {\cdot}8000$. We find

$$\sigma_n/s_n = 1{\cdot}23.$$

Case (ii). Let the correlations be equal to the *gametic* correlations on the Mendelian hypothesis. Then $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}$. Hence $\gamma = 1{\cdot}5$ and we have

$$\xi^2 - {\cdot}875\,\xi + {\cdot}0625 = 0,$$

which leads to $\xi_1 = {\cdot}7966$, and

$$\sigma_n/s_n = 1{\cdot}24.$$

Case (iii). Let the correlations be equal to the *somatic* correlations on the Mendelian hypothesis. Then $\alpha = \frac{1}{3}$, $\beta = \frac{1}{2}$, $\gamma = 2{\cdot}25$ and

$$\xi^2 = 1{\cdot}8125\,\xi + {\cdot}0156 = 0,$$

giving $\xi_1 = 1{\cdot}8041$, and leading to

$$\sigma_n/s_n = 1{\cdot}08.$$

Case (iv). We assumed the fraternal correlation was as high as ·7, following Warren's results for Aphis and Daphnia. The ratio for $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}$ was

$$\sigma_n/s_n = 1{\cdot}19,$$

and no sensible change resulted.

The values of the biometric and Mendelian gametic correlations fit the facts better than those of the Mendelian somatic correlations. But neither give a sufficiently reduced variability when we pass from general population to a special nest. We have at most 1·24 as against 1·8 to 2·1. Two only, and these the indices J and K for the left wing, out of the 26 characters, give values 1·20 and 1·22 comparable with the above theoretical values. We are therefore forced to conclude that (*a*) there is some defect in the above algebraic analysis; the problem is not an easy one, and there may be a slip, or (*b*) the wasps, as Dr Warren supposes in the case of the termites, are markedly influenced by the local environment of the nest, and thus a considerable part of the observed large variability ratio has an environmental and not a hereditary source, or (*c*) the result may be due to winter selection, if we suppose that selection of a twofold character, i.e. that from some unexplained cause the rather larger and rather smaller queens have a somewhat better chance of surviving than the mediocre queens.

Undoubtedly the more prudent course at present is not, however, to manufacture hypotheses, but to wait till further material has been measured and till more light can be thrown on this most interesting point. Unless we were most unfortunate in our selection of the original nest*, Table I shows that remarkably little effect has been produced by either selection or growth on the *mean* values of the indices during the winter months. Yet the variability of these indices is much larger than we can possibly attribute in the present state of our knowledge to the hereditary factor, when compared with their values for an individual nest. We should, *a priori*, conceive that local environment would give the general population a wide range of variability by making a large range of local means. The crux is that our one set of local index means is extraordinarily close to the general population index means. Personally, we admit that we are inclined to attribute the result to the continual inbreeding of the wasp, but we also confess that without assuming values for the hereditary coefficients incompatible with our present knowledge, we cannot reach such a variability ratio as we find indicated for both wasps and termites.

* I.e. just chancing to hit a nest with all its indices closely in agreement with the general population means.

(5) *On the Relative Variability of the General Population and of the Nest Queens. Relative Values.*

In Table III the coefficients of variation of population and nest queens for the 26 characters are placed side by side. We see at once that the variability ratios are as marked when we treat relative as when we treat absolute variation—the

TABLE III.

Coefficients of Variation of Characters for Nest and Population Queens.

	Character	Population Queen		Nest Queen		Ratio *P. Q./N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A*	2·36 ± ·08	2·39 ± ·08	1·05 ± ·04	1·09 ± ·05	2·25	2·19
	B	2·70 ± ·09	2·59 ± ·09	1·36 ± ·07	1·46 ± ·06	1·99	1·77
	C	2·66 ± ·09	2·72 ± ·09	1·05 ± ·04	1·10 ± ·05	2·53	2·47
	D	3·41 ± ·12	3·64 ± ·13	1·31 ± ·06	1·32 ± ·06	2·60	2·76
	E	3·65 ± ·13	3·80 ± ·13	2·13 ± ·09	1·68 ± ·07	1·71	2·26
	F	3·27 ± ·11	3·43 ± ·12	1·45 ± ·06	1·46 ± ·06	2·26	2·35
	G	3·50 ± ·12	3·73 ± ·12	2·65 ± ·11	2·70 ± ·11	1·32	1·38
	Mean	3·08	3·19	1·57	1·54	2·09	2·17
Indices	*H*	3·07 ± ·11	3·09 ± ·11	1·12 ± ·05	1·07 ± ·05	2·74	2·89
	I	2·78 ± ·10	2·88 ± ·10	1·72 ± ·07	1·43 ± ·06	1·62	2·01
	J	2·83 ± ·10	2·47 ± ·09	2·01 ± ·08	2·04 ± ·09	1·41	1·21
	K	1·04 ± ·04	1·04 ± ·04	0·72 ± ·03	0·85 ± ·04	1·44	1·22
	L	1·13 ± ·04	1·15 ± ·04	0·60 ± ·03	0·61 ± ·02	1·88	1·88
	M	2·12 ± ·07	2·10 ± ·07	1·54 ± ·06	1·47 ± ·06	1·38	1·42
	Mean	2·17	2·12	1·28	1·25	1·75	1·77

general results owing to the small changes in the average values of the characters as we pass from nest to general population being closely alike. Although the coefficients of variation for the general population are for absolute sizes double as large as for the single nest, we see that the conclusion of the first wasp paper, *Biometrika*, Vol. v. p. 415, is confirmed. The wing of the queen of *Vespa vulgaris* possesses extraordinarily small variability. If we deal with indices, even smaller values are found than Dr Warren found for his termites, and our population values for absolute sizes are comparable with his nest values. As far as our present experience reaches, the wing of the queen wasp appears to be the least variable organ that has as yet been studied. We may note that the population queen is about as variable as the worker of an individual nest (*Biometrika*, Vol. v. p. 417).

Looking at the results now obtained, it is not possible to assert any marked difference in variability between the two wings, thus the result of the earlier paper (p. 417) is confirmed.

(6) *Coefficients of Correlation of Characters in the General Population and Nest Queens.*

The results are given in Table IV. Examining first the index correlations of the population queens we see that as in the case of the nest queens they are all slight and of small significance. Yet they are now all *positive*, and do

TABLE IV.

Coefficients of Correlation for Characters of Nest and Population Queens.

	Characters correlated	Population Queen		Nest Queen		Ratio *P. Q./N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A* and *D*	·478 ± ·038	·579 ± ·033	·558 ± ·040	·619 ± ·037	·86	·94
	A and *C*	·917 ± ·008	·894 ± ·010	·838 ± ·018	·857 ± ·016	1·09	1·04
	A and *E*	·531 ± ·035	·647 ± ·029	·238 ± ·056	·259 ± ·053	2·23	2·50
	G and *F*	·413 ± ·041	·396 ± ·042	·164 ± ·058	·051 ± ·059	2·52	7·76
	G and *D*	·672 ± ·027	·781 ± ·019	·677 ± ·032	·694 ± ·031	·99	1·13
	B and *E*	·506 ± ·037	·617 ± ·031	·233 ± ·056	·280 ± ·055	2·17	2·20
	B and *F*	·677 ± ·027	·671 ± ·027	·180 ± ·057	·139 ± ·058	3·76	4·83
	Mean	·599	·655	·413	·414	1·95	2·92
Indices	*I* and *K*	·123 ± ·048	·054 ± ·049	− ·021 ± ·059	·037 ± ·059	—	—
	M and *J*	·050 ± ·049	·137 ± ·048	− ·138 ± ·058	− ·185 ± ·057	—	—
	L and *J*	·097 ± ·049	·132 ± ·048	− ·367 ± ·051	− ·186 ± ·057	—	—
	I and *L*	·137 ± ·048	·203 ± ·047	·039 ± ·059	·194 ± ·057	—	—
	Mean	·102	·131	− ·122	− ·035	—	—

not take the somewhat erratic values found in the case of the nest queens. It may be doubted whether, considering their probable errors, any weight whatever can be laid on these correlations. The small variation of these indices and their low correlations demonstrate, however, that they might be of considerable service for classificatory purposes. If we turn to the correlations of the absolute sizes, we see at once a marked difference. With the exception of one pair to be referred to especially below, in which the nest correlation is higher for both wings, and one pair, where in the right wing they are sensibly equal, the correlation is higher, and very markedly higher in the general population, than in the nest queens. Now this is precisely what the theory of correlation would lead us to expect; if we select a small element of a total population we thereby reduce the correlation. Now let us omit the correlations of length and breadth of wing, where clearly some special cause has been at work. We find for the mean correlation of population queens for the remaining six pairs on both sides ·64. The

general reduction of variability for the absolute characters as shown by Table II is about $\frac{1}{2}$*. Now the effect of double selection on the variation of two characters is known to be given by the formula

$$R_{12} = r_{12} \frac{\mu_1 \mu_2}{\sqrt{1 - r_{12}^2 (1 - \mu_1^2)} \sqrt{1 - r_{12}^2 (1 - \mu_2^2)}}.$$

See *Biometrika*, Vol. VI. pp. 111—2.

Putting $r_{12} = \cdot 64$ and $\mu_1 = \mu_2 = \frac{1}{2}$, we find

$$R_{12} = r_{12}/2 \cdot 77.$$

Turning to the last column of Table IV, headed Ratio *P. Q./N. Q.*, and omitting the pair *A* and *D*, we find the mean value of the ratio of the correlation coefficients of the remaining 12 pairs to be 2·68. This is sufficiently near to 2·77 for us to conclude that the decrease in correlation, when we pass from general population to a nest population, is quite compatible with the result which naturally flows when we look upon the nest as a selection of reduced variability made from the general population. It will indeed be noted that the three least correlation ratios involve *A* and *D*, the characters which correlated together give the exceptional case of a *reduced* variation when we pass from the general to a nest population. Can any explanation be offered of this anomalous case? To begin with, the possibility of a stringent winter selection of these characters so that the correlation would be reduced below that of a nest is hardly tenable. Table II shows us that the variability ratios for the *A* and *D* characters are above the average. They ought therefore to show reduced correlation for the nest as compared with the general population. I would draw attention to the fact, however, that *A*, the total length of wing, and *D*, the breadth, are the two characters which are most likely to be affected by wear and tear of the wing. All damaged wings on which the measurement of *A* or *D* was difficult or impossible were, of course, omitted. But it will be clear that the measurement of internal cells is less likely to be influenced by any action of the nature of weathering than these external measurements. The possibility is therefore before us—we do not lay much stress on it—that in the reduced correlation of the length and breadth of the wing, we have the one definite effect of the hibernation. At the same time, the fact that we do not find a reduced correlation in the case of right and left wings for either the characters of length or breadth (see Table V) is somewhat against this hypothesis. It would indicate that the action, whatever its character, is the same for the lengths of both wings or for the breadths of both wings, and can be nothing of the nature of local wear, affecting one wing here and another there.

* Actually (*Biometrika*, Vol. VI. 112) μ should be found from

$$\Sigma_1^2 = \sigma_1^2 \mu_1^2 \{1 - r_{12}^2 (1 - \mu_2^2)\} / \{1 - r_{12}^2 (1 - \mu_1^2)(1 - \mu_2^2)\}$$

by putting Σ_1/σ_1 = variability ratio for the characters taken weighted with the number of times they are taken. But the r_{12}^2 terms have little effect, e.g. if $\Sigma_1/\sigma_1 = \cdot 5$, $\mu_1 = \mu_2 = \cdot 53$, and with this value $R_{12}/r_{12} = 2 \cdot 52$ which is of the same order. In fact the observed mean value lies between the values of R_{12}/r_{12} for $\mu = \cdot 50$ and $\mu = \cdot 53$.

(7) *Comparison of Correlation between Characters in Right and Left Wings for the two Series.*

TABLE V.

Correlations of Right and Left Wings in Nest and Population Queens.

Character	Population Queen	Nest Queen	Ratio *P. Q./N. Q.*
A	·954 ± ·004	·893 ± ·012	1·07
B	·911 ± ·008	·831 ± ·018	1·10
C	·964 ± ·004	·858 ± ·016	1·12
D	·862 ± ·013	·600 ± ·038	1·44
E	·694 ± ·026	·428 ± ·049	1·62
F	·932 ± ·006	·605 ± ·038	1·54
G	·856 ± ·013	·787 ± ·023	1·09
Mean	·882	·714	1·28

If we apply our selection formulae to these cases we must, since r_{12} is large, = ·882, deduce the true value of μ. We have

$$\Sigma_1^2/\sigma_1^2 = \tfrac{1}{4} \text{ about} = \frac{\mu^2\{1 - r_{12}^2(1-\mu^2)\}}{1 - r_{12}^2(1-\mu^2)^2},$$

or, calling $1-\mu^2 = v$,

$$1 - r_{12}^2 v^2 = 4(1-v) - 4r_{12}^2 v(1-v),$$
$$5r_{12}^2 v^2 - 4v(1+r_{12}^2) + 3 = 0,$$
$$v^2 - 1{\cdot}8284v + {\cdot}7713 = 0,$$

of which the required root is $v = {\cdot}6603$, giving $\mu = {\cdot}5828$. Substituting this in the correlation formula

$$R_{12} = r_{12}\frac{\mu^2}{1 - r_{12}^2(1-\mu^2)},$$

we deduce

$$R_{12} = r_{12}/1{\cdot}43.$$

The average correlation ratio 1·43 thus provided is somewhat greater than the observed average value, 1·28. It is, however, greater than four and less than three of the individual values, and it is quite possible that if the process had been applied to each individual character, a closer mean result would have been obtained than by using the mean correlation, ·882. The general agreement is sufficient to show the reduction of correlation found as we pass from general population to nest population is just of the sort of order which the reduction in variability would lead us to anticipate. It is clear, therefore, that the correlation reductions are really what we might expect on the assumption that the nest is a selection from the general population. The outstanding point is the extreme reduction in variability which marks this selection.

It is worth noting that the lowest correlations are those that concern homologous *breadth* measurements on the wings. The highest correlations are those which

concern the total length of the wings and the lengths of cells b, f, and d (see Fig. I, p. 49). If we confine our attention to these homologous lengths of the wasp's wings we see that the range ·911 to ·964 is quite comparable with the values obtained by Whiteley and Pearson* for the first joint of the human fingers, ·904 to ·934; with those obtained by Whiteley and Lewenz† for homologous bones of the two hands, ·793 to ·990; and by Dr Warren‡ for homologous long bones in man, ·86 to ·97. The right and left wings of the wasp appear as closely related as the right and left members in man.

(8) *Conclusions.*

The results reached in this paper really centre on the determination of what meaning we are to give to the high variability ratio between the population of a single nest and the general population. The relatively close agreement of the means of this individual nest with those of the general population for the indices, does not seem to indicate that we are to look for an explanation of the difficulty in differentiated environmental effect on the different nests. *A priori* reasonable values of the heredity coefficients do not appear to allow of a reduction of variability to 50 °/₀ of the general population value. Once the source of this reduction in variability is accounted for, the correlation changes between nest and general population queens are seen to be of the order which should theoretically be associated with it. The only comparable material is that of Dr Warren for Natal termites, which gives a variability reduction of approximately the same value; and this reduction is considered by him to be due in that case to differential environmental effect. It will be of interest to obtain, if possible, a series of autumn queens from the general population, and compare the variability of these with that of the spring queens dealt with in this paper.

Note. It occurred to me after completing the above memoir that selection of *midparents* might give a greater variability than the selection of actual parents in each generation. Testing, however, for the case of immediate parents, this does not appear to be the case. The variability of the offspring:

$$s = \sigma \sqrt{\frac{(1 - r_1^2 - r_2^2 - \epsilon^2 + 2r_1 r_2 \epsilon)}{1 - \epsilon^2}} = \sigma \sqrt{1 - \frac{2r_1^2(1 - \epsilon)}{1 - \epsilon^2}},$$

if the parental correlations be equal. Thus:

$$s = \sigma \sqrt{1 - \left(r_1 \sqrt{\frac{2}{1 + \epsilon}}\right)^2} = \sigma \sqrt{1 - \rho_{01}^2},$$

by p. 54, if ρ_{01} be the correlation of the offspring with the first midparent. Thus the midparent simplification of the analysis does not appear to account for the theoretically large variability in the single nest as compared with observation.

K. P.

* *R. S. Proc.* Vol. 65, pp. 349—350.
† *Biometrika*, Vol. I. p. 358.
‡ *Phil. Trans.* Vol. 189, B, p. 178.

THE OPSONIC INDEX—A MEDICO-STATISTICAL ENQUIRY*.

By W. F. HARVEY, M.A., M.B., D.P.H., Captain I.M.S. and ANDERSON McKENDRICK, M.B., Ch.B., Captain I.M.S†.

Much misunderstanding seems to exist as to the relationship which should hold between the statistician, anxious to scrutinise the validity of inferences drawn from observations, and the observer of the facts. The latter looks with suspicion on the former and is inclined to doubt whether the intrusion of his fellow scientist into his domain is for any more worthy purpose than simply to show that he is totally wrong in his conclusions. The mathematician is apt to be impatient at the want of knowledge of his technical terms displayed by the observer and at the often openly expressed contempt for some of his most cherished elementary principles. But the statistical mathematician must get his data from the doctor —for it is of medical science in particular that we speak here—and the doctor at present is dependent on or would be wise to obtain the assistance of the mathematician for the refining of his conclusions. The doctor may also obtain much help from the criticism of the mathematician regarding the numerical soundness of his control observations. The statistician takes very little account of the non-quantitative data which bulk so largely in support of the claims of medical science. He takes merely certain of the characters which lend themselves easily to analysis, such as case-mortality or its complement case-recovery, attack-rate and the like and on this basis alone examines the validity of the conclusions arrived at. The degrees of correlation between such characters and particular methods of treatment are, through the controversies to which they have given rise, becoming familiar to medical men. But it must be pointed out that these correlations measure only the numerical aspect of single pairs of phenomena, admittedly important though these be. In some cases they do not measure the actual facts under discus-

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† In explanation of the joint authorship of this paper I should say that an earlier unpublished paper on the same subject by Captain McKendrick has been embodied in this paper. Being responsible for the wording of the paper I have used the pronoun "I" in the text. W. F. H.

sion but rather some one quantitative relation of these facts. The improvement in the condition of the patient expressed in terms of feeling-of-well-being, the check in the spread of disease processes, the extension in the duration of life produced by particular methods of treatment have scarcely at all been taken into account because they are not available in a form suitable for statistical treatment. These data may be grouped under the commonly used term, experience. But experience is largely a personal matter and can only carry limited weight in argument or at most appeal to the comparatively few. Professor K. Pearson has shown how the degree of association of non-quantitative characters can be worked out. It is incumbent upon us therefore as medical men to provide material for the calculation of coefficients of contingency, as they are called, as well as for coefficients of correlation in future considerations of efficacy of treatment and the indications afforded by special tests.

Another argument which enters implicitly or explicitly into medical dissertation is the argument from analogy. Logicians warn us against the pitfalls of such a method, and yet it is, when discreetly used, a most valuable adjunct to evidence. Thus the prophylactic value of vaccination against smallpox having been established it seems reasonable to suppose that a similar method would be capable of giving some result in other diseases such as rabies, anthrax, cholera, and typhoid fever. If the evidence for the therapeutic value of inoculation in cases of furunculosis, erysipelas and acne is accepted we may reason further to the trial of specific vaccines in the treatment of other affections. If improvement in the condition of a patient and the development of a feeling of well-being usually synchronise with a rise of the opsonic power of his serum we may—until it is disproved—infer that there is a causal connection between them. Finally if over a wide range of diseases, tuberculosis, staphylococcal, streptococcal, pneumococcal, and gonococcal diseases, glanders, dysentery, typhoid and Malta fevers and others there would seem to be accumulating evidence in favour of the arrival of a new era of treatment we may reasonably anticipate that these hopes will ultimately prove, if properly recorded, to be based on statistically significant foundations. But we must discard the fetish of experience, and set to work at the expense of some labour to record the essence of that experience so that all the world may judge of its real value. The work requires co-operation, and the realisation of this fact may speed the advance of exact measurement in the domain of medicine.

In the following treatment of the subject-matter of this paper I have endeavoured to indicate some of the aspects from which this recently much criticised quantitative measurement may be viewed rather than to deal with it exhaustively.

The phagocytic count affords a peculiarly suitable case for figure analysis. We have here a typical frequency distribution, that is to say, the numerical degree to which a given character is possessed. This is most conveniently shown by stating the actual number of individuals, in a sufficiently large and unselected sample, that possess the different gradations of the character in question. The individuals

are the leucocytes and the character the amount of phagocytosis. We set this forth by stating the number of leucocytes in a count say of 100 which possess the varying numbers of bacteria.

Number of leucocytes	12	16	22	17	11	6	5	3	3	2	1	1	1	Totals 100
Number of contained bacteria	0	1	2	3	4	5	6	7	8	9	10	11	12	311

The number of leucocytes which contain no bacteria is 12; those containing one bacterium are sixteen, and so on. If now we plot in the usual way number of leucocytes against number of contained bacteria we get, when we join up the tops of all the ordinates, what is called a frequency polygon.

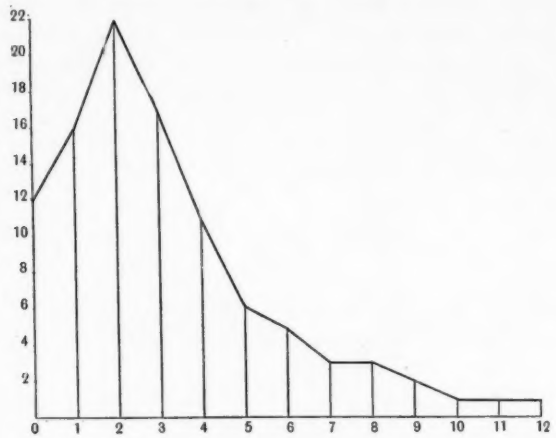


Fig. 1. Frequency Polygon.

The way in which the numbers gradually increase and then gradually decrease suggests that we might be able to replace the polygon with a mathematical curve expressing what is the degree of phagocytosis under the conditions of experiment. But if we counted a second, a third and further hundreds from our slide, we should find that the frequency distribution was never twice the same. We might then be in a position to realise to what extent our method was only a rough approximation to the true state of affairs. We might take very naturally the usual form of average—the Arithmetic Mean—and say we shall deal only with averages of distributions and so get over the difficulty of this variation in our data. But we find that in so doing we have exactly the same difficulty to contend with; the averages, as determined from successive samples taken from the same material, vary just like the distributions. And yet we cannot help feeling that a definite

law of some kind governs the degree of phagocytosis. If we appreciate the point that a deficiency in the number of leucocytes containing any given number of bacteria is balanced by an equal excess distributed amongst the numbers of leucocytes containing different numbers of bacteria and assume that all ordinates representing these magnitudes are interdependent owing to their being governed by some law, we can see how the help of the mathematician becomes indispensable: he will determine, if supplied with sufficiently large numbers to avoid gross error, the smooth curve which will most closely fit the totality of observations. We may then be willing to concede that such a curve will give us a near approach to the condition of things as they ought to be, but which are not so owing to experimental error and to smallness of data.

It is necessary in the first instance to afford the mathematician sufficient data by which to construct his curve but once constructed it may become of the utmost utility as a test of the degree in which we are approximating to the ideal by the method which considerations of convenience lead us to adopt. The same sort of standard of comparison might be set up by the bacteriologist himself if he took the pains to collect very large data. The larger the number of leucocytes counted for example, the fewer would be the irregularities, and finally, when the numbers were very large indeed, we may imagine that our frequency polygon would be identical with the mathematician's curve. But the labour of such a task would be very great, and if it had to be done in many cases would take up too much of the observer's already too short life. It would be folly then to refuse the shorthand expressions of the mathematician in the shape of curve or formula, or his statements as to degree of error, simply because he has deduced these from the data supplied to him on the supposition of the operation of a law governing the varying magnitudes of the observations. In many cases it is quite unnecessary to undertake the labour of fitting a curve, since a single measure like the Arithmetic Mean serves to give all the information which is really necessary. But we must at least know what are the limits of error of our average, and this is almost totally neglected by medical observers.

To return to the example given, we should naturally, in seeking to give expression briefly to what is exhibited here in a somewhat lengthy form, take the Arithmetic Mean of the whole distribution. This is 3·11. In words the average number of bacteria taken up per leucocyte is 3·11. Obviously no leucocytes take up this number of bacteria, but long familiarity with such a measure has made all men acquainted with what is meant by the expression. No leucocyte contains this number nor ever will contain this number; nor can even the majority of the leucocytes be said to contain approximately this number. The Arithmetic Mean then in this case does not correspond to the character possessed by the majority. This is due in reality to the want of symmetry of the distribution. This asymmetry or skewness is typical of all phagocytic distributions which have been as yet investigated from this standpoint, and is itself characteristic of the particular law which governs these distributions. It has an important bearing on the claim

of the Arithmetic Mean to be called the best descriptive average. Another measure of the phagocytic frequency distribution has been proposed, namely the point corresponding to the maximum ordinate or mode. This is in many ways much to be preferred to the average as a descriptive constant for a skew distribution. For one thing it has a more obvious physical meaning, as it represents the state of affairs which most frequently occurs, i.e. is most probable. Moreover the mode is not affected to nearly the same extent as the average by extreme numbers, which may in some cases be due to extraneous causes. The mode is, however, much more difficult to evaluate, and the ultimate test of its utility in ordinary practice will be the ease of its determination. Nevertheless it may be questioned whether, if we are going to search for the single numerical quantity which will best characterise the phagocytic count, either the mean or the mode is the best for the purpose. If the variability of the distribution, the extent to which it ranges and the frequency with which deviations from the mean or mode occur are the essential points of difference between a normal serum effect and an immune serum effect, then it is certain that neither the mean nor the mode will be the best single measure available. We may look into the point later on but it is one which would have to be settled by the statistician.

The stage then which we have reached in this consideration is this, that the Arithmetic Mean of the phagocytic frequency distribution is the most natural if not the most accurate single expression for the totality of our count. The statement, for instance, that the average number of bacteria per leucocyte is 3·11 is no more true than that the Scottish express travels between London and Edinburgh at 50 miles an hour: in fact it is less true, if only for the reason that the express does at times in the course of every journey travel at this speed. At the same time the average does express something which we can perfectly well understand. It has the great convenience of being readily and rapidly evaluated, and also obviates in practice the necessity for deciding, when two or more leucocytes are in juxtaposition, whether a bacterium situated on a boundary is to be placed in the one or the other.

But it may be objected that the opsonic index is not simply the average of the phagocytic count. Such an average varies, as we know, with thickness of bacterial emulsion, age of serum, length of incubation and so on. It was with the idea of getting rid of these variations that Wright and Douglas devised the opsonic index as a useful gauge of degree of phagocytic activity. The index is defined as the ratio of the averages of the counts for test and normal sera, and is a number intended to show how much more, or, as the case may be, less active a test serum is than a standard normal serum. Normal serum activity—supposed to be more or less constant—supplies the unit, and test serum activity is given in terms of this unit. But apart from the possibility of variations in normal sera the unit is variable owing to the impossibility of standardising the bacterial emulsions used in the test. This difficulty is got over by making a fresh evaluation of the unit for each separate group of determinations. The index is, however, at bottom

what it is defined to be, the ratio of two averages, and it is with the value of the particular average used, as a measure of the phagocytic distribution, that we are chiefly concerned. Whether, with varying concentrations of emulsion, longer or shorter incubation, the ratio of the averages of normal and test serum distributions remains the same is quite a separate question and involves the discussion of the comparability of indices estimated under different conditions and at different times. But if we were merely to limit ourselves to an opinion as to whether a given serum is subopsonic, normal or hyperopsonic, we must still know whether it is sufficient for this purpose to compare averages based upon a limited count and what that count ought to be.

We distinguish therefore two questions—in reality they merge into one another:—(1) the question as to whether the average of the phagocytic count as ordinarily carried out affords a characteristic which is practically sufficient to differentiate blood sera; (2) whether, if the average of a test serum is in every case expressed by a separate determination in terms of the average of a normal serum taken under the same conditions, we shall actually get a number which is not affected by peculiarities of the observer or other sources of variation. Question (1) in the first place requires an answer either in the affirmative or the negative, and in the second place necessitates a statement of the approximate limits within which that differentiation is justifiable. Now no one who has worked much with opsonic indices should have much hesitation in saying that they *will* serve to differentiate a highly specific serum from a normal one. The results which we obtain in the case of the glanders bacillus, staphylococci and streptococci, the Micrococcus melitensis, and the organisms of typhoid and Malta fevers are all so striking as to leave no shadow of doubt upon this simple point. It becomes more difficult, however, to dogmatise on the minimum limits within which we may accept difference of index as equivalent to significant differentiation. This is a point on which mathematician and observer might well work hand in hand to reach a conclusion. It will be surmised by workers on the subject that a different answer might have to be given to this question in the case of different bacteria. Such differences—as far as the specific bacterium alone is concerned—may well be connected with the capability of the bacillus to act as an antigen or its capability of being opsonised. The susceptibility of one of the reacting substances (the opsonisable substance) to the action of the other (the opsonin) is, I suspect, one of the elements which enter into the differentiation of phagocytosing leucocytes as regards the number of bacteria ingested by them. If the organism whether through age, quality of culture medium, virulence or some such cause is hard to opsonise, then weakly opsonic sera will not give that differentiation from a normal serum which they would do were the antigen somewhat more susceptible to the influence of the antibody. Dr Greenwood in an admirable paper on this subject has given us some idea of the limits of reliability for differentiation of the index in the case of the tubercle bacillus for counts of 25 leucocytes. This reliability lay outside the range 0·85 to 1·3. It still remains to examine in similar

fashion what the reliability is for counts of 100 instead of 25. The case chosen for illustration, however, is one which bears most hardly upon this question of the reliability of the opsonic index. The tubercle powder used for making the emulsions invariably consists of old, dead and dried bacilli. It is scarcely to be expected that the reactibility (if I may use the term) of the organism under these circumstances would stand very high. The degree of variability therefore which one could hope to get with such a reagent would not be very extended, and this is borne out by experience. The records of the Inoculation Department of St Mary's Hospital, London, now afford a very considerable body of data which would I think well repay the analysis of some kindly mathematician. The records concerning the tuberculo-opsonic index are particularly large, and although by taking this index as the basis of our argument we state the case for reliability in the most depreciatory manner, still its supreme importance requires that it should be considered before all others. The rule at St Mary's Hospital is to take from different persons in the laboratory several normal sera against which to compare the test sera for the day. We have therefore here material for testing variability of normal and test serum indices. Suppose we have, corresponding to any given batch of test sera, three normal sera 1, 2, and 3, then the average of any one of these normal sera distributions may be supposed equally valid as supplying the normal unit for the estimation of the index. Taking the three normal sera by themselves we might calculate a set of "normal" serum indices by dividing each normal average by the other, that is to say 1 by 2, 1 by 3, 2 by 1, 3 by 1, 2 by 3 and 3 by 2. In the same way we might obtain "test" serum indices by dividing the average for each "test" serum by the averages of the several normal sera in turn. By doing this systematically in every case and collating results we should obtain two frequency distributions, and be in a position to state what is the degree of variability amongst normal and test sera, at least as tested by means of an emulsion of old and dried tubercle bacillus. Further if we assume that degree of variability is the basis of the differentiation of an opsonic serum from a normal serum then the degree of variability of the corresponding averages or ratios of averages may give us some idea at any rate of the limits within which their differentiation becomes justifiable. The objection to the procedure is: that we introduce into the "normal" index frequency distribution both indices, and their reciprocals. It would probably be better then to use some other method which would not have this effect. It is essential too, in selecting any one normal average for use in the determination either of normal or of test indices, that it should be altogether a random selection. The plan adopted to obviate the first difficulty and to comply with the requirements of random selection was to take only one normal average as divisor. Amongst the two or three normal averages from which I had to select in any given batch of observations, if I took the first recorded in the one case, then I took the second in the next and the third in the next, or if there was no third went back to the first. This method was followed for both "normal" index and "test" index distributions. In order to compare the variabilities in question, one forms the frequency distributions and then the frequency

curves corresponding to normal and to test indices. The individual indices were based on counts of 100 leucocytes. The distributions were as follows:—

Tuberculo-Opsonic Indices. Normal Sera 30. 11. 08 *to* 6. 3. 09.

Index	Frequency	Index	Frequency	Limits of Grouping
·70	1	1·00	74	·70=·695 to ·715
·72	0	1·02	56	and so on.
·74	0	1·04	61	
·76	1	1·06	39	
·78	2	1 08	37	
·80	4	1·10	24	
·82	2	1·12	18	
·84	12	1·14	11	
·86	12	1·16	7	
·88	16	1·18	5	
·90	20	1·20	3	
·92	48	1·22	1	
·94	43	1·24	1	
·96	49			
·98	63			
		Total	610	

Tuberculo-Opsonic Indices. Test Sera 30. 11. 08 *to* 6. 3. 09.

Index	Frequency	Index	Frequency	Index	Frequency	Limits
·28	1	·76	12	1·24	15	·28=·275 to ·295
·30	0	·78	12	1·26	15	
·32	0	·80	18	1·28	11	
·34	0	·82	29	1·30	9	
·36	0	·84	28	1·32	5	
·38	1	·86	47	1·34	3	
·40	0	·88	36	1·36	6	
·42	1	·90	47	1·38	6	
·44	1	·92	49	1·40	4	
·46	0	·94	47	1·42	4	
·48	0	·96	52	1·44	1	
·50	0	·98	51	1·46	0	
·52	1	1·00	57	1·48	1	
·54	0	1·02	50	1·50	3	
·56	0	1·04	43	1·52	1	
·58	2	1·06	40	1·54	1	
·60	3	1·08	52	1·56	2	
·62	7	1·10	38	1·58	3	
·64	6	1·12	36	1·60	1	
·66	7	1·14	20	1·62	2	
·68	8	1·16	24	1·64	0	
·70	5	1·18	16	1·66	0	
·72	9	1·20	20	1·68	1	
·74	15	1·22	15			
				Total	1000	

These are the distributions as they were worked out. By using a coarser grouping we obtain a more regular and more workable series as given in the following tables:—

Normal Serum.

Index	Frequency	Index	Frequency	Limits
·70	1	1·06	137	·70 = ·695 to ·755
·76	3	1·12	53	
·82	18	1·18	15	
·88	48	1·24	2	
·94	140			
1·00	193			
		Total	610	

Test Serum.

Index	Frequency	Index	Frequency	Limits
·28	1	1·00	158	·28 = ·275 to ·335
·34	0	1·06	135	
·40	2	1·12	94	
·46	1	1·18	60	
·52	1	1·24	45	
·58	5	1·30	25	
·64	20	1·36	15	
·70	22	1·42	9	
·76	39	1·48	4	
·82	75	1·54	4	
·88	130	1·60	6	
·94	148	1·66	1	
		Total	1000	

We may notice now some of the points of difference between the two distributions. In the normal case the range of indices is between ·70 and 1·24, whereas for test sera the range is much greater, being between ·28 and 1·66. But whilst we may be prepared to admit that such indices as ·70 and 1·24 are quite improbable for normal sera, we still want to know the limits within which we are to diagnose a serum normal or abnormal. This point can to some extent be answered from the figures before us without further treatment. Thus, confining our attention to the data for normal sera alone, we note that we may expect roughly 96 to 97 °/₀ of normal sera to afford indices ranging within the limits ·89 and 1·19. So

also we observe, on the same basis, the following expectations regarding the normality of more limited ranges:

Limits	
·83 to ·89	2·95 %
·89 to ·95	7·87 %
·95 to 1·01	22·95 %
1·01 to 1·07	31·64 %
1·07 to 1·13	22·46 %
1·13 to 1·19	8·69 %
Total	96·56 %

Now how do these percentages compare with those found amongst test sera? If our "test" sera indices had all referred to the first examinations of patients, we should then have been in a position to say how frequently any given index occurred with the serum obtained from an individual suspected of being tubercular. With such data we should also have been able to state definitely what were the probabilities for and against a given index being normal or abnormal. This point has been treated of by Noon and Fleming*. The figures given above are unfortunately not those of patients examined only for the first time. They are simply the indices of patients examined from day to day, irrespective of whether they had presented themselves for the first time, were under inoculation treatment, or had received some such special treatment as massage, exercise, Bier's bandage and so on, in order to bring out more distinctly the normality or abnormality of their condition. Indeed a large proportion must have been under treatment at the time of the examinations here recorded. We should therefore expect that a great number of those who were originally subnormal were being maintained at normal by the inoculations, whereas, owing to the well recognised difficulty of maintaining a hypernormal index in tuberculosis, those amongst the originally normal who passed over into the category of the hypernormal were not so numerous. Still the figures as they stand give us a considerable amount of information. They show the much greater extent to which test sera vary as compared with normal sera, and how in spite of the selection implied by the inclusion of an excess of normal individuals there still remains a much greater deviation of indices from the mean than with normal sera. A comparison of the two distributions will also give us some idea of the extreme limits within which a serum can be called normal. It will show us too the relative frequency with which given indices occur in normal and abnormal cases—always remembering that in this comparison the proportions, for the reason given, are likely to be overlaid with odds in favour of normality. With this qualification we set down the figures.

* *Lancet*, April 25, 1908.

Limits	Normal Sera	Test Sera
·83 to ·89	2·95 %	7·5 %
·89 to ·95	7·87 %	13·0 %
·95 to 1·01	22·95 %	14·8 %
1·01 to 1·07	31·64 %	15·8 %
1·07 to 1·13	22·46 %	13·5 %
1·13 to 1·19	8·69 %	9·4 %
Totals	96·56 %	74·0 %

But the best comparison between the distributions is only made after the roughnesses associated with each individual observation are allowed for, that is to say, after we have determined the smooth curve which shall represent the distributions with a minimum of error.

Let I denote the normal serum distribution and II the test serum distribution; then the following are the equations and constants of the curves corresponding to them. Both curves come under the head of Pearson's Type IV, namely

$$y = y_0 \left(1 + \frac{x^2}{a^2}\right)^{-m} e^{-\nu \tan^{-1}\frac{x}{a}}.$$

I. $$y = 167{\cdot}71 \left(1 + \frac{x^2}{(5{\cdot}4950)^2}\right)^{-10{\cdot}5796} e^{2{\cdot}5281 \tan^{-1}\frac{x}{5{\cdot}4950}}.$$

Standard Deviation = 1·30, Mean = 1·0039,
Mode = 1·0080.

II. $$y = 131{\cdot}71 \left(1 + \frac{x^2}{(7{\cdot}9997)^2}\right)^{-5{\cdot}3714} e^{1{\cdot}6666 \tan^{-1}\frac{x}{7{\cdot}9997}}.$$

Standard Deviation = 2·93, Mean = 1·0048,
Mode = 0·9878.

Curve I is represented by 610 observations while Curve II is based on 1000; therefore in order to compare them we must proportionately magnify the first to give an equal area with the second. Better still we may reduce Curve I so that the highest point upon it shall just touch the point corresponding to it in Curve II. The reduction is shown in Fig. 2. Then the area of Curve I as shown now in this reduced form will represent the degree of occurrence of as many indices out of the test sera indices as can possibly be considered normal, i.e. this area within the larger area represents the maximum computation of normal serum indices. The method of reduction adopted gives a somewhat undue preponderance to the normals over and above that already given. We must keep this point in mind in drawing any deductions from the curves. It is taken for granted here that the curve of normal sera indices represents a true random sample.

We may now examine our curves more particularly. Take the index 0·92 as shown on the base line belonging to the two curves of Fig. 2 and we observe that the ordinate corresponding to this point would be nearly bisected by the

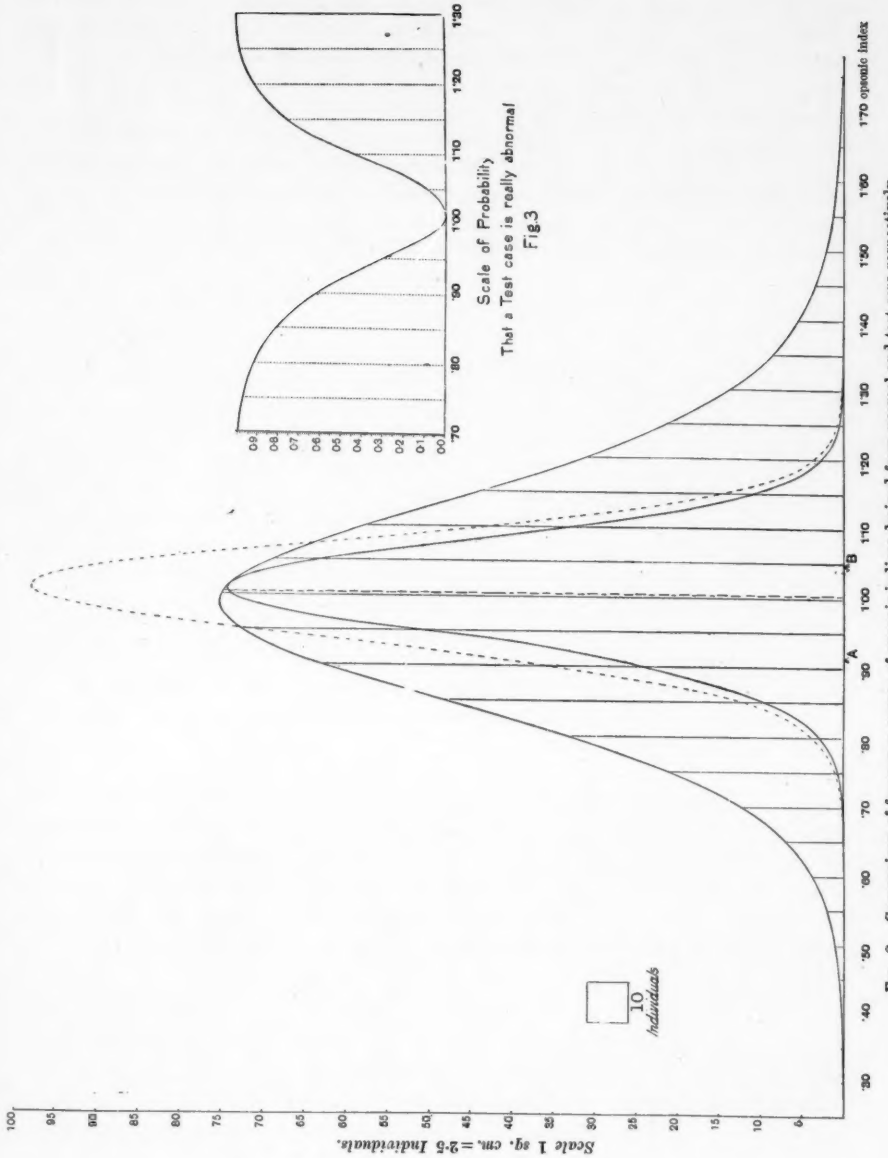


FIG. 2. Comparison of frequency curves of opsonic indices derived from normal and test sera respectively.

A and *B* are the coordinate origins of the equations to the curves.

inner of the two curves. This signifies that for this index the odds in favour of abnormality are not more than those in favour of normality, i.e. the odds are even. In the case of an index unity the probabilities that we are dealing with an abnormal index are seen to be very small, while for indices of ·7 or 1·3 they are very considerable. Fig. 3 shows to scale the actual probabilities for all indices ranging between ·7 and 1·3. In the neighbourhood of the index 1 we see that probability is practically zero. As we proceed outwards in either direction we find the chances of abnormality are becoming greater and greater. The values are as follows:—

Chances in Favour of Abnormality.

Index	Probability	Index	Probability
·70	·990	*1·05*	·081
·75	·965	*1·10*	·428
·80	·914	*1·15*	·750
·85	·801	*1·20*	·907
·90	·614	*1·25*	·976
·95	·303	*1·30*	·991
1·00	·018		

Within the limits ·8 and 1·2 the probability curve slopes rapidly to zero. If then we postulate that for a definite statement on a single examination a probability of at least ·9 is required—and this signifies that our statement would at the worst be wrong once in ten times—we fix our limits of normality at approximately ·8 to 1·2. Outside these we can be reasonably certain that we are dealing with abnormal sera.

These being the facts and the analysis of the facts, it seems reasonable to conclude that even in the admittedly difficult case of the tubercle bacillus we have a means of differentiating within certain limits a hyperopsonic or subopsonic serum from a normal one.

Before proceeding to consider our second point as to the comparability of indices, we may look at the question of simple differentiation from another point of view. We shall leave aside for a moment the index altogether, and fall back upon the frequency distributions from which the averages which enter into the index are calculated. Take the following tuberculo-opsonic distributions, the names attached to which sufficiently explain them.

Number of Bacteria ingested	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Mean	Index
Number of leucocytes:											
1. Normal Serum	10	29	28	18	9	2	2	1	1	2·12 ± ·08	—
2. Test Serum I	12	16	24	30	8	6	1	1	2	2·45 ± ·11	1·16
3. Test Serum II	14	18	26	18	9	6	6	2	1	2·48 ± ·13	1·17

Standard Deviation (S. D.) Normal = 1·53.
" " I = 1·65.
" " II = 1·85.

We may ask ourselves what are the probabilities that the distributions corresponding to Test Serum I and Test Serum II are significantly different from that of the Normal Serum here used and regarded as a type? Let us understand thoroughly what is meant by the use of the expression "significantly different." Had we made a large number of counts of 100 for this normal serum, we should have found that no two of them would be alike. If then we counted at random a large number of such 100's, what are the probabilities that in the long run we should get a distribution like that given in the case of Serum I or Serum II? Put in another form the problem is:—How many successive 100's should we on the average expect to have to count from the slide representing normal serum before we should get a distribution like either of those given by the test sera. Now for distributions like the above, if they contain a sufficiently large number of observations, and still better if they have had an ideal frequency curve fitted to them, the probabilities for and against the chance here referred to can be calculated. In the case we are considering the numbers are much too small for an accurate determination, but they will serve to give us a rough idea of the order of the significance or absence of significance of the differences exhibited by the distributions. Test Serum I works out as significantly different ($P = \cdot 002$) from the normal serum here utilised in 998 cases out of 1000, and Test Serum II ($P = \cdot 005$) in 995 cases out of 1000; in other words for this case it would be safe to the extent named to say that we are dealing with sera producing a different degree of phagocytosis from a normal serum. Let me repeat that owing to the small number of observations the figures are only rough approximations to the order of the chances for and against. Further be it noticed that I limit my comparison to this particular normal serum assumed to be a type, as it would not be safe to state the matter any more strongly without a reference to the degree of variability of normal sera. Now as all work on the subject goes to show that opsonic activity is a specific character, our finding may be said to amount to this: that a diagnosis that the serum of these patients was associated with the presence of tubercular disease or the subjection to treatment with a tubercular vaccine would be correct in some 99 cases out of 100.

I have adverted to the point that the selection of the tuberculo-opsonic index as a test case for the reliability of the inferences which are based upon its estimation, was scarcely a fair one, if the point were generalised to the extent of saying that the adverse findings for this particular index applied to the indices obtained with other bacteria. Take the case of the bacillus of glanders, of which—through the kindness of Dr Fleming—I can afford an instance which was only one of similar results continuing in the particular case for week after week. Indeed, the diagnosis of glanders was based upon the results of opsonic index determinations many months before the actual isolation of the organism set all doubts at rest—if there could be any possible doubt in such a case. At the time of taking the serum, which gave the following phagocytic result, the patient was receiving a glanders vaccine.

Number of Bacteria	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
No. of Leucocytes:																			
1. Normal Serum	13	13	12	10	15	13	10	3	1	3	1	—	—	—	—	—	—	—	—
2. Test Serum	3	5	8	5	9	10	16	10	10	4	5	3	1	5	3	1	1	1	1

Mean 1 = 3·36 ± ·16. S. D. 1 = 2·36.
Mean 2 = 6·66 ± ·27. S. D. 2 = 3·96. Index = 2·00.

We note here the much larger standard deviation which we get in the case of the glanders bacillus as compared with the tubercle bacillus, both as regards normal and test sera. In this case, although the bacillus is used killed, it is not old nor dried. The divergence in this instance between test and normal sera, as judged by standard deviations, is much greater than in the instances chosen for the tubercle bacillus. As regards the order of the probabilities in favour of significant difference between the distributions, we are unable here to consider the case of the test serum distribution arising as a random sample from the normal serum distribution, because the normal serum range for the small numbers here recorded does not extend so far as that of the test serum. We can, however, reverse the problem and ask what are the probabilities against the test serum giving a distribution like that of the normal serum. When we do so, we find that the probability against such being the case is so enormous that the tables provided for the purposes of its evaluation (*Biometrika*, Vol. I. p. 159) do not go far enough to enable us to obtain it. All we can say is that the chances against our normal serum representing merely one random sample of the test serum, are something greater than 999,999 to 1.

If these probabilities give us an indication of the degree of correctness and incorrectness of our inferences regarding opsonic activity, and if we can regard them as representing those for correctness of diagnosis, we should be in a position to compare them with our impressions of the degree of correctness of the usual clinical diagnosis in which some element of doubt exists. Let us take a concrete example. It would, I think, be satisfactory to know that our expressed opinion as to whether a chronically swollen joint represented a gonorrheal or a tubercular infection would be wrong only once in 200, or once in 100, or once in 50, nay even once in 10 times. Now these are just the sort of cases in which an estimation of an opsonic index or, as the case may be, a phagocytic distribution gives us the greatest help.

There are also confirmatory tests which may be, and in fact usually are, applied. One of these is the method of multiple testing. When an important diagnosis has to be made, it is seldom given on merely a single index. The index is determined on several occasions, and if, as usually happens, the indices show characteristic divergences from the normal, the evidence is considered as greatly strengthened for a positive statement. Or again, special methods are applied in order to

accentuate the differences from normal or the fluctuations from an original value, due to the presence of infection. The affected part is massaged, or a Bier's bandage put on above a joint for half an hour, so as to flush it with the retarded blood. If after this procedure the opsonic index shows the fluctuations which might be expected from the entrance of either antibodies or antigens into the blood stream, the diagnosis becomes clear. The following examples will illustrate this point:

(1)

April 8, 1908	Index	
	To Gonococcus	To Tubercle Bacillus
Before Bier	1·40	1·06
½ hour after	1·24	·89
5 hours after ...	1·36	·90
20 hours after ...	1·84	1·00

(2) *Effect of Massage. B. tuberculosis.*

Before	·87
½ hour after	·95
3 hours after	·97
21 hours after	1·19.

(3) *Effect of Exercise. B. tuberculosis*, 25. 5. 08.

Before	·87
Just after	·97
7 hours after	1·12
19 hours after	1·15.

One of the defects of these illustrations is that we are ignorant of the effects produced on the variability of normal serum indices by, say, the application of a Bier's bandage to a normal joint, massage or exercise. The point is one, however, which could be easily settled.

We proceed now to consider how far opsonic indices are comparable with one another. This is a much more difficult question to answer offhand. It would seem certain that if they are, it can only be under some strict limitations of technique. We are dealing in this determination with a number of varying factors. Some of the chief possibilities of variation and problems are:—

(1) *The Leucocyte.* Number in unit volume; degree of admixture of red blood corpuscles; variation in vitality; degree of settling.

(2) *The Bacterium.* Number in unit volume, i.e. the concentration of the emulsion; virulence; whether laboratory culture or autogenous; degree of agglutination by the sera used; age of culture; dead or living; whether diplobacteria or larger aggregations are to be regarded as single ingestions or more.

(3) *The Serum.* Whether, with the quantities of serum emulsion and leucocytes used, we have, so to speak, the opsonin and antigen present in exact combining

proportions; whether the serum agglutinates the red blood corpuscles; age of serum; length of time of action; variability of normal sera, either of one and the same individual at different times, or of different individuals at the same time; heated or unheated.

The factors here enumerated by no means exhaust the list which might be made. Nor are they in any sense classified; many are not mutually exclusive classes. Moreover, as it is the comparability of indices that we are now considering, we postulate throughout equality of conditions for individual observers. Thus, when we say that age of serum may be a factor affecting the opsonic index, it is not intended by this to raise the question as to whether an old serum might be compared with a fresh one—that, we know, it cannot be. But if we have test serum and normal serum taken at precisely the same time, will the index remain the same if estimated at once, and at varying intervals after clotting of the bloods? Or take the case of length of time of action of the serum in the opsonic mixture. Is it all the same whether we give the mixture 15, 20 or 30 minutes in the opsoniser? Will the ratio of the averages at the different periods remain the same? Might we not get certain of the leucocytes more rapidly surfeited under the action of one serum than the other and so put out of action? In such a case additional time for ingestion would not be of any use to these leucocytes. A great number of such questions arise in the mind of every observer, and it is certain that they cannot be answered without set experiment. A most valuable paper which deals with this side of the subject is that by Dr A. Fleming in *The Practitioner* for May, 1908. But there are many problems still to be solved before the answer can be given as to the limits within which indices are comparable. We may, however, illustrate a few of the points, which have been raised by actual examples.

Example 1. What is the bearing of concentration of emulsion upon degree of phagocytosis? The organism here used was the *Bacillus coli* and the emulsion a very thick one. The emulsion was diluted down in varying degrees, and tested at each dilution against the same serum and the same leucocytes with the following result (Table, p. 81).

The result is a curious one. We see that the frequency distribution for a concentrated emulsion exhibits a smaller variation than that given by the same emulsion when considerably diluted—the standard deviations being as 1 to 2 for a tenfold dilution. Nor was there any agglutination of the bacteria obvious enough to account for this result. The probability is that we were dealing here with a toxic effect—the so-called *aggressin* effect. Again, when we reach the very high dilutions, we notice that there is no significant difference between the distributions given by the 80- and the 100-fold dilutions. Perhaps at this point we have reached a degree of separation of leucocytes and bacteria such that the opportunity for phagocytosis is so small that no difference can be brought out between different dilutions in the time allowed for action. It would be interesting

to have a graded series of frequency distributions showing the transition from the inhibitory action of toxins to the maximum development of phagocytosis. Again, it would be interesting to observe the gradual shrinking of the range of phagocytosis with thinning of the emulsion. We may learn from this example that there is probably an optimum concentration for the production of maximum phagocytosis. Doubtless the same phenomenon would be shown by immune sera and by some other organisms than the one here tested. The question immediately arises as to whether this transition would take place at the same point for both an immune and a normal serum. If it did not, we should certainly not get indices which were the same for different degrees of concentration of emulsion, although the normal and immune sera were the same throughout.

Number of Bacteria Ingested	Distribution of Leucocytes with						
	Original Emulsion	5 fold dilution	10 fold dilution	20 fold dilution	40 fold dilution	80 fold dilution	100 fold dilution
0	317	40	43	112	192	632	620
1	345	92	101	180	278	282	282
2	172	112	137	184	221	65	79
3	82	142	139	151	139	16	16
4	47	121	130	128	91	4	2
5	20	99	115	96	49	1	1
6	11	100	111	63	21	—	—
7	4	86	74	44	4	—	—
8	1	64	55	22	4	—	—
9	0	45	32	10	1	—	—
10	1	41	24	4	—	—	—
11	—	17	12	3	—	—	—
12	—	11	9	3	—	—	—
13	—	9	7	—	—	—	—
14	—	5	6	—	—	—	—
15	—	5	2	—	—	—	—
16	—	2	1	—	—	—	—
17	—	2	1	—	—	—	—
18	—	3	0	—	—	—	—
19	—	2	1	—	—	—	—
20	—	2	—	—	—	—	—
Means	1·34	5·03	4·54	3·05	1·94	0·48	0·50
Standard Deviations	1·41	3·38	2·98	2·29	1·62	0·74	0·74

Example 2. The effect of extracellular agglutination of the bacteria of the emulsion by the test serum. The organism used was the *Bacillus Friedlander* and the emulsion was again a concentrated one. The index given by the test serum as determined in the ordinary routine work of the laboratory with a thin emulsion was 2·17. What a contrast to this is presented by the index obtained with a very thick emulsion—viz. 0·75! The determination, it is true, was made on the following day, but age of serum will not account for this amount of lowering of the index.

Number of Bacteria Ingested	Distribution of Leucocytes with	
	Normal Serum	Immune Serum
0	20	41
1	26	52
2	36	57
3	45	42
4	42	30
5	39	24
6	29	20
7	22	8
8	13	3
9	10	7
10	9	2
11	2	6
12	2	3
13	1	1
14	2	1
15	0	0
16	1	1
17	0	1
18	1	0
19	—	0
20	—	0
21	—	0
22	—	1
Means	4·43 ± ·07	3·33 ± ·12
Standard Deviations	2·98	3·18

Opsonic Index 0·75.

In all probability the further dilution of the sera would bring out the real differences between them. This was done at the time but the slides have not yet been counted. The example serves to show how a strongly opsonic serum may yet under certain conditions give an index actually below normal. The reason for this paradox was that strong agglutination of the bacteria took place with the test serum, whereas no visible agglutination took place with the normal serum. The resulting effect was, we may suppose, that for the test serum the agglutination so reduced the concentration of the emulsion as to diminish the phagocytosis below that which took place for a normal serum and a concentrated emulsion. Bacteriolytic action may have contributed to the diminution in the case of the immune serum. With both immune and normal sera a considerable amount of intracellular digestion had taken place. Now although agglutination is often rapid with concentrated emulsions of bacteria, it is much slower for thin emulsions. Opsonic action, on the other hand, is still very rapid even with thin emulsions. With dilution of the emulsion the agglutinating action would be slower and the opsonic action would proceed without this disturbing factor. The same argument would in all likelihood apply to a dilution of a strongly agglutinating serum, even

when the emulsion remained concentrated. It is frequently stated that a highly agglutinating serum may contain no opsonin, but before any such statement is accepted we require to know whether it holds good (*a*) on successive dilution of the serum, (*b*) on successive dilution of the emulsion. Our example serves to show that agglutination may play a large part in vitiating the comparability of indices. But agglutination may be slight and test sera may not be so active as the one here used. Therefore it again becomes necessary to determine, for agglutinating sera of all grades, what limitations agglutination imposes on the comparability of indices. It must also be evident that this factor must be a varying one for different laboratories.

Example 3. One of the remarkable features of an opsonic frequency distribution is the variability or degree of dispersion exhibited. Sir Almroth Wright has suggested that this may be due in part at any rate to unequal opportunity for phagocytosis. Leucocytes and red blood corpuscles are heavy as compared with bacteria, and tend rapidly to settle down. Although the capillary tubes are placed horizontally in the opsoniser, there will nevertheless be a certain amount of settling and deposition in layers. Those leucocytes in the lowest layers will have much less opportunity to ingest bacteria than the topmost ones. Consequently we may expect to get every degree of ingestion from those containing no bacteria up to those containing as many as they can take up in the time, and with the particular concentration of emulsion. Now if we could keep the tubes in constant rotary movement, we might expect on the above hypothesis that the degree of variation would be greatly reduced, and this is what does happen: but instead of finding that all the leucocytes, as on the basis of equal opportunity they should, had taken up more or less the same number of bacteria, we find that the diminution in variability has been brought about by an increase in the number of those leucocytes which have taken up few or no bacteria at the expense of those which took up many. Of course, we could scarcely expect to find in any form of experiment that all leucocytes had taken up nearly the same number of bacteria, for leucocytes are themselves living organisms showing, in any one sample, all the stages of development, maturity and decay, correlated with which we should be prepared to find a variable phagocytic activity. Still the diminution of variability of this experiment was probably to be traced to mechanical interference with phagocytosis due to the turning movements. The tubes were not turned continuously, but given a half-twist every 60 seconds. The organism used was the *Streptococcus foecalis* and the emulsion a thick one. The test serum was highly opsonic, giving in the course of routine examination an index of 2·14 on the day previous to this experiment. See Table on p. 84.

Example 4. To test whether it is immaterial if organisms adhering in couplets or triplets should be counted as 2's or 3's or as single organisms in the estimation of an index. The organism taken was the *Streptococcus foecalis*, which, together with the immune serum, was kindly supplied to me by Dr Matthews. This organism makes a fairly good emulsion and the clumps remaining are not greater

Number of Bacteria Ingested	Frequency Distribution of Leucocytes with			
	Normal Serum	Test Serum	Normal Serum with ½ turn every 1′	Test Serum with ½ turn every 1′
0	32	17	72	48
1	36	18	76	49
2	49	33	86	62
3	45	29	64	50
4	71	27	49	48
5	31	34	43	42
6	38	25	32	36
7	34	28	29	35
8	42	22	14	26
9	25	37	12	33
10	22	27	10	16
11	14	20	4	14
12	20	28	3	8
13	7	24	2	8
14	13	23	2	7
15	5	14	0	6
16	6	16	1	5
17	2	9	0	1
18	5	7	1	1
19	1	9	—	3
20	1	9	—	0
21	0	10	—	0
22	1	4	—	0
23	—	3	—	1
24	—	6	—	1
25	—	4	—	—
26	—	5	—	—
27	—	3	—	—
28	—	1	—	—
29	—	0	—	—
30	—	3	—	—
31	—	1	—	—
32	—	0	—	—
33	—	0	—	—
34	—	1	—	—
35	—	1	—	—
36	—	0	—	—
37	—	0	—	—
38	—	0	—	—
39	—	0	—	—
40	—	0	—	—
41	—	1	—	—
42	—	1	—	—
Means	5·96 ± ·13	9·93 ± ·21	3·47 ± ·09	5·22 ± ·13
Standard Deviations	4·28	6·94	3·00	4·21
Opsonic Index	—	1·67	—	1·50

than couplets or triplets. A count was made of 500 leucocytes in both ways for both normal and immune sera. The results were as follows:—

I. The case where every organism ingested was separately counted whether in a clump or isolated.

Number of Bacteria Ingested	Frequency Distribution 'Leucocytes	
	Normal Serum	Test Serum
0	32	17
1	36	18
2	49	33
3	45	29
4	71	27
5	31	34
6	38	25
7	34	28
8	42	22
9	25	37
10	22	27
11	14	20
12	20	28
13	7	24
14	13	23
15	5	14
16	6	16
17	2	9
18	5	7
19	1	9
20	1	9
21	0	10
22	1	4
23	—	3
24	—	6
25	—	4
26	—	5
27	—	3
28	—	1
29	—	0
30	—	3
31	—	1
32	—	0
33	—	0
34	—	1
35	—	1
36	—	0
37	—	0
38	—	0
39	—	0
40	—	0
41	—	1
42	—	1
Means	5·96 ± ·13	9·93 ± ·21
Standard Deviations	4·28	6·94
Opsonic Index	—	1·67

II. The case where couplets and triplets are counted as single ingestions.

Number of Bacteria Ingested	Frequency Distribution Leucocytes	
	Normal Serum	Test Serum
0	32	17
1	57	39
2	87	44
3	70	41
4	52	55
5	57	48
6	53	37
7	33	47
8	21	32
9	16	33
10	7	25
11	5	16
12	3	15
13	5	18
14	1	8
15	0	8
16	1	6
17	—	7
18	—	4
19	—	2
20	—	2
21	—	2
22	—	0
23	—	2
24	—	0
25	—	0
26	—	0
27	—	2
Means	4·10 ± ·09	6·65 ± ·14
Standard Deviations	2·86	4·74
Opsonic Index	—	1·62

So far as the actual index goes there would not appear to be any great difference produced on counting by either method. Probably a better way of determining the point would be to fit frequency curves to each of the distributions and then test for goodness of fit. The one in which the fit was worse—if there were a marked difference—would represent the worse method of counting. I have carried out this test in one case of a tuberculo-opsonic distribution which, though not conclusive even for the organism used, is at least suggestive. The frequency distributions and the corresponding curves are here given:—

Number of Bacteria Ingested	Frequency Distribution of Leucocytes	
	(1) Couplets and triplets counted as 2's and 3's	(2) Couplets and triplets counted as single ingestion
0	194	194
1	244	271
2	228	249
3	150	146
4	97	91
5	60	57
6	38	22
7	20	2
8	3	2
9	3	3
10	3	3
Means	2·20	2·00
Standard Deviations	1·86	1·67

Both distributions come under the head of Pearson's Type I and are given by the equations

$$(1)\quad y = 268{\cdot}7601\left(1+\frac{x}{1{\cdot}086}\right)^{{\cdot}484}\left(1-\frac{x}{14{\cdot}567}\right)^{6{\cdot}491}.$$

$$(2)\quad y = 297{\cdot}574\left(1+\frac{x}{1{\cdot}825}\right)^{1{\cdot}838}\left(1-\frac{x}{193{\cdot}113}\right)^{194{\cdot}484}.$$

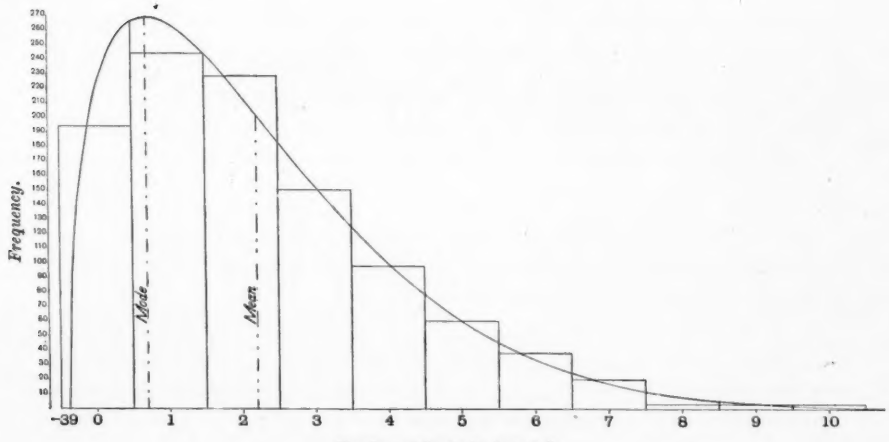


FIG. 4. Couplets and triplets counted as 2's and 3's.

When these curves are tested for goodness of fit to the observations, it is found that this is much better for the case in which couplets and triplets are counted as 2's and 3's than for the case where they are counted as single ingestions, $P = \cdot 51$* as against $P = \cdot 05$. It is quite conceivable that a still better fit would be obtained if we were to regard a couplet for purposes of estimation, say as 1·5 or some other intermediate value between 1 and 2. If such were the case we might conclude that, provided the criterion is sound, this would be the proper value to assign to a couplet inclusion.

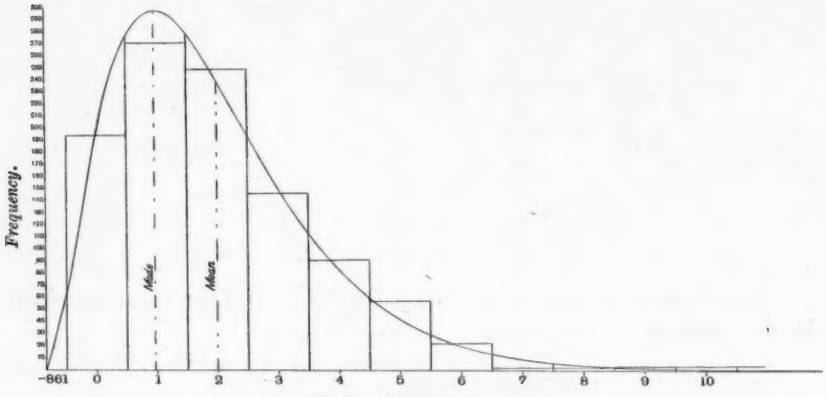


FIG. 5. Couplets and triplets counted as single ingestions.

Other points than those illustrated or mentioned will occur to every worker in these fields. Further, those that are illustrated are single trials, which require repetition both with regard to the particular organism used and to other organisms. They are given here for the purpose of focussing attention on the points which must be considered in discussing the question of the reliability or comparability of the opsonic index as determined by different observers. When, however, such points as the possibility of there being *any* specific opsonic variation demonstrable in tuberculous sera and still more in the case of other sera are disputed, it seems to me that the proof that they are is so overwhelming as to be incontrovertible. All honour then to those who have elaborated so important a method as that here considered; for they have added to the list of quantitative estimations applicable to medical diagnosis and therapeutics one of incalculable value. Its chief drawback is the difficulty of its determination.

It remains now only to consider some of the variations which have been proposed upon the original and very natural measure proposed by Wright and Douglas. Much more criticism seems to have been levelled at the method of measuring the effect of vaccino-therapy than upon the efficacy of vaccino-therapy itself, which seems somewhat of a reversal of their relative importance. One would

* $P = 1$ represents perfect fit.

imagine that those who invented the opsonic index and initiated vaccino-therapy would prefer to have their names more closely associated with the latter than with the former.

Let us take up the discussion, however, on the minor point as to whether an improvement could be made on the method of estimation of the opsonic index. The average is, as we have already stated, but a rough measure of an opsonic frequency distribution. The suggestion has emanated from America that in place of the ratio of averages we should take the ratio of the percentages of leucocytes which have failed to phagocytose at all. The objection which will occur to every one is that this may very likely include a certain number of dead leucocytes. Again, it is a cardinal point in determining the best measure for expressing the features of a frequency distribution that it should be based on the whole number of observations. This the "zero index"—as we may call it—is not. At the same time it must be conceded that the "zero index" is more rapidly determinable than the "average index," and so would allow of the errors associated with its determination being lessened by affording more time to increase the number of observations. It would be specially of advantage where the emulsions used were at all concentrated and where consequently the strain and difficulty of counting the numerous bacteria ingested become very great. It might be of use also in cases where it is not easy to obtain uniform emulsions, and where the leucocytes may contain chains or clumps of bacteria. On the other hand, with concentrated emulsions the number of leucocytes containing no bacteria becomes greatly diminished, and may in fact simply come to represent those which are actually dead. In this case, instead of taking these leucocytes as our gauge of phagocytic power, we might take those containing only one bacterium instead. I have made a small number of observations on the degree of correspondence of the two indices in the case of *B. tuberculosis*. The slides from which the two indices were determined were those which had served for the routine estimations of the laboratory. All the indices were calculated from countings done by myself alone. The distributions of the two were as follows :—

Average Indices	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6	Mean	S. D.
Frequency	0	1	5	4	6	17	17	12	17	13	1	3	2	2	·96 ± ·02	·24

Zero Indices	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6	Mean	S. D.
Frequency	6	5	6	5	10	10	16	10	7	8	5	6	3	3	·91 ± ·02	·35

We note at once a marked difference between the distributions. The "average index" shows a greater concentration about the mean than the "zero index." In fact the zero index shows a greater variability than the other, a result which we should have expected from the fact that the zero index is based on only one observation whereas the other is based on the total number of observations. The

result is that the probable error of the former will be greater than that of the latter. The coefficients of variation are

Zero Index = 38·6,

Average Index = 25·4.

The actual detailed correspondence between the two is given in the following table from which we obtain a correlation coefficient of $r = 0{\cdot}76$.

"Average" Index.

"Zero" Index.	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6	Totals
·3	—	1	3	1	—	1	—	—	—	—	—	—	—	—	6
·4	—	—	1	1	—	1	2	—	—	—	—	—	—	—	5
·5	—	—	—	1	4	—	1	—	—	—	—	—	—	—	6
·6	—	—	—	1	—	3	—	—	1	—	—	—	—	—	5
·7	—	—	1	—	—	2	3	—	2	2	—	—	—	—	10
·8	—	—	—	—	1	4	2	2	—	1	—	—	—	—	10
·9	—	—	—	—	1	4	6	2	3	—	—	—	—	—	16
1·0	—	—	—	—	—	2	1	3	3	—	1	—	—	—	10
1·1	—	—	—	—	—	—	—	4	1	2	—	—	—	—	7
1·2	—	—	—	—	—	—	2	1	3	2	—	—	—	—	8
1·3	—	—	—	—	—	—	—	—	1	4	—	—	—	—	5
1·4	—	—	—	—	—	—	—	—	3	1	—	1	—	1	6
1·5	—	—	—	—	—	—	—	—	—	1	—	1	—	1	3
1·6	—	—	—	—	—	—	—	—	—	—	—	1	2	—	3
Totals	0	1	5	4	6	17	17	12	17	13	1	3	2	2	100

If we designate the "average index" by A and its mean value by $\overline{A}$ and the "zero index" by Z and its mean value by $\overline{Z}$, we have as an equation connecting the two indices regarded as variables

$$A = \overline{A} + {\cdot}53\,(Z - \overline{Z}) \pm {\cdot}01,$$

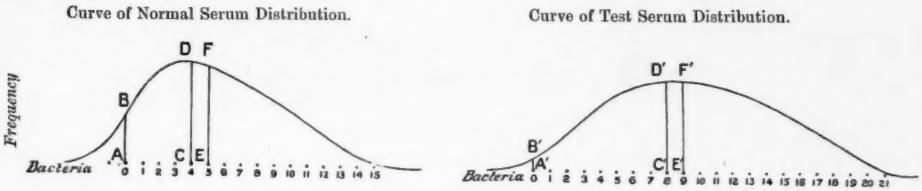
i.e.

$$= {\cdot}96 + {\cdot}53\,(Z - {\cdot}91) \pm {\cdot}01,$$

and the degree of relationship between the one index and the corresponding mean value of the other is shown in the following table:—

Zero Index	Average Index	
	Calculated	Observed
·3	·64	·55
·4	·70	·74
·5	·75	·72
·6	·80	·82
·7	·86	·94
·8	·91	·89
·9	·96	·91
1·0	1·01	1·01
1·1	1·07	1·07
1·2	1·12	1·06
1·3	1·17	1·18
1·4	1·23	1·25
1·5	1·28	1·40
1·6	1·33	1·47

As a matter of fact the zero index probably has its greatest exactness for very thin emulsions, where the number of leucocytes containing no bacteria approximates to the modal ordinate in our frequency curve. This leads me to refer again to a third measure—the mode—which has been proposed as being more descriptive of the phagocytic frequency distribution, and was given by Dr Greenwood (Discussion, Pathological Section, Royal Society of Medicine). There is no doubt that it would have considerable advantages over the arithmetic mean, but it is a difficult value to calculate directly. Further, it would be necessary in every count to set out the frequency distribution, and this is not necessary in the estimation of an arithmetic mean. We may show these different indices in graphic form.



$$\text{Average Index} = \frac{A'E'}{AE}.$$

$$\text{Zero Index} = \frac{AB}{A'B'}.$$

$$\text{Modal Index} = \frac{A'C'}{AC}.$$

If, however, we are to search for some one constant more expressive of a frequency curve than the mean it seems to me that even more characteristic as a measure of dispersion than the mode would be the standard deviation. Our index would then consist of the ratio of the standard deviations. We do not know exactly why all leucocytes do not take up very nearly the same number of bacteria. But the fact is that they do not. The leucocytes, which we may for the moment regard as the prime factor, show differentiation as regards phagocytosis, and the degree of differentiation seems to depend on the degree of opsonic activity of the test sera. We are not to look on this differentiation as something of the nature of personal error or error of technique. It would appear to be an integral part of the phenomenon of phagocytosis. We should look then to a measure of differentiation or variability as giving us the best indication of the degree of activity which we are endeavouring to estimate quantitatively. For this purpose the standard deviation, if the argument is sound, would be a good one to employ. It is quite conceivable that within certain limits the standard deviation of the

frequency distribution for normal serum might be very constant, in which case it might not be necessary to put up a normal serum control on each occasion. It is doubtful, however, whether one would get in this way significant differences for the tubercle bacillus, although it might work with other organisms. In the following tables I give the calculated means and standard deviations together with the frequency distributions and the corresponding indices for two organisms.

Counts were made of 100 leucocytes at a time, and these component distributions are here recorded in order to give an idea of the variability of means, standard deviations, etc. with such small samples. The two organisms used were the *Streptococcus foecalis* and the *Bacillus friedlander*.

I. *Normal Serum. Streptococcus foecalis.*

*B**	*L* (1)*	*L* (2)	*L* (3)	*L* (4)	*L* (5)	Totals
0	4	6	8	5	9	32
1	10	3	11	5	7	36
2	9	12	10	8	10	49
3	6	6	13	7	13	45
4	14	14	10	13	20	71
5	6	9	8	6	2	31
6	9	8	5	7	9	38
7	6	7	9	8	4	34
8	11	7	8	14	2	42
9	6	8	2	4	5	25
10	3	6	2	9	2	22
11	2	4	2	1	5	14
12	6	2	4	4	4	20
13	0	2	2	1	2	7
14	4	3	2	1	3	13
15	1	1	0	2	1	5
16	1	1	1	2	1	6
17	0	0	0	1	1	2
18	2	1	1	1	—	5
19	—	—	1	0	—	1
20	—	—	1	0	—	1
21	—	—	—	0	—	0
22	—	—	—	1	—	1
Totals	100	100	100	100	100	500
Mean	6·17	6·16	5·42	6·65	5·38	5·96
S. D.	4·24	3·99	4·44	4·38	4·21	4·28

* *B* = Number of Bacteria ingested. *L* (1), *L* (2) etc. = successive frequency distributions of ingesting leucocytes.

II. *Immune Serum. Streptococcus foecalis.*

B	*L* (1)	*L* (2)	*L* (3)	*L* (4)	*L* (5)	Totals
0	4	4	1	3	5	17
1	4	4	4	3	3	18
2	7	9	9	5	3	33
3	3	7	3	6	10	29
4	6	6	9	2	4	27
5	11	5	5	8	5	34
6	7	6	3	4	5	25
7	8	5	5	3	7	28
8	4	2	4	6	6	22
9	5	11	4	10	7	37
10	4	4	4	11	4	27
11	6	6	4	2	2	20
12	4	2	11	5	6	28
13	4	5	8	3	4	24
14	3	8	1	6	5	23
15	2	0	6	5	1	14
16	5	2	4	2	3	16
17	1	0	0	7	1	9
18	1	1	3	0	2	7
19	0	3	1	2	3	9
20	2	0	2	1	4	9
21	3	3	2	1	1	10
22	1	1	0	0	2	4
23	0	1	0	1	1	3
24	0	2	0	1	3	6
25	1	0	3	0	0	4
26	0	2	1	1	1	5
27	0	0	0	1	2	3
28	1	0	0	0	—	1
29	0	0	0	0	—	0
30	0	1	2	0	—	3
31	1	—	0	0	—	1
32	0	—	0	0	—	0
33	0	—	0	0	—	0
34	1	—	0	0	—	1
35	0	—	1	0	—	1
36	0	—	—	0	—	0
37	0	—	—	0	—	0
38	0	—	—	0	—	0
39	0	—	—	0	—	0
40	0	—	—	0	—	0
41	1	—	—	0	—	1
42	—	—	—	1	—	1
Totals	100	100	100	100	100	500
Mean	9·58	9·30	10·52	9·95	10·19	9·97
S. D.	7·53	6·81	7·19	6·43	7·11	7·09

Indices on totals: Average = 1·67.
" " Zero = 1·88.
" " S. D. = 1·66.

I. *Normal Serum. B. mallei.*

B	*L* (1)	*L* (2)	*L* (3)	*L* (4)	*L* (5)	Totals
0	5	8	13	10	13	49
1	12	12	15	11	13	63
2	17	15	15	15	12	74
3	19	20	18	19	16	92
4	11	18	16	8	15	68
5	10	16	8	15	13	62
6	11	4	7	7	10	39
7	8	3	4	9	3	27
8	2	1	0	1	1	5
9	2	0	3	1	3	9
10	2	1	1	1	1	6
11	0	1	—	2	—	3
12	0	1	—	0	—	1
13	1	—	—	1	—	2
Totals	100	100	100	100	100	500
Mean	3·86	3·40	3·10	3·72	3·36	3·49
S. D.	2·52	2·25	2·29	2·67	2·36	2·42

II. *Immune Serum. B. mallei.*

B	*L* (1)	*L* (2)	*L* (3)	*L* (4)	*L* (5)	*L* (6)	*L* (7)	*L* (8)	*L* (9)	*L* (10)	Totals
0	3	2	3	1	4	3	3	3	7	0	29
1	5	5	4	3	1	1	5	3	6	6	39
2	8	9	2	5	1	3	6	6	7	1	48
3	5	8	8	9	9	8	6	6	12	6	77
4	9	11	9	7	9	8	6	9	7	8	83
5	10	10	15	13	14	11	6	14	7	6	106
6	15	12	17	14	9	13	15	10	7	16	128
7	10	10	9	11	13	7	10	7	8	12	97
8	10	6	8	9	14	12	10	9	10	14	102
9	4	7	6	7	5	10	8	11	9	6	73
10	5	5	5	9	5	4	10	8	3	7	61
11	3	2	5	3	6	7	5	4	4	4	43
12	1	4	3	3	4	4	1	4	4	6	34
13	5	1	1	1	1	2	1	2	3	3	20
14	3	3	4	1	1	0	1	1	2	2	18
15	1	2	1	1	2	2	3	1	2	0	15
16	1	1	—	1	1	1	3	1	1	2	12
17	1	2	—	0	0	3	0	0	1	1	8
18	1	—	—	1	0	0	1	0	—	—	3
19	—	—	—	0	1	0	—	0	—	—	1
20	—	—	—	1	—	1	—	0	—	—	2
21	—	—	—	—	—	—	—	1	—	—	1
Totals	100	100	100	100	100	100	100	100	100	100	1000
Mean	6·66	6·49	6·51	6·84	6·94	7·39	7·06	6·83	6·30	7·36	6·84
S. D.	3·96	3·95	3·36	3·62	3·55	3·94	3·93	3·74	4·13	3·49	3·79

Indices on Totals: Average = 1·97. Zero = 1·68. S. D. = 1·57.

The irregularity of the distributions is a noticeable feature in the case of the *Streptococcus foecalis* as compared with the *Bacillus friedlander*. The variability is also shown by the standard deviations to be much greater for the former than the latter. The irregularity here exemplified is also to be observed in the case of the Staphylococci. It may have something to do with the method of counting of pairs or threes. A pair of cocci ought very possibly to be counted neither as 1 nor as 2 but, as I have suggested, as 1·5, this last figure being the nearest approximation to the degree of effort put forth in the phagocytosis. Again the irregularity may be indicative of nothing more than smallness of sample.

Conclusions.

1. The opsonic index is a valuable means of gauging the effect of therapeutic inoculation and is useful for diagnosis.

2. The tuberculo-opsonic index is capable of giving in routine practice definite indications of the positive or negative opsonic activity of serum for values which range outside 0·8 and 1·2.

3. Opsonic sera give a much greater degree of differentiation amongst the leucocytes in the case of organisms other than the tubercle bacillus. It is impossible to give limits of normality and abnormality in these cases, because no investigations have been made as regards the variability of normal serum action upon these bacteria.

4. It is doubtful whether the opsonic indices as calculated by different observers are comparable with one another.

I have to thank Sir Almroth Wright and his colleagues for having placed records and material freely at my disposal and for many suggestions as to some of the disturbing factors which enter into the estimation of the opsonic index. The experience of the Vaccino-Therapy Laboratory at St Mary's Hospital is now very large, and the technique of opsonic estimations has been reduced to a very fine art. Finally I owe a deep debt of gratitude to Prof. Karl Pearson for his constant supervision of the work, his suggestions of methods and controls and much help.

ON A NEW METHOD OF DETERMINING CORRELATION BETWEEN A MEASURED CHARACTER *A*, AND A CHARACTER *B*, OF WHICH ONLY THE PERCENTAGE OF CASES WHEREIN *B* EXCEEDS (OR FALLS SHORT OF) A GIVEN INTENSITY IS RECORDED FOR EACH GRADE OF *A*.

By KARL PEARSON, F.R.S.

(1) As an example of the class of cases to which the method of this paper applies I instance that we might be given the ages of candidates for a given examination, and the number of failures at each age, without the individual marks; from these data we might desire to correlate capacity and age. Or, again, we might be given the percentage of first convictions for each age group of the community, and the problem might be to determine the relation between age and the tendency to crime as judged by conviction. Or, age being put on one side, we might desire to correlate any psychical character with anthropometric characters, e.g. the cephalic index in children as a more or less marked racial character with their conscientiousness or shyness, measured by the number of shy or conscientious children at each value of the index.

If the non-measurable, or at least unmeasured character, be classed into a considerable number of groups there is no doubt that the most satisfactory method to adopt is that of the correlation ratio*. The cases we have in view here, however, are those in which no such series of groups has been made, or possibly can be made. As a rule we have, hitherto, fallen back in such cases on a fourfold table method—that is to say, we have been tacitly compelled to drop any advantage that arose from one character having a measured value. Further the result was not unique, depending to some extent on where the division of the measured character was made. The present method is unique, it involves only the discovery of two means and one standard deviation (no product moment, no second standard deviation and no complicated equation having to be worked out); it is in fact singularly brief. It is only for one determination that we have to assume that the Gaussian frequency distribution may be applied with sufficient practical accuracy. This defect (though, I think, to a minor extent) it shares with the fourfold table method; but because it does, the method will serve in the cases to which both can be applied as a useful control method.

* Pearson: "On the Theory of Skew Correlation." *Drapers Research Memoirs* (Dulau & Co).

(2) The basal idea is so extraordinarily simple that one is inclined to believe that it must have been noticed before. I am not able, however, to refer to any previous mention of it.

I start from the hypothesis that the regression is linear. Accordingly, if a volume of the frequency be cut off from the frequency surface by a vertical plane at a given value of the variate B, the vertical through the centroid of this volume cuts the regression line. If $\bar{p}$ and $\bar{q}$ be the coordinates of this point of section measured from the means of the two variates, $\bar{p}$, $\bar{q}$ lies on the regression line and we have, σ_1 and σ_2 being the standard deviations of the two variates and r their correlation,

$$\bar{p} = r\frac{\sigma_1}{\sigma_2}\bar{q}.$$

Hence

$$r = \frac{\bar{p}/\sigma_1}{\bar{q}/\sigma_2} \quad \text{.....(i).}$$

So far there is no assumption of a Gaussian distribution. Now $\bar{p}$ is the mean value of the A-variate, for all the pairs with specially marked B-variate; thus in the example, in the first illustration I have given above, the mean age of all candidates who passed the examination. And σ_1 is the standard deviation of the measured character and can therefore be found, e.g. in the same illustration is the age variability of *all* candidates. Thus the numerator in Equation (i) can always be found. The next point is to consider how the denominator can be discovered. Now the B-variate is not given quantitatively, but we are given the percentage of B beyond the arbitrary division, i.e. in our illustration the number out of the candidates who succeed in passing. We cannot therefore find $\bar{q}/\sigma_2$ by the usual processes of determining a mean. If, however, we assume the B-variate to follow reasonably closely a Gaussian distribution, the percentage of the B-variate gives, by means of the probability-integral tables, the ratio of y/σ_2 for the distance from the mean at which the B-variate is divided, and then

$$\frac{\bar{q}}{\sigma_2} = \frac{\dfrac{N}{\sqrt{2\pi}\sigma_2^2}\displaystyle\int_y^\infty y e^{-\frac{1}{2}\frac{y^2}{\sigma_2^2}}dy}{\dfrac{N}{\sqrt{2\pi}\sigma_2}\displaystyle\int_y^\infty e^{-\frac{1}{2}\frac{y^2}{\sigma_2^2}}dy} = \frac{\dfrac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y/\sigma_2)^2}}{\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{y/\sigma_2}^\infty e^{-\frac{1}{2}y^2}dy}.$$

Here both numerator and denominator are known as soon as y/σ_2 has been found. They are for example the z and $\frac{1}{2}(1-a)$ of Sheppard's Tables (*Biometrika*, Vol. II. p. 182).

Thus by the simple hypotheses that the regression is linear, and that a fairly close value of the mean of the marked part of the B-variate can be found on the assumption that it has a normal distribution, we can readily find numerator and denominator of the value of r as given by Equation (i).

Illustration I. Relation of Anaemia to Age. I take the data from the last *Report* (1909) *of the Education Committee of the London County Council. Report of Medical Officer.* Particulars as to age and "colour of blood" are given for 325 boys at Laxon Street and 364 boys at Sirdar Road schools; and for 367 girls at the former school and 330 girls at the latter school: see p. 19. Sirdar Road, Notting Dale, is described as "one of the poorest schools of London in the midst of a migratory, criminal, and thoroughly degenerate and poverty stricken population." Laxon Street is also in a very poor district; lack of employment, improvidence and drink are exceedingly common in both districts. Either different standards of anaemia were used in the two schools, or the Laxon Street school shows far more anaemia than the Sirdar Road. The ages covered are 7 to 13. Putting "pale" and "very pale" together, the latter "blood colour" only occurring in about 5 p.c. of the children dealt with, we have

BOYS.

Ages.

Colour of Blood		7	8	9	10	11	12	13	Totals
Sirdar Road	Normal	34	29	27	32	31	35	41	229
	Pale ...	17	17	23	19	21	21	17	135
Totals ...		51	46	50	51	52	56	58	364
Laxon Street	Normal	20	23	34	23	25	25	24	174
	Pale ...	22	33	21	30	21	9	15	151
Totals ...		42	56	55	53	46	34	39	325

GIRLS.

Ages.

Colour of Blood		7	8	9	10	11	12	13	Totals
Sirdar Road	Normal	22	32	40	39	37	30	30	230
	Pale ...	4	18	17	15	12	19	15	100
Totals ...		26	50	57	54	49	49	45	330
Laxon Street	Normal	19	23	19	26	14	13	17	131
	Pale ...	34	40	28	40	33	34	27	236
Totals ...		53	63	47	66	47	47	44	367

I find the following results:

Mean age: all Boys: Sirdar Road, 10·6456
Laxon Street, 10·3092
Anaemic Boys: Sirdar Road, 10·5444
Laxon Street, 10·0430

Standard Deviation for ages: all Boys: Sirdar Road, 2·019
Laxon Street, 1·904

For the girls we have:

Mean age: all Girls: Sirdar Road, 10·6667
Laxon Street, 10·3392
Anaemic Girls: Sirdar Road, 10·8000
Normal Girls*: Laxon Street, 10·2634

Standard Deviation for ages: all Girls: Sirdar Road, 1·852
Laxon Sreet, 1·951

I will illustrate the general process on the Sirdar Road boys:

$$\bar{p}/\sigma_1 = (10{\cdot}5444 - 10{\cdot}6456)/2{\cdot}019$$
$$= -{\cdot}1012/2{\cdot}019 = -{\cdot}0512.$$

Again $\frac{1}{2}(1-\alpha) = 135/364 = {\cdot}3709$, thus $\frac{1}{2}(1+\alpha) = {\cdot}6291$. In Sheppard's Tables this corresponds to $x/\sigma = {\cdot}33$ and gives for the corresponding ordinate z the value ·3778.

Hence $\bar{q}/\sigma_2 = z/\frac{1}{2}(1-\alpha) = {\cdot}3778/{\cdot}3709$
$= 1{\cdot}0186.$

Accordingly $r = (\bar{p}/\sigma_1)/(\bar{q}/\sigma_2) = -{\cdot}0512/1{\cdot}0186$
$= -{\cdot}0503.$

The negative sign indicates that anaemia in boys decreases with age, i.e. the age of the anaemic boys is less than that of all boys. The girls at Sirdar Road give a plus correlation $= +{\cdot}0627$. Treated in the same manner the Laxon Street boys give $-{\cdot}1632$. In the case of the girls at the latter school the correlation is $+{\cdot}0373$, the normal girls having a lower age than all girls. Thus we find:

Correlation of Age and Anaemia.

	Boys	Girls
Sirdar Road ...	−·0503	+·0627
Laxon Street ...	−·1632	+·0373

* It is necessary to take the normal and not the anaemic girls as the former are in this school in the minority.

From these results we conclude that:

(i) The relation between anaemia and age (7 to 13) is not very marked.

(ii) Anaemia decreases as the boys grow older and increases as the girls grow older.

(iii) The association of anaemia with age in boys is about trebled in intensity when we pass from Sirdar Road to Laxon Street, or from a school with upwards of 30 °/₀ to one with more than 50 °/₀ of anaemic boys.

The above distributions are perhaps not quite the type we should select as most suitable for the method—the totals for each age are too nearly alike. It would probably, however, be difficult to bring out by any more exact method, *leading to a unique result*, the correlation of age and anaemia. It is obvious that a fourfold table method would present considerable diversity according to the age partition selected and would require far more labour than the 15 minutes requisite to determine a correlation-coefficient by the present method.

Illustration II. The Influence of Age on the Capacity to pass an Examination. I take here the returns for 6156 candidates for the London University Matriculation Examination from the *Calendar* for 1908–9 (pp. 953–955).

Ages of Candidate	Passed	Failed	Totals
16	583	563	1146
17	666	980	1646
18	525	868	1393
19—21	383	814	1197
22—30 (centred at *25*)	214	439	653
Over *30* (centred at *33*)	40	81	121
Totals	2411	3745	6156

The only difficulty here is the centering of the groups "22—30" and "over 30," the official statistics (as usual!) clubbing all these ages, which are really so significant for the frequency distribution, together. After some consideration, I made the mean ages of these groups 25 and 33.

The mean age of the passing candidates was 18·4280 and of the total candidates 18·7865. The S.D. of the latter being 3·2850. Thus $\bar{p} = -\cdot3585$, $\bar{p}/\sigma_1 = -\cdot1091$.

Further $\frac{1}{2}(1-\alpha) = \cdot3917$, giving $\frac{1}{2}(1+\alpha) = \cdot6083$ and $z = \cdot3843$. Thus

$$\bar{q}/\sigma_2 = \cdot9806, \text{ and } r = -\cdot1113.$$

We conclude that there is a small but sensible correlation between youth and the ability to pass this examination. There are some evidences that this holds for other examinations. It would be interesting to consider this problem at greater length. While the complexity of the brain may be greater at 28 than 18, the

mean weight appears to be greater at the latter age, and possibly in mere examination activity quantity may reckon more than quality in brains as well as it frequently does in marking.* It might be of value to determine the partial correlation when allowance was made for the number of times the candidate had entered.

Illustration III. Conscientiousness and Cephalic Index. It is well known that the cephalic index remains fairly constant from about 2 or 3 years old onwards. It is further a marked racial character. Folklore associates with various grades of honesty the local races of this country. It seemed accordingly worth while testing whether my school records gave any association between cephalic index and conscientiousness. The children measured were divided into those whose conscientiousness was "Keen," "Dull" or "Betwixt." The latter class were about 14°/₀ of the total and we divided them equally between the "keen" and "dull" classes. The result is given in the table below. We obtained for 1534 boys:

Mean Cephalic Index of the "Dull" group 78·8876.
Mean Cephalic Index of all Boys 78·8801.
Standard Deviation for all Boys 3·2929.

Conscientiousness and Cephalic Index of Boys.

Cephalic Index	Conscientiousness			Cephalic Index	Conscientiousness		
	Keen	Dull	Totals		Keen	Dull	Totals
66	1	—	1	*79*	127	62	189
67	—	—	—	*80*	122	58	180
68	1	1	2	*81*	99	44	143
69	4	1	5	*82*	86	24	110
70	5	1	6	*83*	55	29	84
71	7	2	9	*84*	28	21	49
72	11	6	17	*85*	18	11	29
73	26	10	36	*86*	16	7	23
74	35	18	53	*87*	8	1	9
75	58	25	83	*88*	3	—	3
76	81	40	121	*89*	—	1	1
77	110	58	168	*90*	1	1	2
78	145	64	209	*91*	2	—	2
				Totals	1049	485	1534

Further, $\frac{1}{2}(1-\alpha) = \cdot 3162$, $\frac{1}{2}(1+\alpha) = \cdot 6838$ giving $z = \cdot 3558$.

Thus we find $\bar{p}/\sigma_1 = +\frac{\cdot 0075}{3\cdot 2929} = +\cdot 0023$,

$\bar{q}/\sigma_2 = 1\cdot 1252$.

And finally $r = +\cdot 0020$.

This correlation therefore confirms the view already expressed*, that there exists very little relationship between psychical characters and head-measurements in man. In this case the correlation is zero, considering the probable error which must be sensible in the second place of figures.

Illustration IV. Effect of Enlarged Glands and Tonsils on the Weight of Children. The medical inspectors of the London County Council classify the state of the glands under two systems, 0 = no enlargement, and + = enlargement, and again under 1 good state, and 2 defective, and 3 bad state; the tonsils are only classed under 0 and +. I owe to Mr David Heron the following reductions for a typical L.C.C. school.

Weight	Glands		Tonsils		Totals
	Good	Bad	Good	Bad	
14	2	—	2	—	2
16	3	5	5	3	8
18	15	26	33	8	41
20	20	40	36	24	60
22	28	47	50	25	75
24	34	30	47	17	64
26	30	31	49	12	61
28	29	20	41	8	49
30	30	30	47	13	60
32	21	14	24	11	35
34	18	11	22	7	29
36	18	5	15	8	23
38	6	7	10	3	13
40	5	2	4	3	7
42	7	3	8	2	10
44	1	—	1	—	1
46	—	—	—	—	—
48	3	—	2	1	3
50	1	—	1	—	1
52	1	—	1	—	1
...	—	—	—	—	—
62	1	—	1	1	2
Totals	274	271	399	146	545

Mean weight of Boys in school = 27·7522.

Standard Deviation = 6·7502.

Mean weight of those with Bad Glands = 27·3737.

Mean weight of those with Bad Tonsils = 27·1212.

For glands:

$$\tfrac{1}{2}(1-\alpha) = \cdot 4972, \quad \tfrac{1}{2}(1+\alpha) = \cdot 5028, \quad z = \cdot 3988.$$

* K. Pearson, "Relation of Intelligence to Size and Shape of the Head," *Biometrika*, Vol. v. pp. 105—146.

For tonsils:

$$\tfrac{1}{2}(1-\alpha) = \cdot 2679, \quad \tfrac{1}{2}(1+\alpha) = \cdot 7321, \quad z = \cdot 3293.$$

$$\text{Correlation of } \textit{good} \text{ glands and weight} = \frac{\cdot 3785}{6\cdot 7502} \times \frac{\cdot 4972}{\cdot 3988} = \cdot 070.$$

$$\text{Correlation of } \textit{good} \text{ tonsils and weight} = \frac{\cdot 6310}{6\cdot 7502} \times \frac{\cdot 2679}{\cdot 3293} = \cdot 076.$$

Thus bad glands and tonsils are slightly, but I think significantly associated, with less weight in the child. The correlations are not, however, as large as we might *a priori* have anticipated that they would be.

Illustration V. Effect of the Employment of Mothers on the Height of their Children. I owe the data on this point to Miss E. M. Elderton, who has been working on the Glasgow School Board Returns. The first table gives the age of sons for mothers employed and not employed; the second gives the height of sons for mothers employed and not employed. I take it that the categories employed and not employed correspond to a continuous variable, which for want of a better name, I may term, stress on mother. Practically all these mothers may be said to have their household work. In some cases—apparently relatively few in Glasgow —they do other work of a variety of kinds. This extra work must increase the claims on the mother's energy, and we might expect it to show in the physical welfare of the child. This point will be fully discussed by Miss Elderton in her forthcoming memoir. I only use the data here as illustration of the new method.

Employment of Mothers and Age of Sons.

Age of Sons	Mothers, Employed	Mothers, not Employed	Totals
4·5— 5·5	48	942	990
5·5— 6·5	185	3137	3322
6·5— 7·5	247	3656	3903
7·5— 8·5	253	3947	4200
8·5— 9·5	263	3754	4017
9·5—10·5	242	3639	3851
10·5—11·5	186	3575	3761
11·5—12·5	204	3428	3632
12·5—13·5	187	3451	3638
13·5—14·5	45	1422	1467
Totals ...	1860	30951	32811

Mean age of all Boys $= 9\cdot 5563.$

Standard Deviation of all Boys $= 2\cdot 4806.$

Mean age of sons of employed mothers $= 9\cdot 3790.$

$$\tfrac{1}{2}(1-\alpha) = \cdot 0567, \quad \tfrac{1}{2}(1+\alpha) = \cdot 9433, \quad z = \cdot 1139.$$

$$r_{ae} = -\frac{\cdot 1773}{2\cdot 4806} \times \frac{\cdot 0567}{\cdot 1139} = -\cdot 0356.$$

Thus the mothers are somewhat more likely to be employed if their sons are younger. This probably only means that the more well-to-do parents allow their children to stay longer at school.

Employment of Mothers and Height of Sons.

Height of Sons	Mothers		Totals	Height of Sons	Mothers		Totals
	Employed	Not Employed			Employed	Not Employed	
27·5	—	1	1	*51·5*	205	3860	4065
29·5	1	3	4	*53·5*	165	3296	3461
31·5	1	8	9	*55·5*	70	2207	2277
33·5	4	41	45	*57·5*	39	1234	1273
35·5	12	154	166	*59·5*	8	518	526
37·5	46	514	560	*61·5*	2	158	160
39·5	111	1261	1372	*63·5*	—	51	51
41·5	193	2430	2623	*65·5*	—	21	21
43·5	230	3376	3606	*67·5*	1	3	4
45·5	288	3738	4026	*69·5*	—	2	2
47·5	239	3959	4198	*71·5*	—	1	1
49·5	245	4114	4359	*73·5*	—	1	1
				Totals	1860	30951	32811

Mean Height of all Boys $= 48{\cdot}4827.$

Standard Deviation of all Boys $= 5{\cdot}3043.$

Mean Height of sons of employed mothers $= 47{\cdot}0474.$

$$\tfrac{1}{2}(1-\alpha) = {\cdot}0567, \quad \tfrac{1}{2}(1+\alpha) = {\cdot}9433, \quad z = {\cdot}1139.$$

$$r_{he} = -\frac{1{\cdot}4353}{5{\cdot}3043} \times \frac{{\cdot}0567}{{\cdot}1139} = -{\cdot}1347.$$

Miss Elderton finds by the product-moment method that the correlation of age and height for these boys is

$$r_{ah} = {\cdot}8452.$$

Finding the partial correlation coefficient between employment of mother and height of sons for a constant age of offspring, we have

$$\rho_{he} = \frac{r_{he} - r_{ha}\, r_{ae}}{\sqrt{1 - r_{ha}^2}\sqrt{1 - r_{ae}^2}}$$

$$= -{\cdot}1958.$$

There is thus a quite sensible correlation for a given age of child between its stature and the increasing stress due to employment of mother. Miss Elderton will, however, indicate in her memoir some difficulties in the interpretation of this result.

Conclusions. The illustrations will have indicated that the new process of determining correlation can be applied to a great variety of problems. In none of the cases dealt with was the correlation very high, but this is purely a result of the material selected, which was chosen, not from any knowledge of the existence of correlation, but to indicate the type of problem to which the new method can be applied. Hitherto such problems could only be treated by the fourfold table method. The examples given show how much more expeditious is the new process, and further how it frees us from all doubt that exists in the old method as to the suitable position for the division of the measured variate. That its probable error will be less than that of the fourfold table method will I have little doubt be demonstrated, when its value has been worked out. The method is, however, so convenient and so frequently of service that I have not delayed its publication until the leisure came to determine the probable error.

ON INHERITANCE IN THE PRODUCTION OF BUTTER FAT.

By H. L. RIETZ, Ph.D., University of Illinois, Urbana, Illinois, U.S.A.

(1) *Introduction.*

It is the main object of this paper to present an investigation into inheritance in the production of butter fat. As far as I am aware, no statistical investigation has been published dealing with this question, notwithstanding its importance in a great industry and also its purely scientific interest.

Production of butter fat is one of the chief functions of the cow in the service of man. We are, in the present paper, concerned with the inheritance of this function, which is perhaps highly correlated with certain characters of the organism of the cow, but little or nothing is known of the quantitative value of such correlations.

The source of the data is the Advanced Register of the Holstein-Friesian Association of America. The performance of an individual in the production of butter fat is not constant from day to day, but varies considerably. The records taken from the above register are in pounds of butter fat made in seven consecutive days, and are given along with the age of the cow at the time of the test. The tests for the register are all made, at present, under the supervision of representatives of State Agricultural Experiment Stations. In the earlier years of the register, certain private tests were accepted. Throughout our work, we have taken only records attested by authorized officials of the Association, who are, in general, representatives of State Agricultural Experiment Stations. In beginning this work over a year ago, there were available 17 volumes representing records of 17 successive years and Volume 18 has appeared for use in the latter part of the work. However, it turns out that Volumes 1—10 give very few pairs of variates that satisfy conditions which it seems desirable to impose. For this reason, only data from Volumes 11—18 are used.

In the register, a cow often has a record when her dam has none, and in many more cases when her granddam has none. This is due in part to the fact that it is becoming more fashionable for breeders to enter their cows in the Advanced

Register, but also to the fact that there are certain entrance requirements to be satisfied. If we had records of all the offspring of certain grandparents with records, we should doubtless find many of the offspring not eligible to the Advanced Register for the latter reason.

(2) *Requirements for Admission to Advanced Registry.*

The minimum requirements of butter fat production to admit a cow to entry in the Advanced Register are as follows:

If a cow calves at two years old or under, 7·2 lbs. of butter fat in seven consecutive days. If the cow calves at three years old, 8·8 lbs. fat in seven consecutive days. If the cow calves at four years old, 10·4 lbs. fat in seven consecutive days. If the cow calves at five years old or over, 12 lbs. fat are required in seven consecutive days. If the cow calves between two and three years, or between three and four years, or between four and five years old, every day of increased age adds to the requirements for the years 0·00439 lb. butter fat.

Our data are therefore not a random sample of pure bred Holstein-Friesian cows, but a selected group which meets certain requirements. Since ancestors as well as offspring must meet these minimal requirements, our problem presents an illustration of the double selection recently dealt with by Pearson*. A correction for the influence of selection will be applied (§ 9) after we obtain statistical constants from the selected groups given in the register.

(3) *Correlation of Age and Production* (Table I.).

Anticipating a high correlation between age and production, we first took the year books 1902—1906 from which can be obtained easily data necessary to determine the correlation between age and production. From the means of arrays in Table I., it follows that up to five years old, the regression is almost "truly linear," and that, by dividing the table into two parts, near the five year mark, it gives two tables of nearly linear regression in each. For this reason, we attempt to describe the population by separating the data into two parts at the 4·75 year point. It results that, for the group of cows under 4·75 years old, the correlation coefficient is

$$r = 0{\cdot}662 \pm 0{\cdot}007.$$

For the group of cows over 4·75 years old,

$$r = 0{\cdot}030 \pm 0{\cdot}016$$

if the fifteen most extreme variates with respect to age are excluded from the calculation. If these fifteen are included

$$r = 0{\cdot}004 \pm 0{\cdot}026.$$

From these results, we are able to assert a high correlation between age and production up to 4·75 years, but we are unable to assert that any correlation exists between age and production of cows over 4·75 years old.

* *Biometrika*, Vol. VI., Part I., pp. 111, 112.

From a slightly different table, we calculated the correlation coefficient between age and production of butter fat by including cows up to 5·25 years old, and certain additional cows that entered the records on a milk test rather than on production of butter fat. This gave

$$r = 0{\cdot}688 \pm 0{\cdot}006.$$

The correlation is thus slightly increased by including cows between 4·75 and 5·25 years old, while the regression coefficient of production on age is slightly decreased.

For the group under 4·75 years old, regression of production on age is

$$r\frac{\sigma_p}{\sigma_a} = 0{\cdot}6623\,\frac{2{\cdot}513}{0{\cdot}8752} = 1{\cdot}901 \text{ with a probable error of } 0{\cdot}021.$$

For the group under 5·25 years,

$$r\frac{\sigma_p}{\sigma_a} = 0{\cdot}6879\,\frac{2{\cdot}713}{1{\cdot}026} = 1{\cdot}819 \text{ with a probable error of } 0{\cdot}018.$$

From the variability of arrays corresponding to different ages, we make the following observation:

The coefficient of variability increases gradually from youth to three years old, and then gradually decreases to maturity. This may be expressed roughly by saying that the coefficient of variability is greatest near the mean of the total period of growth.

In reference to lack of "smoothness" in the frequency distribution with respect to age, it should be noted that at each of the ages 2·5, 3·5, 4·5, 5·5, 6·5, 7·5, 9·5, 11·5 there are fewer variates than at the year marks between which each of them lies. For example, at 4·5 there are 284 while at 4 there are 498, and at 5 years 312. This seems to show simply a decided tendency on the part of breeders to have their cows tested nearer the year marks than the half-year marks. Ages are given to the day.

(4) *Mature and immature Cows.*

From the results on correlation between age and production, it appears that the data should be divided into two parts near the age of 4 years 9 months—the cows in one division being regarded as mature, and in the other division as immature.

The important question at once suggests itself as to whether inheritance, as measured by the correlation coefficient, is the greater when offspring and parents are at the same stage in the period of growth or when they are mature.

(*a*) *Offspring and dams under four years old at time of test* (Table II.). The table represents records when the offspring and dams differ by less than one year in age at time of their respective tests, and are less than four years old. Tabulated with the records of offspring in the Advanced Register are, in general, the most remarkable records of dams and granddams. To correlate records of offspring

with the most remarkable records of ancestry would make the latter appear to be better producers than the former. In the case of dams and offspring under consideration, we have examined the back records of dams, so as to correlate the records when dams and corresponding offspring are most nearly equal in age.

If M, σ, V, and r represent mean, standard deviation, coefficient of variability, and coefficient of correlation respectively, the results are:

	M	σ	V	
Offspring,	$10{\cdot}687 \pm 0{\cdot}057$,	$2{\cdot}136 \pm 0{\cdot}040$,	$19{\cdot}99 \pm 0{\cdot}38$,	$r = 0{\cdot}344 \pm 0{\cdot}023$.
Dams,	$10{\cdot}605 \pm 0{\cdot}057$,	$2{\cdot}141 \pm 0{\cdot}040$,	$20{\cdot}19 \pm 0{\cdot}38$,	

(*b*) *Offspring and dams over four years old at time of test* (Table III.). If the dividing line between mature and immature cows is drawn at 4·75 years as appears desirable from Table I., too few variates for our purpose fall into the division of mature cows. By making the dividing line at four years, the numbers in the two groups are nearly equal. In the above study of immature cows, the parent and corresponding offspring may differ in ages at the time of their respective tests by any amount less than one year; and there is no reason why a small change in the dividing line should affect the point as to whether the heredity coefficient is the larger during growth or after maturity.

For these reasons, we have, for the present purpose, classed cows above four years old as mature. The results are:

	M	σ	V	
Offspring,	$15{\cdot}316 \pm 0{\cdot}070$,	$2{\cdot}543 \pm 0{\cdot}049$,	$16{\cdot}60 \pm 0{\cdot}33$,	$r = 0{\cdot}284 \pm 0{\cdot}025$.
Dams,	$15{\cdot}368 \pm 0{\cdot}070$,	$2{\cdot}646 \pm 0{\cdot}051$,	$17{\cdot}22 \pm 0{\cdot}34$,	

(*c*) It may be noted that the group of cows under four years old yields a coefficient 0·344, while the group over four years old gives $r = 0{\cdot}284$; but the value $r = 0{\cdot}344$ is, as we shall show presently, much greater than the correlation for a group of parents and offspring of one age, say two or three years old. We have used a method of correspondence which much increases the coefficient for cows in the period of growth. For the purpose of establishing this point, we have first made a direct test based, to be sure, on only 219 pairs of variates (Table IV.). This table represents records when animals are under 2·25 years old at the time of their respective tests. This material gives

$$r = 0{\cdot}145 \pm 0{\cdot}045.$$

While the probable error is large, we shall in (*d*) present an argument to show that for offspring and dams two or three years old, it is highly probable that $r \gtrsim 0{\cdot}145$. This will be shown by regarding the result $r = 0{\cdot}344$ as arising from a combination of material of two sub-groups.

(*d*) *Combination of sub-groups of a population.* To illustrate the combination of sub-groups, we first combine data from Tables II. and III. to form a table in which corresponding dams and offspring differ by less than one year in age at

times of their respective tests when they are under four years old, and no regard is given to age when they are over four years old. The results are:

	M	σ	V	
Offspring,	$12{\cdot}913 \pm 0{\cdot}063$,	$3{\cdot}292 \pm 0{\cdot}045$,	$25{\cdot}46 \pm 0{\cdot}36$,	$r = 0{\cdot}651 \pm 0{\cdot}011$.
Dams,	$12{\cdot}915 \pm 0{\cdot}063$,	$3{\cdot}380 \pm 0{\cdot}045$,	$26{\cdot}17 \pm 0{\cdot}38$,	

This combined group thus yields a value of the correlation coefficient greater than the sum of the correlation coefficients of the sub-groups.

It has been shown* that whether any given mixture increases or reduces correlation depends entirely on the signs of the difference of the means of the sub-groups. What is to be expected under the conditions of our problem is easily seen; that is, when the means and standard deviations of the two systems of variates for each sub-group are substantially equal.

Let n_1, m_1, m_1', σ_1, σ_1', r_1 be the number of variates, means, standard deviations, and correlation coefficient respectively of a sub-group A_1, and n_2, m_2, m_2', σ_2, σ_2', r_2 be corresponding values for a second sub-group A_2, and n, m, m', σ, σ', r for the combined group. Then

$$n\sigma^2 = n_1\sigma_1^2 + n_2\sigma_2^2 + \frac{n_1 n_2}{n}(m_1 - m_2)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

$$n\sigma'^2 = n_1\sigma_1'^2 + n_2\sigma_2'^2 + \frac{n_1 n_2}{n}(m_1' - m_2')^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

$$nr\sigma\sigma' = n_1\sigma_1\sigma_1' r_1 + n_2\sigma_2\sigma_2' r_2 + \frac{n_1 n_2}{n}(m_1 - m_2)(m_1' - m_2') \quad \ldots\ldots\ldots(3).$$

Make $$m_1 = m_1', \quad m_2 = m_2', \quad \sigma_1 = \sigma_1', \quad \sigma_2 = \sigma_2' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

The relations (4) are substantially satisfied by material in (*a*) and (*b*). From (1), (2), (3), (4),

$$r = r_1 + (1 - r_1)\frac{n_1 n_2}{n^2}\frac{(m_1 - m_2)^2}{\sigma^2} + (r_2 - r_1)\frac{n_2\sigma_2^2}{n\sigma^2} \quad \ldots\ldots\ldots\ldots\ldots(5),$$

$$r = r_2 + (1 - r_2)\frac{n_1 n_2}{n^2}\frac{(m_1 - m_2)^2}{\sigma^2} + (r_1 - r_2)\frac{n_1\sigma_1^2}{n\sigma^2} \quad \ldots\ldots\ldots\ldots\ldots(6).$$

It may be observed, incidentally, from (5) and (6), that the correlation coefficient for the combined group formed of two sub-groups under conditions (4) is always greater than the smaller of the correlation coefficients of the sub-groups; but what we propose to determine from (5) and (6) is an upper limit for the correlation in the case of cows and dams of a fixed age. The same factors which produce the large value 0·651 in combining material of Tables II. and III. enter, to a considerable degree, into the study of cows under four years old. It appears justifiable to assume that the correlation coefficient 0·344 from Table II. is equal to or greater than the correlation coefficient which arises from combining into one

* Pearson: *Phil. Trans.*, A., Vol. 192, p. 278.

table material of two tables, in one of which offspring and dams are two (1·75—2·25) and in the other three (2·75—3·25) years old. Take $r_1 \equiv r_2$, then from (6),

$$0{\cdot}344 \gtrless r_2 + (1 - r_2)\frac{n_1 n_2}{n^2}\frac{(m_1 - m_2)^2}{\sigma^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7).$$

Values of n_1, n_2, n, m_1, m_2 are obtained from the arrays of Table I., and we assign to σ a value 2·34 unfavourable to a small value of r_2. This gives

$$0{\cdot}344 \gtrless r_2 + (1 - r_2)\frac{(983)(545)}{(1528)^2}\frac{(2{\cdot}36)^2}{(2{\cdot}34)^2},$$

$$r_2 \gtrless 0{\cdot}145.$$

When this estimate and the value 0·145 for the group of cows under 2·25 years old are compared with the value 0·284 for mature cows, it looks as if inheritance in the production of butter fat is much greater after maturity than in youth, but we avoid drawing conclusions until the corrections for selection are applied (see § 8).

(5) *Correlation of Granddams and Offspring—mature Cows* (Tables VI. and VII.).

The requirements for entrance to the Advanced Register are stated in § 2. A cow may have more than one record, and that one is regarded as "most remarkable" which exceeds most the required production of the age at which she is entered. We shall take the most remarkable records in dealing with granddams, instead of the records when the members of a pair are most nearly equal in age at the time of their respective tests, since the laborious task of finding back records is unnecessary to determine the correlation, as I have found in treating dams and offspring for comparison with the methods previously used in this paper.

Granddams have no official records in many cases where dams and offspring have such records, and there is difficulty in securing adequate numbers from our data for determining the correlation of granddams and offspring. To overcome this difficulty, we have, in order to determine correlation for mature cows, adopted a method of transmuting production of immature cows to mature cows. For this purpose we multiply the production of any immature cow of age s by $\frac{M_5}{M_s}$, where M_s is the mean production of cows of age indicated by the subscript. A good determination of $\frac{M_5}{M_s}$ is obtained from arrays of Table I. The means, standard deviations, and coefficient of correlation obtained from these transmuted values are comparable in an average way to the statistical constants obtained from mature cows.

For the sake of numerical comparison, we have determined the correlation of dams and maternal granddams (Table V.) by transmuting to mature cows. The results are:

	M	σ	
Dams,	15·383 ± 0·073,	2·377 ± 0·052,	$r = 0{\cdot}301 \pm 0{\cdot}028.$
Maternal Granddams,	15·423 ± 0·077,	2·495 ± 0·055,	

These results agree satisfactorily with those drawn from Table III., where the relationship is the same as in this case.

With transmuted values, we obtain the following coefficients of correlation (Tables VI., VII.):

Maternal Granddams and Offspring, $r = 0{\cdot}138 \pm 0{\cdot}024$.

Paternal Granddams and Offspring, $r = 0{\cdot}086 \pm 0{\cdot}017$.

While it is impossible to measure the potential character of butter fat production in the male, we aim to determine the extent to which he transmits this character from his dam to his female offspring. It may be observed that the correlation coefficient of maternal granddams and offspring is 0·138 while that of paternal granddams and offspring is 0·086. It is, however, to be further observed in this connection that the table of paternal granddams contains a considerable number of scattered extreme variates. If these were omitted the correlation coefficient would be appreciably increased. Furthermore, aside from entrance requirements, there is a strong tendency to select paternal granddams of high production so that selection may enter as a factor. For these reasons, we hesitate to attach significance to the difference $0{\cdot}138 - 0{\cdot}086 = 0{\cdot}052$ although possibly it indicates prepotency of females over males in the transmission of this character.

(6) *Selection of Sires.*

In breeding problems, the artificial selection of sires is comparable to assortative mating in man. The question thus arises: To what extent is the production of butter fat by any cows correlated with the production by dams of sires to whom the cows are mated? In answer to this question, we obtain the result (Table VIII.)

$$r = 0{\cdot}140 \pm 0{\cdot}025$$

for the correlation in question.

(7) *Further Correlations for immature Cows.*

In beginning this work, I expected to apply in a straightforward way the theory of multiple correlation by carrying the work back to granddams, and at the same time to take the age element into account. Then, in the usual notation, the regression equation could be written in the form

$$x_1 = a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 + a_{18}x_8,$$

where x_1, x_3, x_5, x_7 refer to production of offspring, dams, maternal granddams, and paternal granddams respectively; and x_2, x_4, x_6, x_8 refer to ages at times of respective tests of offspring, dams, maternal granddams, and paternal granddams. In Tables IX. to XII. is collected material for the correlation of first records of offspring with most remarkable records of ancestors when no regard is given to age except that all animals are under five years old. From Table IX., we obtain for the correlation of offspring and dams

$$r_{13} = 0{\cdot}132 \pm 0{\cdot}023.$$

Although the upper limit for immature cows is thus extended to five years, the data are too meagre to give reliable results in the determination of the correlation of granddams and granddaughters tested during the period of growth. However, the material is collected in Tables X. to XII. as it may be of interest in showing the poor records of the granddaughters of some fancy granddams, and the effect of these extreme variates on the correlation coefficient. These figures naturally lead to the question for the breeder as to whether the probability of progress with a considerable number of good animals is not greater than with an equal investment in fancy animals.

From Tables X. and XI.,

$$r_{15} = 0{\cdot}062 \pm 0{\cdot}046,$$
$$r_{17} = 0{\cdot}005 \pm 0{\cdot}026.$$

If we exclude the fancy paternal granddams at and above 22,

$$r_{17} = 0{\cdot}091 \pm 0{\cdot}026.$$

The variates excluded arise from only six paternal granddams; this shows how adversely a few extreme paternal granddams may influence the correlation coefficient when these ancestors are weighted with their offspring. On the question of the selection of sires, we have (Table XII.)

$$r_{37} = 0{\cdot}082 \pm 0{\cdot}048.$$

(8) *Corrections for Effects of Selection on the foregoing Results.*

The entrance requirements of the Advanced Register are stated in § 2. The material used in our study of heredity is subject to the form of double selection treated by Pearson*. To make the application, let s be the standard deviation of the group of cows which meets the requirements of admission to the Advanced Register, σ the standard deviation of pure bred Holstein-Friesian cows from which those which meet the requirements are selected, s_1 the standard deviation of dams which meet the requirements, σ_1 the standard deviation of cows from which the selection is made. In our material, a cow may meet the requirements when her dam does not, or a dam may meet the requirements when her offspring do not meet them. We can enter a pair in the correlation table when and only when both variates meet the minimal requirements.

Let $\mu = \frac{s}{\sigma}$, $\mu_1 = \frac{s_1}{\sigma_1}$, then the formula

$$R = r \frac{\mu\mu_1}{\sqrt{1 - r^2(1-\mu^2)}\sqrt{1 - r^2(1-\mu_1^2)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

gives the means of correcting the foregoing heredity coefficients for selection, where R is the correlation coefficient for the selected group, and r for the total group.

* *Biometrika*, Vol. VI., pp. 111, 112.

Neglecting other forms of selection than that of meeting requirements for entry,

$$s = s_1,\ \sigma = \sigma_1,\ \mu = \mu_1.$$

Then from (1)

$$r = \frac{-\mu^2 \pm \sqrt{\mu^4 + 4R^2(1-\mu^2)}}{2R(1-\mu^2)} \quad \text{.....(2).}$$

The value of s for cows of various ages is known from Table I., but it is necessary to evaluate σ in order to obtain μ. To determine σ we make use of the fact that the frequency distributions which give s are truncated. The requisite theory for obtaining the standard deviation of a normal distribution from a truncated portion of it has been given by Pearson*.

Let N, n be the number in the total group and in the "tail" respectively, h the distance from the mean of the total distribution at which the distribution is truncated, ν_1' the deviation of the mean of the selected group from the minimal requirement, Σ the standard deviation of the tail about its own mean,

$$\mu'_p = \int_{h'}^{\infty} x^p e^{-\frac{1}{2}x^2}\, dx,$$

where $h' = \frac{h}{\sigma}$. Then

$$\frac{n}{N} = \mu_0',\quad \sigma = \frac{\nu_1'\mu_0'}{\mu_1' - h'\mu_0'} = \psi_2\nu_1' \quad \text{.....(3)}$$

are the formulae to be used for our purpose. Further relations are given for the determination of N from the tail, but, bearing on the application of these formulae, it is pointed out that the effect of random sampling on the frequency at the stump of the tail is very influential in modifying the results. It is obvious that, in our problem, the frequencies near the stump are likely to be less than what we should obtain from the truncated portion of a random sample of the total group, for the reason that a cow is likely to be left out when she can barely meet the requirements. Hence, instead of trying to determine n/N from the "tail," we determine this ratio by direct observation, as we have the means of making a fair estimate of the number of cows N in milk from which the n are selected. The basis of the method is found in the fact that the Holstein-Friesian Association maintain a Herd Book in which animals are registered, in general, when they are calves. Substantially all the female calves that are born are registered by such owners as use the Advanced Registry. By thus going to the Herd Book, we find the number of the female calves registered together with the age of calf and the Herd Book number of the dam, so that we can find the age of the dam. As a cow is in milk for each calf registered, we double the number of female calves registered to obtain an estimate of the number of cows in milk. We have excluded a few owners who buy and sell extensively, as the transfer of animals might prove a source of error. We have searched the records for the year May 15, 1906 to May 15, 1907, and the results may be exhibited in the following form:

* *Loc. cit.* pp. 63—68.

	0—2·5 years	2·5—3·5 years	3·5—4·5 years	4·5—	Total under 4·5 years
Cows in milk	618	494	430	1416	1542
Number in Advanced Registry	413	346	269	620	1028
$\mu_0 = \frac{n}{N}$	0·67	0·70	0·63	0·44	0·67

With these estimates for $\mu_0 = n/N$, we can obtain σ for various classes of cows from data in Table I.

(*a*) *Value of* σ *for mature cows.*

$$\nu_1' = 14{\cdot}90 - 12{\cdot}00 = 2{\cdot}90,$$
$$\mu_0 = 0{\cdot}44,$$
$$h' = 0{\cdot}151,$$
$$\psi_2 = 1{\cdot}342,$$
$$\sigma = 3{\cdot}89.$$

(*b*) *Value of* σ *for immature cows.* For immature cows the requirements increase uniformly with age from two years old to five years old according to the relation

$$y = 1{\cdot}6\,x + 4,$$

where x is age in years and y is production in pounds. If the total frequency surface with respect to age and production is

$$z = f(x, y)$$

the selected portion is separated from the rest of the surface by the plane $y = 1{\cdot}6\,x + 4$. The theory for obtaining the constants of the total surface from such a portion of it has not, so far as I am aware, been developed. Perhaps it can be developed, without great difficulty, for the normal surface, but our distribution as to age is so far from normal that I think a method of dealing with arrays of the correlation table will give more reliable results than to assume the distribution normal.

(*c*) *Cows under* 2·25 *years.* For cows two years old or under, the requirement is 7·2 and for one 2·25 years old it is 7·6. For this small range in age we shall accept the requirement for age 2·125, that is 7·4, as locating the stump of the tail. From the combination of arrays in Table I., for cows under 2·25,

$$\Sigma = 1{\cdot}48, \text{ and } m = 9{\cdot}33,$$
$$\mu_0 = \frac{n}{N} = 0{\cdot}67,$$
$$h' = -0{\cdot}44,$$
$$\nu_1' = 9{\cdot}33 - 7{\cdot}4 = 1{\cdot}93,$$
$$\sigma = 1{\cdot}99.$$

(*d*) *Cows* 3 *years old* (2·5—3·5).

$$\Sigma = 2{\cdot}21,$$

$$\nu_1' = 11{\cdot}75 - 8{\cdot}8 = 2{\cdot}95,$$

$$\mu_0 = \frac{n}{N} = 0{\cdot}70,$$

$$h' = -0{\cdot}525,$$

$$\sigma = 2{\cdot}89.$$

(*e*) *Cows* 4 *years old* (3·5—4·0).

$$\Sigma = 2{\cdot}10,$$

$$\nu_1' = 2{\cdot}84,$$

$$h' = -0{\cdot}33,$$

$$\sigma = 3{\cdot}06.$$

(*f*) *Corrected standard deviation for the entire group of cows under* 4·5 *years old.* This standard deviation I obtain from the standard deviations of arrays up to 4·5. That is to say, we pool together the arrays with their corrected means and standard deviations. This gives for all cows under 4·5 years old

$$\sigma_i = 2{\cdot}94.$$

(*g*) *Ratio of variability of selected group to that of total group.* Let μ represent the ratio, then μ has the following values:

For group over 4·5 years old, $\mu = 0{\cdot}57$.

For group under 2·25 years old, $\mu = 0{\cdot}75$.

For group under 4·5 years old, $\mu = 0{\cdot}85$.

(*h*) *Values of corrected correlations.* By means of (2) we now give the values of some corrected correlation coefficients.

Dams and offspring:

For mature cows, $r = 0{\cdot}63$ from $R = 0{\cdot}284$ of selected group.

For cows under 2·25 years old, $r = 0{\cdot}25$ from $R = 0{\cdot}145$.

For cows under 5 years old without regard to age in selecting pairs, $r = 0{\cdot}18$ from $R = 0{\cdot}132$ for selected group.

Granddams and offspring:

For maternal granddams (mature), $r = 0{\cdot}38$ from $R = 0{\cdot}138$.

For paternal granddams (mature), $r = 0{\cdot}25$ from $R = 0{\cdot}086$.

Correlation between dams and dams of sires with which they are mated (mature), $r = 0{\cdot}39$ from $R = 0{\cdot}14$.

With the formula for the selection of one character only, we obtain 0·695 as the corrected correlation of age and production from 0·662.

It should be understood that these corrected values are to be regarded as approximate results only. In finding the percentage of cows that meet the standards, I think perhaps our method gives values for these percentages rather too small than too large, owing to the fact that breeders are likely to keep from the test animals that can barely meet the requirements. If this be the case, our corrected values of standard deviation and correlation are somewhat larger than they should be.

(9) *Mature and immature cows.* To return to the consideration of mature and immature cows (§ 4), it appears that, both before and after applying the correction for selection, the coefficient of heredity is greater for mature than for very young cows. We shall next make use of the partial correlation coefficient to obtain a sort of average value for the correlation coefficient during the period of growth. Following the usual notation, let the subscripts 1, 2, 3, 4 mark production of offspring, age of offspring at time of test, production of dams, and age of dams at time of test respectively. Then the desired value is given by

$$\rho_{13} = \frac{r_{13}(1-r_{24}^2) + r_{12}(r_{24}r_{34} - r_{23}) + r_{14}(r_{23}r_{24} - r_{34})}{\sqrt{1-r_{24}^2-r_{23}^2-r_{34}^2+2r_{23}r_{34}r_{24}}\,\sqrt{1-r_{24}^2-r_{12}^2-r_{14}^2+2r_{12}r_{14}r_{24}}}.$$

The values to be substituted in this formula are $r_{13} = 0{\cdot}18$ from the material of Table IX., corrected for selection; $r_{12} = r_{34} = 0{\cdot}695$ from Table I. corrected for selection; $r_{24} = 0{\cdot}033$ from Table XIII.; $r_{14} = 0$, $r_{23} = 0$.

$$\rho_{13} = 0{\cdot}38.$$

We have then 0·38 for a sort of average value of the correlation coefficient at fixed ages in the period of growth, 0·145 and 0·25 for uncorrected and corrected coefficients of cows under 2·25 years old, 0·28 and 0·63 for uncorrected and corrected coefficients after maturity. These results seem convincing of the fact that *inheritance in production of butter is, in general, much more pronounced after maturity than at a fixed stage in the period of growth.*

(10) *Summary of results.*

(*a*) The correlation of age and production of butter fat by cows under 4·75 years is approximately $\frac{2}{3}$, and the regression is linear.

(*b*) The correlation of age and production of cows over 4·75 years is substantially zero.

(*c*) The coefficient of variability in production increases gradually from youth to three years old, and then decreases to maturity.

(*d*) Production of butter fat is a function transmitted to a marked extent. The coefficient of heredity between cows and dams in the performance of this

function is probably at least as large as the coefficient of mean parental inheritance of physical characters in man*.

(*e*) Inheritance in production is more pronounced in mature than in immature cows.

(*f*) A comparison of the results corrected for selection with the uncorrected values illustrated the great influence of selection in modifying the correlation coefficient.

(11) *Description of Tables.*

(*a*) Ages in Tables are expressed in years.

(*b*) Production is expressed in pounds of butter fat made in seven consecutive days.

(*c*) In Table II., about one-tenth of the variates are re-entries, where we mean by a re-entry that a cow has a previous record. In Table IV., one-half the variates are re-entries. In Tables IV., X.–XIII., there are given only first records of offspring.

(*d*) In some Tables, the class values are 10·75, 11·75, ... instead of integral values. This grouping arises out of the fact that we first grouped in $\frac{1}{2}$ pound groups and found these groups narrower than desirable.

(*e*) The term "offspring" as used in Tables VI., VII., X., XI. means offspring of the second generation removed, that is, granddaughters.

Illinois Agricultural Experiment Station.
March 18, 1909.

* [It is worth while noticing also that whereas the parental correlation for mature cows is about ·63 that for grandparents on the average $= \frac{1}{2}(\cdot38 + \cdot25) = \frac{1}{2}(\cdot63)$ or, within the limits indicated by the probable errors, may be taken as *half* the parental correlation. Considering the difficulties of correcting for selection pointed out by Dr Rietz, it is not unjustifiable to suppose these relationships are in close accordance with the values of inheritance found for man. Ed.]

TABLE I (see pp. 120 and 121).

TABLE II.

Correlation of Dams and Offspring for Group under Four Years Old. Members of Corresponding Pairs less than one year different in Age.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Totals
7	—	1	3	2	2	—	—	—	—	—	—	—	—	—	—	8
8	1	22	14	25	13	3	4	—	—	—	—	—	—	—	—	82
9	4	25	23	23	19	12	7	4	2	1	—	—	—	—	—	120
10	4	24	28	25	28	8	7	2	6	1	—	—	—	—	—	133
11	1	12	19	15	16	17	14	6	5	—	1	—	—	—	—	106
12	3	4	11	19	15	17	5	4	4	2	—	—	—	—	—	84
13	—	5	9	9	5	7	2	5	2	1	—	1	—	—	1	47
14	—	1	2	6	3	9	8	2	3	1	1	—	—	—	—	36
15	—	1	—	—	2	2	2	—	1	—	—	—	—	—	—	8
16	—	1	—	2	2	2	—	—	1	—	—	—	—	—	—	8
17	—	1	—	—	3	—	2	1	1	—	1	1	—	—	—	10
18	—	—	—	2	—	—	—	—	—	—	—	1	—	—	—	3
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
21	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	13	97	109	128	108	77	51	24	26	6	3	3	—	—	1	646

TABLE III.

Correlation of Dams and Offspring. Mature Cows.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Totals
11	1	3	1	3	2	1	1	1	—	1	—	—	—	—	—	14
12	1	8	13	12	6	9	5	—	3	1	—	—	1	—	—	59
13	3	18	18	8	13	11	6	3	3	1	—	—	—	—	—	84
14	—	14	18	14	15	11	12	8	2	4	1	—	—	—	—	99
15	—	12	16	16	11	11	8	3	3	7	3	—	—	—	—	90
16	1	8	13	16	18	11	8	2	2	5	6	2	1	—	—	93
17	—	3	6	11	13	4	8	3	4	6	4	1	—	—	—	63
18	—	3	5	2	8	5	5	2	5	3	2	1	—	—	—	41
19	—	—	2	1	4	3	6	2	2	—	1	2	—	—	1	24
20	1	—	1	7	3	3	3	—	4	—	1	—	—	—	—	23
21	—	—	1	2	—	1	—	—	—	1	—	1	—	—	—	6
22	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	2
23	—	—	—	1	1	—	—	—	1	—	1	2	—	—	—	6
24	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	2
25	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
28	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	7	70	94	94	94	70	63	25	30	30	19	9	2	—	1	608

TABLE I. *Correlation of*

Production in Pounds

Age in Years.	7	7·5	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	16	16·5
1·0	—	**1**	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1·5	4	16	**10**	12	10	—	—	4	—	—	—	—	—	—	—	—	—	—	—	—
2·0	5	112	124	142	145	**120**	93	73	44	49	25	23	13	7	3	2	3	—	—	—
2·5	—	8	35	49	55	51	**69**	39	48	39	17	18	15	5	8	4	1	2	1	—
3·0	—	—	1	12	44	54	64	44	34	**49**	40	31	40	28	29	23	17	14	3	4
3·5	—	—	—	—	6	18	31	37	32	39	39	**26**	29	19	16	18	10	12	7	5
4·0	—	—	—	—	—	—	—	34	48	55	48	60	41	**35**	41	35	20	16	17	18
4·5	—	—	—	—	—	—	—	—	13	28	25	28	30	36	**20**	22	18	7	13	12
5·0	—	—	—	—	—	—	—	—	—	—	21	26	37	38	27	20	**22**	28	22	18
5·5	—	—	—	—	—	—	—	—	—	—	18	20	22	19	14	10	**14**	13	11	9
6·0	—	—	—	—	—	—	—	—	—	—	17	24	31	27	23	29	**21**	26	17	13
6·5	—	—	—	—	—	—	—	—	—	—	6	13	10	7	14	18	**9**	12	12	5
7·0	—	—	—	—	—	—	—	—	—	—	9	22	21	24	19	22	**5**	10	15	6
7·5	—	—	—	—	—	—	—	—	—	—	3	14	19	15	5	6	**11**	9	5	2
8·0	—	—	—	—	—	—	—	—	—	—	9	16	8	6	15	13	**8**	2	11	9
8·5	—	—	—	—	—	—	—	—	—	—	5	9	7	5	5	10	**3**	4	7	4
9·0	—	—	—	—	—	—	—	—	—	—	2	6	7	4	8	9	**6**	3	3	1
9·5	—	—	—	—	—	—	—	—	—	—	2	6	3	5	5	5	**2**	2	1	—
10·0	—	—	—	—	—	—	—	—	—	—	—	5	7	8	6	3	**5**	—	**3**	2
10·5	—	—	—	—	—	—	—	—	—	—	3	4	2	—	1	**2**	1	2	4	1
11·0	—	—	—	—	—	—	—	—	—	—	2	3	1	1	—	1	1	**1**	1	3
11·5	—	—	—	—	—	—	—	—	—	—	—	2	**3**	4	1	—	2	1	1	—
12·0	—	—	—	—	—	—	—	—	—	—	1	3	2	4	—	1	**2**	1	2	—
12·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	**1**	1	1
13·0	—	—	—	—	—	—	—	—	—	—	2	—	—	**1**	—	—	1	1	—	—
13·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	—	—	—
14·0	—	—	—	—	—	—	—	—	—	—	**1**	—	**1**	—	—	—	1	—	—	—
14·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—
15·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
Totals	9	137	170	215	260	243	257	231	219	259	297	359	349	299	264	254	183	167	157	113

Age and Production.

of Butter Fat.

17	*17·5*	*18*	*18·5*	*19*	*19·5*	*20*	*20·5*	*21*	*21·5*	*22*	*22·5*	*23*	Totals	Mean of Arrays	σ of Arrays	V of Arrays
—	—	—	—	—	—	—	—	—	—	—	—	—	1			
—	—	—	—	—	—	—	—	—	—	—	—	—	56	8·25 ±0·08	0·86	
—	—	—	—	—	—	—	—	—	—	—	—	—	983	9·390±0·30	1·483±0·023	15·80±0·25
1	—	—	—	—	—	—	1	—	—	—	—	—	466	10·24 ±0·05	1·644±0·036	16·05±0·36
5	4	—	—	—	3	—	—	1	1	—	—	—	545	11·75 ±0·06	2·214±0·046	18·85±0·40
6	2	2	—	—	1	1	—	1	—	—	—	—	357	12·32 ±0·07	2·095±0·053	17·00±0·45
6	7	6	3	2	2	1	1	1	—	1	—	—	498	13·24 ±0·06	2·070±0·045	15·63±0·34
12	3	5	3	3	1	3	1	—	—	—	1	—	284	13·93 ±0·08	2·116±0·060	15·19±0·43
9	13	10	5	3	4	3	—	4	—	—	2	—	312	14·84 ±0·08	2·124±0·057	14·31±0·40
13	9	5	6	1	3	4	—	—	2	1	—	—	194	14·88		
11	9	3	5	5	1	4	5	3	—	2	1	—	277	14·93		
7	5	2	1	3	4	1	—	—	1	—	—	—	130	14·99		
17	4	4	5	2	4	2	2	1	1	—	—	—	195	14·96		
6	4	1	2	3	2	2	1	2	—	—	—	—	112	14·83		
8	8	—	3	—	2	2	—	—	1	—	—	—	121	14·87		
1	2	3	2	1	—	—	—	2	—	—	—	—	70	14·79		
7	2	1	1	2	2	3	2	—	—	—	1	—	70	15·01 +0·44		
3	1	3	2	—	3	—	—	1	—	—	—	—	44	15·09 ±0·58		
5	—	2	—	1	2	2	—	1	—	—	—	1	53	15·24 ±0·48		
1	—	—	—	—	—	—	—	1	—	—	—	—	22	14·52 ±0·74		
—	—	1	1	1	—	—	1	1	—	—	—	—	19	15·55 ±1·15		
1	—	1	—	—	1	—	—	—	—	—	—	—	17	14·50		
1	—	—	—	—	—	—	1	2	—	—	—	—	20	15·05		
—	—	—	—	—	—	—	—	—	—	—	—	—	4	15·50		
—	—	—	—	—	—	—	—	—	—	—	—	—	5	13·60		
—	—	—	—	—	—	—	—	—	—	—	—	—	4	13·63		
—	—	—	—	—	—	—	—	—	—	—	—	—	3	13·33		
—	—	—	—	—	—	—	—	—	—	—	—	—	1			
—	—	—	—	—	—	—	—	—	—	—	—	—	—			
—	—	—	—	—	—	—	—	—	—	—	—	—	1			
120	73	49	39	27	35	28	15	21	6	4	5	1	4765	—	—	—

TABLE IV.

Correlation of Dams and Offspring when Both are Under 2·25 Years Old.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	Totals
7	—	1	2	4	2	—	—	—	—	—	9
8	2	15	9	10	6	1	—	—	—	—	43
9	3	18	21	5	6	2	2	1	—	—	58
10	2	17	12	12	11	—	—	—	—	1	55
11	—	6	10	2	2	5	3	—	1	—	29
12	1	1	3	2	4	1	—	—	1	—	13
13	—	4	1	—	2	1	—	—	—	—	8
14	—	—	1	2	—	—	—	—	—	—	3
15	—	—	—	—	—	—	—	—	—	—	—
16	—	—	—	1	—	—	—	—	—	—	1
Totals	8	62	59	38	33	10	5	1	2	1	219

TABLE V.

Correlation of Maternal Granddams and their Offspring. Dams.

Production of Maternal Granddams in Pounds of Butter Fat.

Production of Dams in Pounds of Butter Fat.	*10·75*	*11·75*	*12·75*	*13·75*	*14·75*	*15·75*	*16·75*	*17·75*	*18·75*	*19·75*	*20·75*	*21·75*	*22·75*	*23·75*	*24·75*	*25·75*	*26·75*	*27·75*	Totals
10·75	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
11·75	—	2	9	8	6	3	4	—	—	1	—	—	—	—	—	—	—	—	33
12·75	—	9	11	17	15	6	5	2	3	1	—	—	—	—	—	—	—	—	69
13·75	—	5	12	11	16	10	7	3	—	4	—	—	—	—	—	—	—	—	68
14·75	—	1	10	13	10	10	10	10	3	1	2	—	—	1	—	—	—	—	71
15·75	—	5	8	8	8	16	7	4	—	3	3	1	—	—	—	1	—	—	64
16·75	—	3	12	15	3	18	10	5	2	6	3	1	—	—	—	—	1	—	79
17·75	—	1	2	9	6	4	6	3	4	1	—	1	—	—	—	—	—	—	37
18·75	—	—	—	5	1	1	8	—	2	2	1	1	—	—	—	—	—	—	21
19·75	—	1	—	3	3	—	4	—	—	3	6	—	—	—	—	—	—	—	20
20·75	—	—	2	—	1	—	1	—	—	—	2	—	—	—	—	—	—	—	6
21·75	—	—	—	1	—	—	—	1	—	—	1	—	—	—	—	—	—	—	3
22·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23·75	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
24·75	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	—	27	67	91	71	68	63	28	14	22	18	4	—	1	—	1	1	—	476

Production of Offspring in Pounds of Butter Fat.

TABLE VI.

Correlation of Maternal Granddams and Offspring. Mature Cows.

Production of Maternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.

	11	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	Totals
11	—	3	4	1	1	4	1	—	2	2	—	—	—	—	—	18
12	1	12	27	16	14	7	12	4	3	6	—	1	—	—	—	103
13	2	11	22	14	12	12	10	2	2	5	—	1	1	—	—	94
14	3	14	23	25	27	24	12	5	4	6	6	—	—	—	—	149
15	3	2	18	18	18	14	7	4	7	6	3	1	—	1	—	102
16	—	11	20	26	17	24	13	3	3	7	2	—	—	—	—	126
17	—	7	5	13	5	10	7	2	6	2	1	1	—	—	—	59
18	—	2	5	7	7	3	6	7	5	1	1	1	—	—	1	46
19	1	3	3	6	6	4	3	3	—	3	—	—	—	—	—	32
20	—	—	3	2	4	6	4	1	2	1	—	3	1	—	—	27
21	—	1	—	1	6	1	1	—	—	—	—	—	—	—	—	10
22	—	—	—	2	1	—	5	—	—	—	—	—	—	—	—	8
23	—	—	2	—	—	1	—	—	1	—	—	—	—	—	—	4
24	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
25	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
26	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
29	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	10	67	132	131	119	110	81	31	35	41	13	8	2	1	1	782

TABLE VII.

Correlation of Paternal Granddams and Offspring. Mature Cows.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.

	10·75	*11·75*	*12·75*	*13·75*	*14·75*	*15·75*	*16·75*	*17·75*	*18·75*	*19·75*	*20·75*	*21·75*	*22·75*	*23·75*	*24·75*	*25·75*	*26·75*	*27·75*	*28·75*	*29·75*	Totals
10·75	—	—	1	3	1	1	—	1	1	1	2	1	1	—	—	—	—	—	—	—	13
11·75	—	5	16	22	16	14	10	7	9	11	8	7	3	—	1	—	—	—	—	1	130
12·75	—	7	31	12	21	28	15	22	21	36	25	9	4	—	—	—	1	—	—	2	234
13·75	1	2	35	32	26	24	28	28	37	34	32	21	7	1	1	—	5	3	—	2	319
14·75	1	3	26	21	22	26	28	32	23	25	17	13	3	4	1	—	2	—	—	—	247
15·75	—	4	19	21	9	23	25	16	11	19	17	9	1	—	—	—	1	—	—	—	175
16·75	—	—	9	9	12	19	6	21	20	17	9	8	1	1	—	—	—	—	—	1	133
17·75	—	1	3	7	6	12	10	7	13	16	10	11	2	1	—	3	1	—	—	—	103
18·75	—	—	4	—	2	4	5	7	5	8	5	3	—	—	—	—	—	1	—	—	44
19·75	—	—	2	1	2	6	5	4	2	2	2	2	—	—	—	—	1	1	—	—	30
20·75	—	—	2	—	2	2	2	2	1	2	1	1	—	—	—	—	—	—	—	—	15
21·75	—	—	1	1	—	—	—	—	3	1	2	1	—	1	—	—	—	—	—	—	10
22·75	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	2
23·75	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
24·75	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
25·75	—	—	—	1	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	3
26·75	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
27·75	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
28·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
29·75	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	22	150	131	119	160	135	147	147	174	132	87	22	8	4	3	11	5	—	6	1465

TABLE VIII.

Correlation of Paternal Granddams and Dams.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Dams in Pounds of Butter Fat.

	10·75	11·75	12·75	13·75	14·75	15·75	16·75	17·75	18·75	19·75	20·75	21·75	22·75	23·75	24·75	25·75	26·75	Totals
10·75	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	2
11·75	—	—	6	9	3	3	6	9	5	1	10	2	—	—	—	—	—	54
12·75	—	1	13	3	9	11	9	12	18	6	17	2	—	—	—	—	1	102
13·75	—	—	11	8	7	14	17	12	18	14	16	5	4	—	1	—	2	129
14·75	—	1	11	13	2	18	15	20	16	13	12	6	4	—	1	1	—	133
15·75	—	1	9	7	7	16	10	11	9	11	6	11	2	—	—	—	—	100
16·75	—	—	4	5	9	5	7	2	13	9	12	3	1	—	—	—	—	70
17·75	—	1	3	1	1	6	6	1	4	3	5	3	2	—	—	—	1	37
18·75	—	—	—	2	3	4	2	1	5	3	9	6	—	—	—	—	—	35
19·75	—	—	1	2	—	2	2	4	2	3	3	—	2	—	—	—	—	21
20·75	—	1	—	—	—	—	3	2	—	1	5	10	—	1	—	—	—	23
21·75	—	—	2	—	—	—	—	—	—	1	3	—	—	—	—	—	—	6
22·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23·75	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	—	6	61	50	41	79	77	74	90	65	99	48	15	1	2	1	4	713

TABLE IX.

Correlation of Dams and Offspring. Cows under Five Years Old but no other Restriction in regard to Age.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.

	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Totals
7	—	1	1	4	3	4	2	—	1	—	—	—	—	—	—	—	—	—	16
8	1	12	6	14	22	13	14	13	6	5	7	2	—	1	1	—	—	—	117
9	1	10	13	12	12	21	22	23	9	10	3	6	4	—	—	—	—	—	146
10	1	18	9	22	24	18	23	12	21	9	7	3	2	—	—	—	—	—	169
11	1	8	10	13	15	16	17	9	13	8	5	6	2	1	—	1	1	1	127
12	1	5	4	11	5	11	13	13	11	9	5	4	2	2	—	—	—	—	96
13	—	5	2	3	5	7	7	9	9	4	1	2	2	1	1	—	—	—	58
14	—	—	3	4	1	5	6	2	5	3	4	—	—	2	—	—	1	—	36
15	—	3	1	1	3	1	4	3	4	1	3	—	—	1	—	—	—	—	25
16	—	2	—	1	1	1	1	1	3	—	2	—	—	—	1	—	—	—	13
17	—	1	—	—	1	—	2	—	3	—	—	2	—	—	—	—	—	—	9
18	—	—	—	—	2	—	—	—	1	1	1	—	—	—	—	—	—	—	5
19	1	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	4
Totals	6	66	49	85	94	97	112	85	87	50	38	25	12	8	3	1	2	1	821

Production of Offspring in Pounds of Butter Fat.

TABLE X.

Correlation of Maternal Granddams and their Offspring.

Production of Maternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Totals
7	—	—	—	1	1	—	1	—	1	—	—	—	—	—	—	4
8	1	—	3	2	4	2	2	2	3	1	1	—	—	—	—	21
9	—	3	4	6	6	6	5	1	3	2	4	1	—	—	1	42
10	—	2	6	2	6	10	6	10	6	1	1	1	1	1	—	53
11	—	2	1	2	5	2	2	3	2	4	1	1	2	—	1	28
12	—	2	3	4	1	2	3	2	—	—	3	—	1	—	—	21
13	—	2	1	5	3	2	—	—	—	3	4	—	—	—	—	20
14	—	—	1	—	—	2	2	3	1	1	1	1	—	—	—	12
15	—	—	1	1	—	1	1	1	—	1	—	—	—	—	—	6
16	—	—	—	3	1	—	1	—	—	1	—	—	—	—	—	6
17	—	—	—	1	—	—	—	1	1	1	—	—	—	—	—	4
18	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	1	11	20	27	27	27	23	23	18	15	15	4	4	1	2	218

TABLE XI.

Correlation of Paternal Granddams and their Offspring.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	Totals
7	—	1	—	—	1	—	2	1	2	3	2	—	—	—	—	—	1	—	—	—	—	13
8	1	6	3	8	7	6	10	5	11	10	9	2	1	—	2	4	2	—	—	—	4	91
9	1	5	4	10	4	11	13	—	17	14	18	3	1	—	2	4	3	—	—	—	1	111
10	—	8	4	14	8	5	7	9	9	9	21	5	2	1	3	2	4	—	—	—	1	112
11	1	4	2	9	6	5	10	3	5	10	16	—	5	1	2	5	6	—	—	—	1	91
12	—	5	3	8	7	4	10	2	10	5	20	6	4	1	2	4	—	—	—	—	1	92
13	—	2	4	6	4	7	7	7	6	4	21	3	5	2	1	—	1	—	—	—	—	80
14	—	3	1	4	1	—	5	6	4	5	11	3	1	1	—	—	2	—	—	—	—	47
15	1	2	1	1	2	4	2	5	2	1	3	1	2	—	—	—	—	—	—	—	—	27
16	—	—	—	2	—	1	—	1	—	2	1	—	1	—	—	—	—	—	—	—	—	8
17	—	1	1	—	1	—	—	1	—	—	1	—	3	2	1	—	—	—	—	—	—	11
18	—	1	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	—	—	4
19	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
20	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	2
Totals	4	39	23	62	41	43	66	40	68	63	126	23	26	8	13	19	19	—	—	—	8	691

TABLE XII.

Correlation of Paternal Granddams and Dams.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Dams in Pounds of Butter Fat.

	7	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	Totals
7	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	1	—	—	—	—	3
8	—	—	—	4	2	1	—	1	4	—	4	—	1	—	—	—	—	—	—	—	1	18
9	—	—	—	1	—	2	1	1	1	2	4	—	—	—	1	—	—	—	—	—	2	15
10	1	1	—	1	3	2	—	1	8	2	3	2	—	—	1	1	—	—	—	—	—	26
11	—	—	—	1	—	1	1	—	3	1	10	—	—	—	2	—	1	—	—	—	1	21
12	—	—	1	3	—	4	2	—	3	1	6	2	—	—	—	—	1	—	—	—	1	24
13	—	3	—	1	1	—	—	2	5	3	2	—	2	1	—	2	1	—	—	—	2	25
14	1	—	—	2	—	1	1	2	—	1	5	3	3	—	1	1	2	—	—	—	—	23
15	—	—	—	—	—	1	2	2	—	—	—	1	1	—	—	1	1	—	—	—	—	9
16	—	1	—	2	—	1	1	2	—	2	3	1	—	—	2	—	1	—	—	—	1	17
17	—	1	1	1	—	—	—	1	—	—	—	—	—	—	—	3	—	—	—	—	—	7
18	—	—	—	—	—	—	—	—	—	—	2	1	1	—	—	—	—	—	—	—	—	4
19	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	3
27	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	2	6	2	16	6	13	9	14	24	12	41	11	8	1	7	8	8	—	—	—	8	196

TABLE XIII.

Correlation of Age of Cows at First Record and Age of Corresponding Dams at Time of their "Most Remarkable Record." All Animals under Five Years Old.

Age of Dams—Years.

Age of Offspring—Years.

	1·75	*2·25*	*2·75*	*3·25*	*3·75*	*4·25*	*4·75*	Totals
1·75	12	30	10	25	21	25	27	150
2·25	26	70	27	53	56	82	56	370
2·75	5	11	7	11	25	22	22	103
3·25	6	9	3	16	10	19	13	76
3·75	3	9	1	13	13	10	8	57
4·25	4	5	4	6	4	7	9	39
4·75	3	2	3	6	5	2	5	26
Totals	59	136	55	130	134	167	140	821

TABLES TO FACILITATE THE COMPUTATION OF THE PROBABLE ERRORS OF THE CHIEF CONSTANTS OF SKEW FREQUENCY DISTRIBUTIONS.

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THE general theory of the probable errors of the constants of skew frequency distributions was originally given with illustrations by Pearson and Filon, *Phil. Trans.* Vol. 191, A (1898), pp. 229—311. The values there deduced depend on the form of special frequency curve adopted, and involve considerable arithmetical work for each individual case. In these frequency investigations the fundamental constants are the well-known β_1 and β_2. Every frequency character expressible in terms of β_1 and β_2 can have its probable error determined, provided we know the probable errors of β_1 and β_2 and the correlation in deviations between β_1 and β_2. General expressions for the probable errors (or the S.D.'s Σ_{β_1}, Σ_{β_2}) of β_1 and β_2 as well as the correlation $R_{\beta_1\beta_2}$ of deviations in β_1 and β_2, together with the probable error of the criterion (or its S.D. $= \Sigma_K$) were first given by Pearson, *Phil. Trans.* Vol. 198, A (1902). These involve a knowledge of β_6, β_5, β_4 and β_3*, further constants of the distribution which can only be found if the numerical values of μ_5, μ_6, μ_7 and μ_8 have been in some way determined. Now it has been shown that with the total frequencies usual in practice these high moments are subject to very large percentage errors†, rendering their use extremely undesirable, even if we could overcome our natural repugnance to the great labour of calculating them.

But we have to bear in mind that the *exact* value of a probable error is not usually desired. What we more often require is a rough determination of its

* $\beta_3 = \mu_3\mu_6/\mu_2^4$, $\beta_4 = \mu_6/\mu_2^3$, $\beta_5 = \mu_7\mu_3/\mu_2^5$, $\beta_6 = \mu_8/\mu_2^4$.

† Pearson: "On the General Theory of Skew Correlation and Non-Linear Regression," *Drapers' Company Research Memoirs* (Dulau & Co., 1905), p. 8.

magnitude in order that we may appreciate whether a certain quantity is or is not really significant. Accordingly it had been the practice of the Biometric Laboratory, if a frequency was only moderately a-normal, to use the values of the higher β's in determining probable errors, which would flow from the assumption of normality; if on the other hand the distribution had values of β_1 and $\beta_2 - 3$ differing a good deal from zero, to assume that the higher moments might be obtained from a skew distribution of Type III. (i.e. a distribution for which $2\beta_2 - 3\beta_1 - 6 = 0$). The justification of such hypotheses lies in the fact that if our data are to be of much value, the probable errors must themselves be small, hence in calculating these errors it is legitimate to insert into the formulae for them values for the β's that only differ from their true values by small quantities. Such insertion can only introduce second order, and therefore for our purposes usually unimportant, changes in the probable error. This point is emphasised when we remember the large percentage errors of the high β's. For example, it is usual to take the probable error of a standard deviation $\sigma = \cdot 67449\, \sigma/\sqrt{2n}$, but its true value $= \cdot 67449 \dfrac{\sigma}{\sqrt{2n}} (1 + \frac{1}{2}\eta)^{\frac{1}{2}}$; the former really results from assuming the kurtosis, $\eta = \beta_2 - 3$, to have the value zero of the normal curve. There are very few arguments made from probable error which would be seriously affected if the probable error were altered by 25 per cent. of its value, or if η took values from $-\cdot 875$ to $1 \cdot 125$, i.e. we might give β_2 any value between $2 \cdot 125$ and $4 \cdot 125$ to get in practice a sufficiently close result.

Now the object of the present paper is to extend this idea by applying a method of determining β_3, β_4, β_5 and β_6 still more exact than the methods indicated above. It is well-known that the frequency curves in common use are deduced from the integral of

$$\frac{1}{y}\frac{dy}{dx} = \frac{ax + b}{c_0 + c_1 x + c_2 x^2},$$

and that this really assumes the condition that the coefficients of higher terms in the denominator on the right, e.g. c_3, c_4, etc., are all zero. These conditions involve a finite difference relation between the successive moments, first published in 1903*, and enable us to determine any higher moment from the first three μ_2, μ_3 and μ_4. Such a finite difference momental equation actually exists for all probability frequency distributions of the hypergeometrical series type, which cover so wide a range of chance problems.

It is practically impossible to determine in a large percentage of cases whether the higher moments do or do not within their probable errors obey this finite difference relation, for the reason above stated, i.e. the high values of their probable errors. The present tables assume that they do; in other words β_3, β_4,

* *Biometrika*, Vol. II., p. 281.

β_5 and β_6 are calculated from β_1 and β_2, on the assumption that the values of them obtained from the finite difference momental formula are sufficiently accurate to use in Pearson's formulae for Σ_{β_1}, Σ_{β_2}, $R_{\beta_1\beta_2}$ and Σ_{sk}, etc.

The following are the finite difference β-formulae used:

$$\beta_n \text{ (even)} = (n+1)\left\{\frac{\beta_{n-1}}{2} + \left(1+\frac{\alpha}{2}\right)\beta_{n-2}\right\}\Big/\left(1-\frac{n-1}{2}\alpha\right),$$

$$\beta_n \text{ (odd)} = (n+1)\left\{\frac{\beta_1}{2}\beta_{n-1} + \left(1+\frac{\alpha}{2}\right)\beta_{n-2}\right\}\Big/\left(1-\frac{n-1}{2}\alpha\right),$$

where $\alpha = (2\beta_2 - 3\beta_1 - 6)/(\beta_2 - 3)$.

The process of calculation adopted was as follows:

Fundamental values of β_1, β_2 were adopted; these are indicated in Table VI., and the resulting values of β_3, β_4, β_5, β_6 calculated by the above formula were then found to seven figures. These are tabulated to six figures as they may be of service for the determination of other constants as occasion arises.

The values of κ_2 for the different values of β_1 and β_2 being known, it was seen that a very simple diagram would permit of a statistician ascertaining at once from his values of β_1 and β_2 the type of his frequency distribution. In fact this diagram brings out very suggestively the normal curve "point" (G), the Types II., III., V. and VII.* "lines" and the Types I., IV. and VI. "areas" of occurrence. By aid of this diagram and a reasonable consideration of the probable errors of his β_1 and β_2 the statistician can readily determine within what limits he is justified in using any special type of frequency curve for given data.

The following values of analytic constants for the fundamental values were then found $\sqrt{N}\Sigma_{\beta_1}$ (Table I.), $\sqrt{N}\Sigma_{\beta_2}$ (Table II.), $R_{\beta_1\beta_2}$ (Table III.), and intermediate values deduced by interpolation. It is believed that these interpolated values recorded to two places of decimals only are amply sufficient for the practical uses to which these tables will be put.

From these tables were found, again using interpolation, the probable errors of the two chief desired physical constants: (i) d, the distance from mean to mode or the modal divergence, and (ii) Sk, the skewness of the distribution. In Table IV. we have the values of $\sqrt{N}\Sigma_d/\sigma$ and in Table V. the values of $\sqrt{N}\Sigma_{sk}$ provided. The actual quantities tabled in Tables I., II., IV. and V. are such that when multiplied by $\cdot 67449/\sqrt{N}$ we obtain immediately the required probable errors. The value of this numerical factor can at once, however, be extracted from Winifred Gibson's Tables for computing probable errors†.

* It is convenient to call $y = y_0\left(1+\frac{x^2}{a^2}\right)^{-n}$ Type VII., see *Biometrika*, Vol. IV., p. 174.

† *Biometrika*, Vol. IV., p. 385.

In the actual construction of the tables fundamental values of β_1 and β_2 were taken covering the usual range of these quantities in actual practice. The tables are, however, limited by certain considerations, which are not without suggestiveness for theoretical frequency discussions. Along a certain line, indicated in our diagram, β_6 becomes infinite. But this is statistically impossible. Hence, either we never get frequency distributions having such values of β_1 and β_2, or if we do the finite difference moment formula cannot approximate in such cases to the true state of affairs, and we must introduce an additional coefficient c_3 into the denominator of the right-hand side of the fundamental differential equation. The discussion and classification of such curves are now in hand. On the diagram the areas where they are absolutely needful are marked *Heterotypic*.

It is believed that the diagrams and tables now published will prove extremely useful to the biometrician. Having determined his β_1 and β_2, he will be able to see at a glance whether his frequency distribution may be safely treated by the usual types or is heterotypic. If it be one of the usual types, he will know at once how to classify it. Next an examination of Tables I., II., III. and IV., or such examination with a short interpolation, will give him with sufficient practical accuracy the probable errors of β_1, β_2, the modal divergence (d) and the skewness (Sk). Should other and more elaborate probable errors be required, they will be deducible from Tables I., II. and III., or by using Table VI., for all the usually desired constants depend upon the quantities therein tabulated.

The two diagrams represent, (A), a practical working diagram covering the customary range of β_1 and β_2 and, (B), a diagram on a small scale showing the whole nature of the distribution of the type curves from the theoretical standpoint.

The upper part of either diagram is bounded by the line $4\beta_2 - 3\beta_1 = 0$, because it has been shown that β_2 is of necessity $> \frac{3}{4}\beta_1$*. The normal curve is represented by a point $\beta_1 = 0$, $\beta_2 = 3$ marked G on the diagram.

The transition Types III. and V. are given by values of β_1, β_2 on the line

$$2\beta_2 - 3\beta_1 - 6 = 0,$$

and on the cubic

$$\beta_1(\beta_2 + 3)^2 = 4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6),$$

respectively.

Type II. is a particular case of Type I. for $\beta_1 = 0$, and Type VII. a particular case of Type IV. for $\beta_1 = 0$; they are thus represented by portions of the vertical running through the "normal point" G. While the upper boundary of the diagram lies in the U-curve part of Type I., to be discussed below, the lower boundary is fixed by the line $8\beta_2 - 15\beta_1 - 36 = 0$ along which the finite difference moment formula first fails. This matter needs a little consideration.

* See Pearson: *Phil. Trans.* Vol. 186, A, pp. 343—414.

DIAGRAM A.

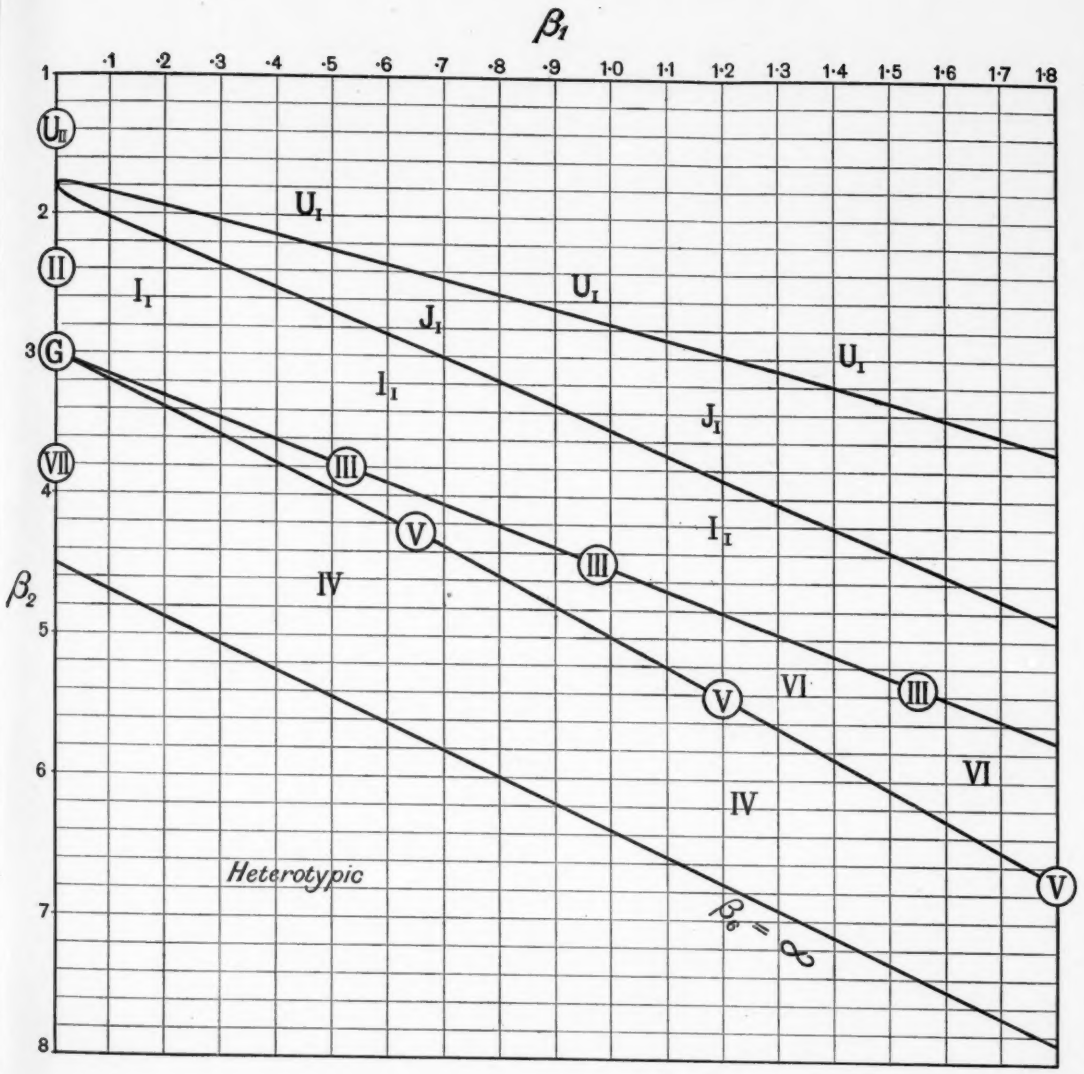
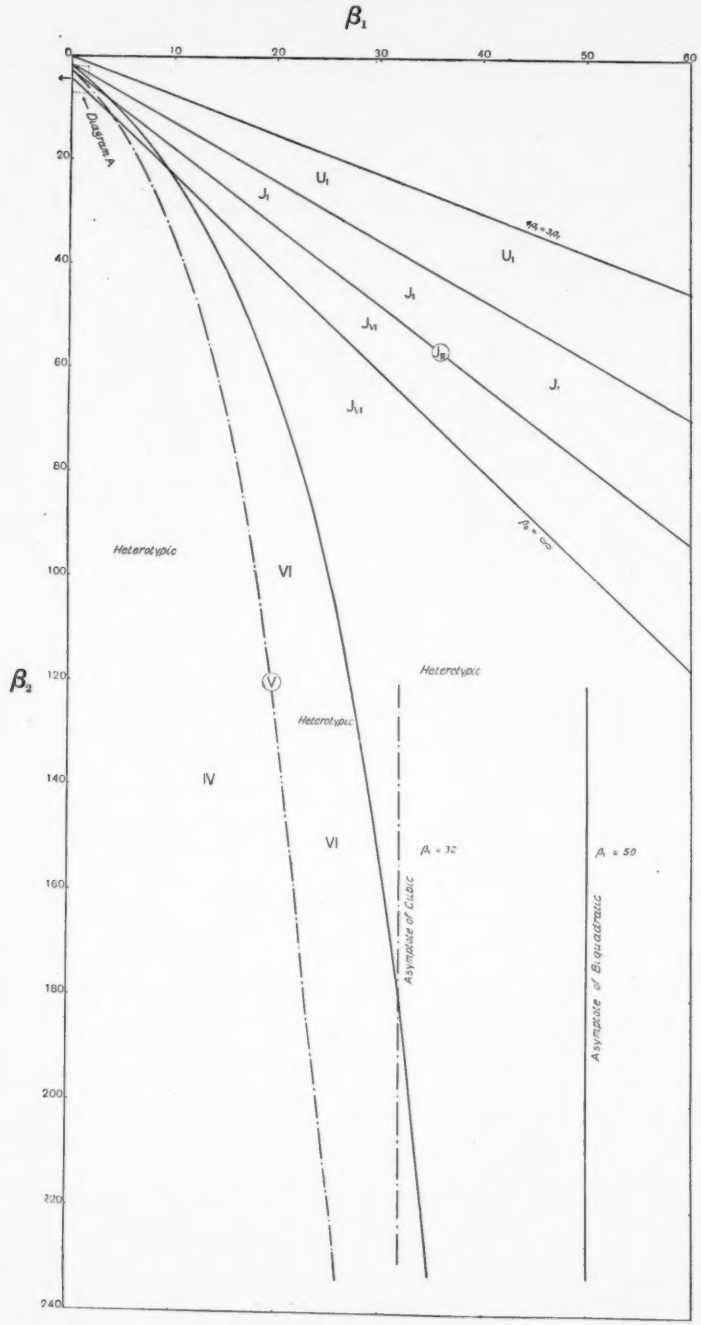

β_1
·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8 ·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8
1
2
3
4
5
6
7
8
β_2
U_{II}
II
G
VII
U_I
I_I
J_I
III
V
IV
VI
Heterotypic
$\beta_6 = \infty$

DIAGRAM B.


β1
0
10
20
30
40
50
60
β2
Diagram A
U1
J1
U1
J1
JII
JI
JII
Heterotypic
VI
Heterotypic
Heterotypic
IV
VI
Heterotypic
β1 = 32
β1 = 50
Asymptote of Cubic
Asymptote of Biquadratic

The equation to Type IV. is

$$y = y_0 \left\{1 + \frac{x^2}{a^2}\right\}^{-\frac{1}{2}(r+2)} e^{-\nu \tan^{-1}(x/a)},$$

with the reduction formula*

$$\mu_{n+1} = \frac{n}{r-n}\left(\frac{a^2\mu_{n-1}}{\cos^2\phi} - 2a\tan\phi\,\mu_n\right),$$

where $\tan\phi = \nu/r$ and $r = 6(\beta_2 - \beta_1 - 1)/(2\beta_2 - 3\beta_1 - 6)$. It is clear that when $r = n$, μ_{n+1} becomes infinite. Now the probable errors of β_1 and β_2 require us to go as far as μ_8. Hence r must be greater than 7 if we are to use this formula, and this gives us at once the limiting line

$$8\beta_2 - 15\beta_1 - 36 = 0.$$

Of course the difference formula would also fail for μ_5, μ_6 or μ_7, if r were equal to 4, 5 or 6. But all the resulting lines lie outside the above line, which is all we need take into account. The failure of the difference formula is easily seen if we remember that

$$\frac{1}{y}\frac{dy}{dx} = \frac{a_0 + a_1 x}{c_0 + c_1 x + c_2 x^2}$$

leads at once for this type to

$$\int_{-\infty}^{+\infty} x^n (a_0 + a_1 x)\, y\, dx = \int_{-\infty}^{+\infty} (c_0 + c_1 x + c_2 x^2)\, x^n\, dy$$

or
$$N(a_0\mu_n + a_1\mu_{n+1}) = -N(c_0 n\mu_{n-1} + c_1(n+1)\mu_n + c_2(n+2)\mu_{n+1}) + \left[y(c_0 x^n + c_1 x^{n+1} + c_2 x^{n+2})\right]_{-\infty}^{+\infty}.$$

The difference formula above follows from supposing the term between brackets to vanish at the limits.

But this it will not do unless

$$\left[\frac{x^{n+2}}{(a^2 + x^2)^{\frac{1}{2}(r+2)}}\right]_{x=\infty} = 0,$$

or if a be finite, unless r be $> n$, or to apply present results $r > 7$.

Of course for any real data μ_8 may become large, but it cannot actually become infinite. Fairly good fits—owing to the agreement of the first four moments—may be found even near the line $8\beta_2 - 15\beta_1 - 36 = 0$, but if we want to get the probable errors of β_1 and β_2 in this neighbourhood, it is best to calculate the higher moments and β_3, β_4, β_5 and β_6 from the actual data. Outside this line

* See *Biometrika*, Vol. II., p. 281.

we have marked the area as *Heterotypic*, because theoretically we need to introduce further terms into the denominator of our expression, i.e. to use c_3.

We now pass to a consideration of the subtypes of Type I. The equation is*

$$y = y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2},$$

where m_1 and m_2 are roots of the quadratic

$$m^2 - m(r-2) + \epsilon - r + 1 = 0$$

and

$$r = 6(\beta_2 - \beta_1 - 1)/(3\beta_1 - 2\beta_2 + 6),$$

$$\epsilon = \frac{r^2}{4 + \frac{1}{4}\beta_1 (r+2)^2/(r+1)}.$$

Now m_1 and m_2 will either be both positive or both negative if $\epsilon - r + 1$ is positive, or the curve $\epsilon - r + 1 = 0$ separates the area of J-curves or modeless curves from the area of modal curves (I_1 curves) and the area of anti-modal curves or U-curves.

$\epsilon - r + 1 = 0$ is the biquadratic

$$\beta_1 (8\beta_2 - 9\beta_1 - 12)/(4\beta_2 - 3\beta_1) = (10\beta_2 - 12\beta_1 - 18)^2/(\beta_2 + 3)^2.$$

This biquadratic was traced by expressing it in the form:

$$\beta_1 = 4(1 + 2\alpha)^2 (2 + \alpha)/(2 + 3\alpha),$$

and then finding β_1 for a series of values of α and determining β_2 from the equation

$$\alpha = (2\beta_2 - 3\beta_1 - 6)/(\beta_2 + 3).$$

Within the loop of the biquadratic all curves are J-curves, and the term "skewness" loses its essential meaning. Within this area, it will be noticed, our tables do not give the probable error of the skewness or of the modal divergence. Above this loop and up to the line $4\beta_2 - 3\beta_1 = 0$, we are in the range of U-curves and the skewness signifies the ratio to the standard deviation of the distance from mean to anti-mode. Below the loop we are in the customary Type I. area with m_1 and m_2 both positive.

Our second Diagram, B, shows what becomes of the biquadratic limiting the J-shaped curves. It first meets the Type III. line, and at this point Type III. curves become J-curves† and cease to have a true mode distinct from the asymptote value. The biquadratic then passes into the Type VI. area and Type VI. curves become J-curves beyond this. It never crosses, however, into

* Pearson: *Phil. Trans.* Vol. 186, A, pp. 367—371.

† Pearson: *Phil. Trans.* Vol. 186, A, p. 374 and Plate 9, Fig. 5.

Type IV. area, for the cubic which bounds Type IV. asymptotes to the vertical line $\beta_1 = 32$, and the biquadratic asymptotes to the vertical line $\beta_1 = 50$ as shown on the diagram.

Of course much of Diagram B extends beyond the values of β_1 and β_2 that we are familiar with in actual frequency distributions. For practical purposes Diagram A drawn to a much larger scale suffices, and guides the statistician to the appropriate type and to the probable errors. Nevertheless Diagram B will indicate many points of much theoretical interest, and serves to show where failure in curve fitting is likely to arise.

TABLE I.

Values of $\sqrt{N}\,\Sigma_{\beta_1}$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	0·00	0·58	0·93	1·15	1·37	1·57	1·77	1·97	2·17	2·38	2·58	2·80	3·02	3·24	3·46	3·71
2·1	0·00	0·59	0·95	1·12	1·30	1·50	1·70	1·90	2·10	2·29	2·48	2·69	2·91	3·12	3·34	3·57
2·2	0·00	0·60	0·97	1·13	1·30	1·48	1·67	1·86	2·05	2·22	2·41	2·61	2·81	3·01	3·22	3·44
2·3	0·00	0·62	0·99	1·15	1·32	1·49	1·67	1·84	2·02	2·19	2·36	2·55	2·74	2·93	3·12	3·32
2·4	0·00	0·64	1·02	1·20	1·37	1·54	1·70	1·86	2·02	2·18	2·34	2·51	2·68	2·85	3·03	3·22
2·5	0·00	0·66	1·05	1·26	1·45	1·61	1·76	1·91	2·05	2·19	2·33	2·49	2·65	2·81	2·97	3·14
2·6	0·00	0·69	1·10	1·34	1·54	1·72	1·86	2·00	2·13	2·25	2·37	2·50	2·64	2·78	2·92	3·08
2·7	0·00	0·73	1·15	1·42	1·64	1·83	1·96	2·09	2·22	2·32	2·42	2·53	2·65	2·77	2·90	3·05
2·8	0·00	0·77	1·22	1·51	1·75	1·94	2·07	2·20	2·32	2·41	2·50	2·60	2·70	2·80	2·93	3·05
2·9	0·00	0·81	1·30	1·61	1·87	2·06	2·20	2·33	2·44	2·53	2·62	2·70	2·79	2·89	2·99	3·09
3·0	0·00	0·87	1·40	1·73	2·01	2·20	2·34	2·47	2·57	2·67	2·76	2·84	2·92	3·00	3·09	3·18
3·1	0·00	0·94	1·53	1·86	2·17	2·35	2·51	2·64	2·75	2·85	2·94	3·00	3·08	3·15	3·23	3·30
3·2	0·00	1·02	1·67	2·02	2·33	2·52	2·71	2·84	2·95	3·05	3·14	3·22	3·27	3·33	3·40	3·46
3·3	0·00	1·12	1·82	2·20	2·50	2·71	2·92	3·06	3·18	3·28	3·37	3·44	3·50	3·55	3·60	3·65
3·4	0·00	1·24	1·99	2·38	2·68	2·93	3·14	3·30	3·43	3·53	3·63	3·70	3·75	3·79	3·83	3·87
3·5	0·00	1·37	2·16	2·57	2·89	3·17	3·39	3·56	3·69	3·81	3·91	3·98	4·02	4·06	4·10	4·12
3·6	0·00	1·50	2·33	2·78	3·11	3·43	3·65	3·84	3·99	4·12	4·22	4·29	4·33	4·37	4·40	4·41
3·7	0·00	1·64	2·50	2·99	3·36	3·70	3·93	4·14	4·31	4·44	4·54	4·61	4·66	4·70	4·72	4·74
3·8	0·00	1·78	2·67	3·20	3·62	3·97	4·23	4·46	4·64	4·77	4·87	4·95	5·00	5·05	5·07	5·09
3·9	0·00	1·93	2·86	3·43	3·89	4·25	4·54	4·79	4·97	5·11	5·23	5·32	5·38	5·43	5·46	5·48
4·0	0·00	2·10	3·07	3·69	4·17	4·55	4·87	5·13	5·32	5·48	5·62	5·72	5·79	5·84	5·88	5·89
4·1	—	—	3·29	3·87	4·47	4·87	5·21	5·49	5·69	5·87	6·03	6·15	6·23	6·28	6·32	6·33
4·2	—	—	3·53	4·19	4·79	5·21	5·58	5·88	6·10	6·30	6·46	6·60	6·69	6·75	6·80	6·81
4·3	—	—	3·78	4·52	5·13	5·58	5·97	6·29	6·54	6·75	6·93	7·07	7·18	7·25	7·29	7·31
4·4	—	—	4·05	4·85	5·49	5·98	6·40	6·74	7·01	7·24	7·42	7·57	7·68	7·76	7·80	7·83
4·5	—	—	4·33	5·18	5·88	6·42	6·87	7·23	7·52	7·75	7·95	8·10	8·21	8·29	8·34	8·37
4·6	—	—	—	—	—	—	7·37	7·76	8·07	8·30	8·51	8·66	8·76	8·85	8·91	8·95
4·7	—	—	—	—	—	—	7·90	8·31	8·64	8·90	9·11	9·25	9·35	9·44	9·50	9·54
4·8	—	—	—	—	—	—	8·46	8·88	9·24	9·54	9·75	9·89	9·99	10·08	10·14	10·18
4·9	—	—	—	—	—	—	9·05	9·47	9·86	10·21	10·42	10·58	10·69	10·78	10·84	10·80
5·0	—	—	—	—	—	—	9·66	10·08	10·50	10·90	11·19	11·33	11·44	11·53	11·60	11·64
5·1	—	—	—	—	—	—	—	—	—	—	—	—	12·26	12·36	12·42	12·43
5·2	—	—	—	—	—	—	—	—	—	—	—	—	13·10	13·26	13·29	13·29
5·3	—	—	—	—	—	—	—	—	—	—	—	—	13·98	14·15	14·18	14·18
5·4	—	—	—	—	—	—	—	—	—	—	—	—	14·91	15·05	15·10	15·11
5·5	—	—	—	—	—	—	—	—	—	—	—	—	15·90	15·98	16·05	16·07
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE I.—(*continued*).

Values of $\sqrt{N}\,\Sigma_{\beta_1}$.

β_1															
0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
3·96	4·21	4·47	4·73	5·00	5·27	5·55	5·83	6·12	6·41	6·71	7·01	7·31	7·62	7·94	*2·0*
3·80	4·03	4·27	4·53	4·80	5·07	5·34	5·62	5·90	6·18	6·48	6·77	7·07	7·37	7·69	*2·1*
3·66	3·88	4·11	4·36	4·63	4·88	5·15	5·42	5·69	5·96	6·25	6·54	6·84	7·14	7·45	*2·2*
3·52	3·74	3·96	4·20	4·46	4·71	4·96	5·22	5·48	5·75	6·02	6·31	6·61	6·91	7·21	*2·3*
3·41	3·62	3·83	4·05	4·29	4·54	4·78	5·03	5·28	5·55	5·82	6·10	6·38	6·68	6·97	*2·4*
3·32	3·51	3·71	3·92	4·15	4·38	4·61	4·85	5·10	5·36	5·62	5·89	6·16	6·45	6·74	*2·5*
3·25	3·42	3·60	3·80	4·01	4·23	4·45	4·68	4·92	5·17	5·43	5·68	5·94	6·22	6·51	*2·6*
3·20	3·35	3·51	3·69	3·89	4·10	4·32	4·54	4·76	5·00	5·24	5·48	5·73	6·00	6·28	*2·7*
3·18	3·32	3·47	3·63	3·80	4·00	4·21	4·41	4·62	4·84	5·07	5·30	5·53	5·79	6·06	*2·8*
3·19	3·32	3·45	3·60	3·75	3·92	4·11	4·30	4·49	4·70	4·91	5·12	5·34	5·59	5·85	*2·9*
3·27	3·38	3·49	3·61	3·74	3·87	4·03	4·21	4·39	4·58	4·78	4·98	5·19	5·42	5·68	*3·0*
3·38	3·47	3·57	3·67	3·77	3·89	4·02	4·16	4·31	4·48	4·66	4·85	5·05	5·28	5·53	*3·1*
3·53	3·60	3·68	3·76	3·84	3·93	4·03	4·15	4·28	4·43	4·59	4·75	4·92	5·15	5·40	*3·2*
3·70	3·75	3·81	3·88	3·95	4·02	4·10	4·19	4·28	4·42	4·56	4·69	4·84	5·04	5·28	*3·3*
3·90	3·93	3·97	4·03	4·08	4·14	4·20	4·26	4·34	4·45	4·56	4·66	4·78	4·97	5·18	*3·4*
4·14	4·17	4·19	4·22	4·26	4·30	4·34	4·39	4·45	4·52	4·60	4·68	4·79	4·93	5·12	*3·5*
4·42	4·44	4·45	4·47	4·49	4·51	4·54	4·57	4·62	4·67	4·74	4·78	4·81	4·95	5·09	*3·6*
4·74	4·75	4·76	4·76	4·77	4·78	4·79	4·81	4·84	4·87	4·90	4·92	4·95	5·04	5·13	*3·7*
5·10	5·10	5·09	5·08	5·08	5·07	5·07	5·06	5·06	5·08	5·09	5·11	5·14	5·18	5·22	*3·8*
5·49	5·48	5·46	5·44	5·42	5·40	5·37	5·35	5·33	5·32	5·32	5·33	5·34	5·35	5·37	*3·9*
5·89	5·88	5·86	5·83	5·80	5·76	5·72	5·69	5·65	5·62	5·60	5·58	5·57	5·57	5·59	*4·0*
6·33	6·32	6·30	6·26	6·21	6·16	6·11	6·06	6·02	5·98	5·94	5·91	5·88	5·86	5·86	*4·1*
6·80	6·79	6·76	6·71	6·65	6·60	6·54	6·48	6·42	6·36	6·31	6·27	6·24	6·21	6·18	*4·2*
7·30	7·28	7·25	7·19	7·13	7·07	7·01	6·93	6·87	6·80	6·74	6·67	6·62	6·57	6·53	*4·3*
7·83	7·80	7·76	7·71	7·65	7·58	7·51	7·44	7·37	7·28	7·20	7·12	7·05	6·98	6·92	*4·4*
8·38	8·36	8·32	8·28	8·21	8·14	8·07	7·99	7·90	7·81	7·71	7·61	7·51	7·42	7·34	*4·5*
8·96	8·95	8·91	8·86	8·79	8·72	8·64	8·55	8·45	8·35	8·24	8·13	8·00	7·90	7·80	*4·6*
9·57	9·57	9·53	9·47	9·40	9·33	9·24	9·14	9·04	8·93	8·82	8·69	8·55	8·42	8·31	*4·7*
10·20	10·23	10·16	10·10	10·05	9·97	9·88	9·77	9·67	9·55	9·42	9·28	9·14	9·00	8·87	*4·8*
10·91	10·92	10·87	10·80	10·74	10·66	10·57	10·44	10·32	10·18	10·04	9·90	9·76	9·63	9·50	*4·9*
11·66	11·65	11·61	11·55	11·48	11·39	11·29	11·17	11·04	10·90	10·77	10·62	10·46	10·30	10·14	*5·0*
12·45	12·43	12·38	12·32	12·24	12·14	12·03	11·91	11·78	11·64	11·50	11·34	11·15	10·96	10·82	*5·1*
13·28	13·25	13·20	13·13	13·04	12·92	12·80	12·67	12·54	12·40	12·24	12·07	11·88	11·72	11·54	*5·2*
14·16	14·12	14·07	13·98	13·87	13·76	13·63	13·47	13·35	13·20	13·02	12·84	12·66	12·48	12·30	*5·3*
15·09	15·06	15·00	14·90	14·78	14·65	14·51	14·36	14·22	14·05	13·81	13·67	13·48	13·29	13·11	*5·4*
16·06	16·02	15·96	15·87	15·76	15·63	15·49	15·33	15·17	15·00	14·81	14·61	14·40	14·18	13·97	*5·5*
—	—	17·02	16·91	16·79	16·67	16·51	16·34	16·18	15·95	15·70	15·50	15·30	15·07	14·84	*5·6*
—	—	18·14	17·99	17·88	17·75	17·58	17·40	17·23	16·94	16·70	16·47	16·26	16·04	15·77	*5·7*
—	—	19·34	19·13	19·02	18·87	18·69	18·48	18·26	17·98	17·74	17·50	17·26	17·01	16·76	*5·8*
—	—	20·57	20·36	20·20	20·03	19·84	19·62	19·39	19·11	18·84	18·59	18·32	18·05	17·78	*5·9*
—	—	21·86	21·65	21·45	21·25	21·03	20·79	20·54	20·29	20·02	19·76	19·47	19·18	18·90	*6·0*
—	—	—	—	—	—	22·36	22·18	21·92	21·61	21·31	20·97	20·61	20·30	20·13	*6·1*
—	—	—	—	—	—	23·77	23·61	23·32	23·00	22·63	22·22	21·82	21·50	21·29	*6·2*
—	—	—	—	—	—	25·33	25·09	24·74	24·38	24·00	23·55	23·13	22·78	22·50	*6·3*
—	—	—	—	—	—	26·95	26·64	26·27	25·86	25·43	26·00	24·52	24·12	23·82	*6·4*
—	—	—	—	—	—	28·61	28·18	27·73	27·30	26·89	26·46	26·06	25·65	25·24	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	27·67	27·21	26·75	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	29·40	28·90	28·35	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	31·15	30·61	29·94	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	33·02	32·41	31·72	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	34·89	34·16	33·59	*7·0*

β_2

TABLE II.

Values of $\sqrt{N}\Sigma_{\beta_2}$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	1·41	1·60	1·74	1·93	2·11	2·28	2·44	2·60	2·77	2·94	3·12	3·29	3·46	3·65	3·86	4·05
2·1	1·57	1·76	1·89	2·00	2·10	2·20	2·35	2·51	2·68	2·86	3·05	3·24	3·44	3·64	3·84	4·04
2·2	1·75	1·94	2·07	2·16	2·20	2·28	2·40	2·53	2·68	2·85	3·04	3·23	3·43	3·63	3·83	4·03
2·3	1·95	2·16	2·28	2·35	2·42	2·49	2·58	2·67	2·78	2·92	3·09	3·27	3·45	3·64	3·84	4·03
2·4	2·18	2·39	2·53	2·60	2·72	2·82	2·90	2·96	3·02	3·10	3·22	3·35	3·50	3·68	3·86	4·03
2·5	2·46	2·68	2·83	2·97	3·09	3·19	3·27	3·31	3·32	3·36	3·47	3·53	3·63	3·75	3·88	4·03
2·6	2·78	3·03	3·24	3·38	3·52	3·60	3·66	3·69	3·69	3·70	3·75	3·78	3·83	3·87	3·95	4·07
2·7	3·17	3·48	3·71	3·87	3·98	4·03	4·08	4·12	4·11	4·09	4·07	4·06	4·06	4·06	4·07	4·15
2·8	3·64	4·02	4·26	4·42	4·52	4·58	4·60	4·59	4·57	4·52	4·44	4·39	4·34	4·32	4·31	4·34
2·9	4·22	4·65	4·94	5·11	5·20	5·22	5·18	5·13	5·07	4·99	4·90	4·80	4·70	4·63	4·60	4·61
3·0	4·90	5·48	5·76	5·89	5·95	5·93	5·86	5·76	5·65	5·53	5·41	5·30	5·20	5·12	5·05	5·00
3·1	5·75	6·41	6·72	6·88	6·90	6·82	6·70	6·54	6·38	6·22	6·07	5·92	5·79	5·69	5·60	5·53
3·2	6·77	7·55	7·90	8·00	7·97	7·83	7·63	7·42	7·21	7·03	6·86	6·70	6·53	6·39	6·26	6·14
3·3	8·00	8·83	9·22	9·30	9·22	9·02	8·80	8·53	8·29	8·05	7·83	7·60	7·38	7·20	7·02	6·84
3·4	9·37	10·28	10·68	10·76	10·67	10·46	10·20	9·91	9·62	9·31	9·01	8·73	8·44	8·18	7·92	7·66
3·5	10·85	11·75	12·31	12·52	12·46	12·25	11·95	11·60	11·24	10·86	10·45	10·03	9·63	9·26	8·90	8·54
3·6	12·67	13·74	14·40	14·78	14·53	14·21	13·80	13·38	12·95	12·55	12·10	11·60	11·06	10·54	10·02	9·55
3·7	14·78	15·98	16·78	17·09	16·93	16·53	16·05	15·58	15·09	14·61	14·08	13·49	12·74	12·02	11·36	10·80
3·8	17·50	18·83	19·83	20·03	19·78	19·36	18·76	18·22	17·64	16·98	16·25	15·30	14·42	13·60	12·88	12·27
3·9	20·80	22·50	23·68	23·81	23·34	22·67	21·98	21·14	20·29	19·45	18·58	17·54	16·50	15·54	14·77	14·06
4·0	24·74	26·83	28·47	28·05	27·24	26·29	25·25	24 18	23·03	22·02	21·01	20·01	19·04	18·12	17·23	16·36
4·1	—	—	35·00	34·17	32·88	31·36	29·77	28·13	26·60	25·12	23·82	22·64	21·54	20·53	19·56	18·62
4·2	—	—	43·3	41·4	39·2	37·2	35·2	33·2	31·2	29·2	27·4	26·0	24·7	23·4	22·3	21·3
4·3	—	—	55·3	51·6	48·0	44·6	41·2	38·6	36·2	33·8	31·8	30·1	28·5	26·9	25·6	24·3
4·4	—	—	72·7	66·0	59·7	54·1	49·5	45·7	42·1	39·2	36·8	34·8	32·9	31·0	29·2	27·7
4·5	—	—	96·5	82·7	72·7	65·3	59·8	54·7	50·8	47·2	44·0	41·0	38·5	36·2	34·1	32·0
4·6	—	—	—	—	—	—	75·0	68·0	62·2	56·9	52·2	48·2	45·1	42·2	39·6	37·2
4·7	—	—	—	—	—	—	101·3	87·2	76·8	68·3	62·0	56·9	52·7	49·1	45·9	42·8
4·8	—	—	—	—	—	—	140·0	115·2	96·2	82·6	72·7	66·1	60·9	56·7	52·9	49·3
4·9	—	—	—	—	—	—	204·5	150·8	122·3	102·5	89·1	80·2	72·4	66·6	61·4	56·7
5·0	—	—	—	—	—	—	325·7	206·0	154·2	126·8	110·1	96·9	86·6	78·1	71·2	65·6
5·1	—	—	—	—	—	—	—	—	—	—	—	—	103·6	94·4	85·9	78·0
5·2	—	—	—	—	—	—	—	—	—	—	—	—	130·4	116·4	104·0	91·0
5·3	—	—	—	—	—	—	—	—	—	—	—	—	175·2	144·8	124·4	109·6
5·4	—	—	—	—	—	—	—	—	—	—	—	—	224·4	178·0	151·0	132·6
5·5	—	—	—	—	—	—	—	—	—	—	—	—	340·8	246·0	195·3	163·2
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE II.—(*continued*).

Values of $\sqrt{N}\,\Sigma_{\beta_2}$.

β_1															
0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
4·24	4·43	4·62	4·81	5·00	5·19	5·38	5·56	5·75	5·94	6·12	6·30	6·49	6·67	6·84	*2·0*
4·23	4·41	4·59	4·77	4·96	5·15	5·34	5·53	5·72	5·90	6·08	6·27	6·47	6·65	6·83	*2·1*
4·22	4·39	4·56	4·74	4·93	5·12	5·31	5·50	5·69	5·87	6·05	6·24	6·44	6·63	6·82	*2·2*
4·19	4·35	4·51	4·69	4·87	5·05	5·23	5·42	5·61	5·80	5·99	6·18	6·38	6·58	6·78	*2·4*
4·18	4·34	4·50	4·67	4·85	5·03	5·21	5·39	5·58	5·77	5·96	6·15	6·35	6·54	6·74	*2·5*
4·20	4·35	4·50	4·67	4·84	5·01	5·20	5·36	5·54	5·72	5·91	6·11	6·30	6·49	6·68	*2·6*
4·26	4·38	4·52	4·68	4·84	5·01	5·18	5·34	5·51	5·67	5·85	6·05	6·25	6·44	6·62	*2·7*
4·40	4·50	4·60	4·72	4·86	5·03	5·19	5·34	5·49	5·65	5·83	6·02	6·21	6·39	6·58	*2·8*
4·63	4·67	4·73	4·82	4·93	5·05	5·20	5·35	5·50	5·66	5·82	6·00	6·18	6·36	6·54	*2·9*
4·98	4·97	4·99	5·03	5·10	5·18	5·28	5·39	5·52	5·66	5·82	5·98	6·15	6·33	6·51	*3·0*
5·47	5·42	5·38	5·34	5·36	5·37	5·41	5·48	5·58	5·70	5·83	5·97	6·14	6·32	6·52	*3·1*
6·03	5·92	5·83	5·75	5·67	5·62	5·60	5·62	5·68	5·78	5·90	6·03	6·18	6·34	6·52	*3·2*
6·67	6·51	6·35	6·22	6·09	6·00	5·90	5·94	5·92	5·95	6·01	6·12	6·25	6·37	6·53	*3·3*
7·41	7·17	6·95	6·77	6·61	6·48	6·29	6·26	6·24	6·22	6·22	6·26	6·34	6·47	6·61	*3·4*
8·22	7·92	7·64	7·38	7·17	6·99	6·84	6·72	6·63	6·57	6·54	6·53	6·56	6·61	6·71	*3·5*
9·14	8·80	8·51	8·23	7·98	7·70	7·53	7·40	7·22	7·09	6·99	6·98	6·95	6·92	6·93	*3·6*
10·34	9·94	9·58	9·25	8·96	8·66	8·36	8·14	7·90	7·75	7·61	7·51	7·42	7·34	7·23	*3·7*
11·77	11·29	10·82	10·37	9·98	9·62	9·31	9·03	8·73	8·51	8·29	8·11	7·94	7·78	7·60	*3·8*
13·42	12·85	12·31	11·79	11·30	10·86	10·41	10·02	9·64	9·34	9·03	8·77	8·52	8·30	8·10	*3·9*
15·58	14·84	14·10	13·42	12·79	12·20	11·64	11·13	10·65	10·21	9·83	9·51	9·20	8·92	8·67	*4·0*
17·72	16·85	16·01	15·21	14·44	13·70	13·00	12·34	11·73	11·17	10·67	10·24	9·87	9·58	9·32	*4·1*
20·2	19·2	18·3	17·3	16·4	15·5	14·7	14·0	13·3	12·6	12·0	11·5	11·0	10·5	10·3	*4·2*
23·1	22·0	20·9	19·8	18·7	17·6	16·7	15·8	15·0	14·2	13·5	12·8	12·3	11·8	11·3	*4·3*
26·3	25·0	23·8	22·5	21·3	20·1	19·0	18·0	17·1	16·1	15·3	14·6	13·9	13·2	12·6	*4·4*
30·1	28·4	26·8	25·3	23·9	22·6	21·4	20·3	19·3	18·3	17·3	16·4	15·6	14·8	14·1	*4·5*
34·7	32·5	30·5	28·8	27·3	25·6	24·2	22·9	21·7	20·6	19·5	18·4	17·5	16·7	16·1	*4·6*
40·0	37·4	35·0	32·8	30·9	29·2	27·6	26·1	24·7	23·3	22·0	20·9	19·8	18·8	18·1	*4·7*
46·1	43·1	40·3	37·7	35·3	33·2	31·4	29·7	28·0	26·4	25·0	23·6	22·3	21·2	20·3	*4·8*
52·4	48·8	46·8	43·1	40·2	37·8	35·6	33·6	31·6	29·8	28·1	26·6	25·1	23·8	22·7	*4·9*
60·6	56·1	52·3	48·8	45·5	42·6	40·0	37·6	35·4	33·4	31·5	29·8	28·1	26·6	25·2	*5·0*
71·2	65·1	60·6	56·5	52·6	49·1	45·8	43·1	40·5	38·0	35·6	33·6	31·7	30·0	28·4	*5·1*
83·0	76·4	70·5	65·4	60·6	56·3	52·5	49·3	46·3	43·4	40·4	38·0	35·6	33·6	31·7	*5·2*
98·8	89·6	81·9	75·6	70·2	65·0	60·2	56·2	52·5	48·9	45·5	42·6	39·9	37·4	35·2	*5·3*
118·4	105·2	96·0	87·6	80·4	74·0	68·3	63·4	58·8	54·7	51·0	47·7	44·5	41·5	38·9	*5·4*
141·4	124·0	111·2	99·6	91·2	84·0	77·4	71·2	65·7	61·2	56·9	52·9	49·4	46·2	43·5	*5·5*
—	—	131·2	117·4	105·2	96·0	87·3	79·3	72·8	67·8	63·2	58·6	54·8	51·4	48·8	*5·6*
—	—	160·0	142·4	126·4	113·4	102·2	93·0	84·4	77·3	71·1	65·6	60·8	57·2	54·7	*5·7*
—	—	199·2	175·8	154·8	134·2	119·6	107·0	97·2	88·4	80·6	74·4	69·4	64·9	61·5	*5·8*
—	—	266·0	221·6	192·8	163·6	142·8	128·0	114·6	104·0	94·6	86·0	79·6	74·4	70·2	*5·9*
—	—	378·1	284·0	231·5	198·2	171·6	151·5	136·2	123·8	112·8	103·4	94·8	87·5	81·4	*6·0*
—	—	—	—	—	—	206·3	186·3	167·5	150·0	134·2	121·5	111·0	101·8	92·8	*6·1*
—	—	—	—	—	—	264	232	205	280	160	141	128	116	107	*6·2*
—	—	—	—	—	—	350	297	251	216	188	164	148	132	122	*6·3*
—	—	—	—	—	—	510	376	308	263	225	196	172	152	138	*6·4*
—	—	—	—	—	—	889	524	387	313	264	229	200	177	161	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	237	204	184	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	286	249	220	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	363	305	268	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	485	392	333	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	747	510	416	*7·0*

β_2

TABLE III.

Values of $R_{\beta_1\beta_2}$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	0·00	·570	·706	·770	·823	·863	·894	·917	·935	·949	·960	·968	·976	·983	·989	·992
2·1	0·00	·557	·685	·755	·798	·838	·870	·895	·914	·936	·948	·959	·969	·977	·983	·986
2·2	0·00	·551	·672	·728	·771	·814	·847	·874	·896	·919	·934	·948	·960	·968	·975	·980
2·3	0·00	·550	·663	·719	·765	·799	·829	·859	·880	·900	·919	·935	·948	·957	·965	·971
2·4	0·00	·551	·660	·712	·752	·787	·814	·843	·867	·886	·905	·920	·936	·945	·954	·962
2·5	0·00	·554	·659	·706	·745	·776	·805	·834	·858	·878	·893	·908	·924	·933	·941	·949
2·6	0·00	·557	·662	·706	·742	·773	·799	·825	·851	·871	·883	·898	·913	·921	·930	·938
2·7	0·00	·557	·668	·710	·744	·773	·800	·825	·846	·863	·876	·889	·902	·911	·920	·928
2·8	0·00	·556	·674	·716	·750	·779	·803	·826	·842	·858	·871	·883	·893	·901	·910	·919
2·9	0·00	·550	·680	·724	·760	·787	·810	·830	·844	·857	·868	·878	·887	·895	·903	·912
3·0	0·00	·542	·684	·738	·774	·796	·816	·835	·847	·857	·867	·875	·883	·890	·898	·906
3·1	0·00	·534	·687	·744	·781	·808	·825	·840	·850	·858	·867	·874	·882	·889	·897	·903
3·2	0·00	·524	·688	·746	·786	·811	·830	·842	·852	·860	·868	·875	·882	·889	·896	·902
3·3	0·00	·512	·688	·747	·788	·814	·832	·845	·855	·863	·870	·876	·882	·888	·895	·901
3·4	0·00	·501	·686	·748	·790	·816	·833	·848	·858	·865	·872	·878	·883	·889	·895	·900
3·5	0·00	·490	·681	·747	·790	·815	·833	·849	·860	·867	·873	·879	·884	·890	·895	·900
3·6	0·00	·477	·676	·745	·788	·813	·832	·850	·860	·867	·874	·880	·886	·891	·896	·900
3·7	0·00	·462	·670	·741	·784	·810	·831	·848	·859	·867	·874	·881	·887	·892	·897	·901
3·8	0·00	·450	·662	·736	·779	·803	·828	·845	·858	·866	·874	·882	·888	·893	·898	·901
3·9	0·00	·438	·654	·720	·770	·796	·822	·841	·856	·866	·875	·882	·889	·894	·899	·903
4·0	0·00	·422	·645	·713	·760	·788	·816	·837	·853	·865	·873	·881	·888	·894	·899	·903
4·1	—	—	·630	·702	·748	·780	·807	·830	·849	·862	·871	·880	·887	·892	·897	·901
4·2	—	—	·608	·682	·733	·770	·793	·822	·842	·857	·867	·877	·884	·890	·894	·899
4·3	—	—	·580	·658	·712	·753	·784	·811	·832	·848	·860	·871	·878	·885	·890	·897
4·4	—	—	·540	·623	·688	·732	·770	·796	·819	·837	·851	·863	·872	·880	·887	·894
4·5	—	—	·481	·590	·657	·709	·749	·780	·804	·824	·841	·853	·865	·874	·882	·890
4·6	—	—	—	—	—	—	·716	·754	·784	·808	·828	·842	·856	·868	·877	·886
4·7	—	—	—	—	—	—	·674	·723	·759	·788	·812	·830	·846	·860	·870	·879
4·8	—	—	—	—	—	—	·615	·681	·727	·761	·791	·815	·834	·849	·861	·872
4·9	—	—	—	—	—	—	·532	·620	·680	·728	·766	·795	·818	·835	·850	·862
5·0	—	—	—	—	—	—	·362	·534	·628	·687	·731	·767	·798	·822	·837	·851
5·1	—	—	—	—	—	—	—	—	—	—	—	—	·768	·799	·820	·837
5·2	—	—	—	—	—	—	—	—	—	—	—	—	·730	·768	·799	·820
5·3	—	—	—	—	—	—	—	—	—	—	—	—	·679	·729	·769	·799
5·4	—	—	—	—	—	—	—	—	—	—	—	—	·608	·686	·736	·774
5·5	—	—	—	—	—	—	—	—	—	—	—	—	·496	·601	·674	·724
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE III.—(*continued*).

Values of $R_{\beta_1\beta_2}$.

β_1

0·80	0·85	0·90	0·95	1·00	1·05	1·10	1·15	1·20	1·25	1·30	1·35	1·40	1·45	1·50	β_2
·993	·995	·997	·999	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	2·0
·989	·991	·994	·996	·998	·998	·999	·999	·999	1·000	1·000	1·000	1·000	1·000	1·000	2·1
·983	·986	·989	·992	·995	·996	·997	·998	·998	·999	1·000	1·000	1·000	1·000	1·000	2·2
·976	·980	·984	·988	·992	·993	·994	·995	·997	·998	·999	·999	·999	1·000	1·000	2·3
·968	·973	·978	·983	·987	·989	·991	·993	·995	·996	·998	·998	·999	·999	1·000	2·4
·958	·965	·972	·977	·982	·985	·988	·990	·992	·994	·996	·997	·998	·999	1·000	2·5
·947	·956	·964	·970	·976	·980	·984	·986	·988	·991	·993	·995	·997	·998	·999	2·6
·937	·947	·957	·963	·968	·973	·977	·980	·983	·986	·989	·992	·995	·997	·998	2·7
·928	·939	·949	·955	·960	·965	·970	·974	·978	·981	·985	·989	·992	·994	·996	2·8
·921	·932	·942	·947	·952	·957	·963	·968	·972	·976	·980	·984	·988	·990	·992	2·9
·915	·923	·931	·937	·943	·948	·954	·960	·966	·971	·975	·979	·983	·986	·988	3·0
·909	·915	·922	·929	·936	·942	·947	·953	·959	·965	·970	·974	·978	·981	·984	3·1
·907	·912	·918	·924	·930	·936	·941	·946	·952	·958	·963	·968	·973	·977	·980	3·2
·906	·909	·914	·919	·925	·930	·935	·940	·946	·951	·956	·961	·966	·971	·975	3·3
·905	·908	·912	·916	·920	·925	·930	·935	·940	·945	·950	·954	·958	·964	·974	3·4
·904	·907	·910	·914	·918	·922	·926	·931	·936	·940	·944	·948	·952	·958	·965	3·5
·904	·907	·910	·914	·918	·921	·924	·928	·932	·935	·938	·942	·946	·952	·959	3·6
·905	·907	·910	·914	·917	·920	·923	·927	·930	·933	·935	·937	·940	·946	·953	3·7
·905	·908	·911	·914	·917	·920	·922	·925	·928	·930	·932	·934	·936	·941	·948	3·8
·906	·909	·911	·914	·917	·919	·921	·924	·927	·929	·931	·933	·935	·939	·944	3·9
·906	·909	·912	·914	·917	·919	·921	·923	·926	·928	·930	·932	·934	·936	·940	4·0
·905	·908	·911	·914	·917	·919	·921	·923	·925	·927	·930	·931	·932	·933	·934	4·1
·905	·907	·910	·913	·916	·919	·921	·923	·924	·926	·929	·929	·930	·930	·929	4·2
·903	·906	·910	·913	·916	·918	·920	·922	·924	·926	·929	·928	·928	·927	·924	4·3
·900	·904	·908	·912	·916	·918	·920	·922	·923	·926	·928	·927	·927	·925	·922	4·4
·897	·902	·906	·910	·915	·918	·920	·922	·923	·926	·928	·927	·926	·923	·920	4·5
·893	·898	·903	·908	·913	·916	·919	·920	·922	·925	·927	·926	·925	·923	·920	4·6
·887	·894	·900	·905	·910	·913	·917	·919	·921	·924	·926	·925	·925	·923	·922	4·7
·881	·890	·896	·901	·906	·910	·914	·917	·920	·923	·925	·926	·926	·925	·925	4·8
·874	·884	·890	·895	·901	·907	·911	·915	·919	·922	·925	·926	·927	·927	·928	4·9
·863	·875	·883	·889	·896	·903	·908	·913	·918	·922	·925	·927	·928	·930	·932	5·0
·851	·864	·875	·882	·890	·898	·905	·911	·917	·922	·925	·928	·931	·933	·936	5·1
·837	·852	·866	·875	·884	·892	·901	·909	·916	·921	·924	·928	·933	·937	·941	5·2
·820	·839	·853	·865	·876	·885	·895	·904	·913	·918	·923	·929	·935	·940	·945	5·3
·798	·818	·837	·853	·867	·877	·888	·898	·908	·915	·921	·928	·935	·941	·947	5·4
·764	·792	·817	·837	·854	·867	·880	·890	·900	·910	·918	·925	·933	·940	·947	5·5
—	—	·789	·815	·835	·852	·868	·880	·890	·904	·911	·917	·926	·935	·944	5·6
—	—	·750	·786	·811	·835	·854	·869	·880	·892	·901	·909	·917	·927	·938	5·7
—	—	·701	·748	·783	·811	·835	·852	·866	·879	·890	·897	·905	·915	·928	5·8
—	—	·640	·700	·748	·781	·810	·828	·846	·861	·875	·883	·892	·901	·913	5·9
—	—	·544	·639	·703	·746	·778	·802	·825	·842	·857	·867	·879	·886	·893	6·0
—	—	—	—	—	—	·741	·769	·796	·820	·837	·852	·866	·872	·873	6·1
—	—	—	—	—	—	·691	·727	·762	·792	·815	·836	·852	·858	·856	6·2
—	—	—	—	—	—	·628	·678	·724	·761	·790	·818	·838	·845	·842	6·3
—	—	—	—	—	—	·526	·606	·675	·724	·763	·793	·818	·831	·834	6·4
—	—	—	—	—	—	·354	·526	·619	·680	·726	·761	·791	·814	·831	6·5
—	—	—	—	—	—	—	—	—	—	—	—	·761	·790	·832	6·6
—	—	—	—	—	—	—	—	—	—	—	—	·721	·760	·837	6·7
—	—	—	—	—	—	—	—	—	—	—	—	·670	·727	·845	6·8
—	—	—	—	—	—	—	—	—	—	—	—	·600	·683	·857	6·9
—	—	—	—	—	—	—	—	—	—	—	—	·468	·602	·876	7·0

TABLE IV.

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75	0·80
2·0	3·54	—	—	—	—	—	—	—	—	—	3·03	2·44	2·10	1·80	1·58	1·42	1·30
2·1	2·15	4·36	—	—	—	—	—	—	—	—	—	—	—	3·10	2·53	2·16	1·88
2·2	1·87	2·75	9·65	—	—	—	—	—	—	—	—	—	—	—	3·91	3·17	2·60
2·3	1·64	1·86	3·00	—	—	—	—	—	—	—	—	—	—	—	—	—	3·78
2·4	1·46	1·58	2·07	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·46	1·67	2·08	2·87	4·04	5·21	—	—	—	—	—	—	—	—	—	—
2·6	1·28	1·37	1·58	1·98	2·60	3·42	4·43	6·72	—	—	—	—	—	—	—	—	—
2·7	1·25	1·30	1·50	1·83	2·34	2·98	3·75	5·06	7·48	—	—	—	—	—	—	—	—
2·8	1·23	1·28	1·43	1·71	2·11	2·60	3·17	4·12	5·28	7·45	—	—	—	—	—	—	—
2·9	1·22	1·27	1·38	1·60	1·90	2·27	2·69	3·20	3·84	4·78	6·65	—	—	—	—	—	—
3·0	1·23	1·26	1·34	1·51	1·73	1·98	2·29	2·63	3·06	3·58	4·28	5·18	6·43	8·24	10·89	—	—
3·1	1·25	1·27	1·32	1·44	1·58	1·76	2·00	2·23	2·54	2·94	3·42	3·93	4·52	5·50	6·76	8·66	—
3·2	1·27	1·28	1·30	1·38	1·48	1·60	1·75	1·92	2·13	2·37	2·72	3·12	3·54	4·21	5·07	6·22	8·00
3·3	1·29	1·29	1·28	1·32	1·39	1·47	1·55	1·68	1·83	2·03	2·27	2·57	2·90	3·39	4·04	4·66	5·53
3·4	1·30	1·29	1·28	1·29	1·31	1·37	1·45	1·54	1·63	1·79	2·00	2·24	2·51	2·88	3·36	3·86	4·50
3·5	1·31	1·30	1·29	1·27	1·25	1·30	1·37	1·45	1·54	1·66	1·83	2·03	2·26	2·55	2·89	3·18	3·61
3·6	1·32	1·31	1·30	1·26	1·22	1·26	1·32	1·40	1·50	1·61	1·74	1·89	2·08	2·31	2·56	2·86	3·24
3·7	1·31	1·31	1·31	1·26	1·22	1·25	1·30	1·37	1·46	1·57	1·69	1·82	1·97	2·14	2·34	2·62	2·95
3·8	1·30	1·31	1·32	1·28	1·25	1·27	1·32	1·38	1·46	1·55	1·65	1·76	1·88	2·03	2·20	2·43	2·69
3·9	1·29	1·33	1·35	1·33	1·30	1·32	1·36	1·41	1·48	1·56	1·64	1·73	1·84	1·96	2·11	2·27	2·49
4·0	1·27	1·37	1·40	1·39	1·39	1·40	1·42	1·46	1·51	1·58	1·65	1·73	1·83	1·94	2·06	2·19	2·36
4·1	—	—	1·47	1·48	1·50	1·51	1·53	1·55	1·57	1·61	1·66	1·75	1·85	1·94	2·03	2·13	2·26
4·2	—	—	1·58	1·62	1·64	1·65	1·65	1·65	1·65	1·66	1·70	1·77	1·85	1·94	2·03	2·12	2·22
4·3	—	—	1·75	1·77	1·78	1·79	1·78	1·76	1·75	1·75	1·78	1·83	1·90	1·97	2·05	2·13	2·23
4·4	—	—	1·98	1·97	1·95	1·94	1·93	1·90	1·88	1·89	1·92	1·96	2·01	2·07	2·13	2·20	2·29
4·5	—	—	2·27	2·20	2·15	2·11	2·10	2·09	2·09	2·10	2·12	2·15	2·18	2·22	2·26	2·32	2·38
4·6	—	—	—	—	—	—	2·44	2·40	2·38	2·36	2·34	2·36	2·38	2·40	2·42	2·45	2·48
4·7	—	—	—	—	—	—	2·93	2·85	2·78	2·71	2·67	2·65	2·64	2·63	2·62	2·61	2·61
4·8	—	—	—	—	—	—	3·74	3·52	3·33	3·16	3·07	3·00	2·95	2·90	2·85	2·81	2·80
4·9	—	—	—	—	—	—	5·44	4·64	4·16	3·87	3·63	3·45	3·32	3·24	3·11	3·05	3·03
5·0	—	—	—	—	—	—	10·66	6·83	5·53	4·84	4·37	4·04	3·79	3·62	3·47	3·37	3·31
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·46	4·21	3·99	3·85	3·74
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·05	4·73	4·47	4·24
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·19	5·66	5·27	4·92
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·96	7·00	6·24	5·74
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·81	10·89	8·87	7·64	6·81
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE IV.—(*continued*).

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_1														
0·85	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
1·20	1·13	1·07	1·02	·97	·92	·87	·83	·80	·76	·72	·68	·64	·60	*2·0*
1·64	1·49	1·38	1·29	1·21	1·14	1·08	1·03	·99	·94	·90	·86	·82	·78	*2·1*
2·22	1·97	1·80	1·66	1·54	1·45	1·38	1·32	1·26	1·20	1·14	1·08	1·02	·96	*2·2*
3·10	2·63	2·46	2·15	1·98	1·85	1·75	1·67	1·58	1·50	1·41	1·32	1·24	1·16	*2·3*
—	3·72	3·20	2·81	2·56	2·36	2·22	2·09	1·96	1·84	1·73	1·62	1·51	1·39	*2·4*
—	—	—	3·94	3·40	3·08	2·87	2·66	2·47	2·29	2·12	1·96	1·80	1·64	*2·5*
—	—	—	—	—	4·32	3·82	3·48	3·14	2·86	2·60	2·34	2·10	1·91	*2·6*
—	—	—	—	—	—	—	4·82	4·20	3·66	3·18	2·78	2·46	2·19	*2·7*
—	—	—	—	—	—	—	—	5·73	4·82	3·88	3·28	2·87	2·49	*2·8*
—	—	—	—	—	—	—	—	—	6·63	4·81	3·94	3·31	2·80	*2·9*
—	—	—	—	—	—	—	—	—	—	6·15	4·80	3·79	3·12	*3·0*
—	—	—	—	—	—	—	—	—	—	—	—	4·29	3·48	*3·1*
—	—	—	—	—	—	—	—	—	—	—	—	—	3·88	*3·2*
6·80	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*
5·38	6·55	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*
4·21	4·95	6·00	7·33	9·36	12·16	—	—	—	—	—	—	—	—	*3·5*
3·74	4·34	5·12	6·03	7·17	8·80	11·52	—	—	—	—	—	—	—	*3·6*
3·34	3·80	4·32	5·01	5·78	6·65	8·28	11·12	—	—	—	—	—	—	*3·7*
3·00	3·35	3·74	4·23	4·72	5·41	6·36	8·00	10·22	—	—	—	—	—	*3·8*
2·74	3·00	3·32	3·66	4·04	4·50	5·18	6·16	7·43	9·32	—	—	—	—	*3·9*
2·55	2·77	3·02	3·29	3·60	3·98	4·50	5·12	5·92	6·91	8·00	9·23	11·08	—	*4·0*
2·42	2·60	2·79	3·00	3·27	3·61	4·06	4·61	5·20	5·90	6·80	7·86	9·48	11·58	*4·1*
2·35	2·50	2·67	2·86	3·09	3·38	3·71	4·13	4·60	5·15	5·86	6·72	7·76	9·10	*4·2*
2·35	2·48	2·62	2·77	2·96	3·21	3·49	3·82	4·18	4·59	5·14	5·82	6·66	7·59	*4·3*
2·39	2·50	2·61	2·73	2·89	3·10	3·32	3·55	3·83	4·16	4·60	5·16	5·82	6·60	*4·4*
2·45	2·53	2·63	2·74	2·87	3·02	3·20	3·39	3·61	3·87	4·21	4·66	5·18	5·86	*4·5*
2·52	2·60	2·69	2·79	2·89	3·01	3·15	3·31	3·48	3·67	3·95	4·31	4·77	5·36	*4·6*
2·64	2·69	2·77	2·85	2·93	3·02	3·13	3·25	3·39	3·55	3·78	4·10	4·50	4·99	*4·7*
2·81	2·82	2·86	2·93	3·00	3·08	3·17	3·27	3·38	3·50	3·68	3·94	4·26	4·66	*4·8*
3·02	3·03	3·04	3·08	3·12	3·16	3·22	3·30	3·39	3·50	3·63	3·81	4·07	4·35	*4·9*
3·28	3·26	3·25	3·26	3·28	3·31	3·34	3·39	3·44	3·51	3·61	3·73	3·90	4·09	*5·0*
3·64	3·55	3·51	3·49	3·47	3·47	3·48	3·49	3·52	3·56	3·61	3·68	3·77	3·90	*5·1*
4·04	3·90	3·81	3·74	3·68	3·65	3·63	3·62	3·61	3·62	3·63	3·65	3·69	3·76	*5·2*
4·60	4·35	4·18	4·05	3·95	3·88	3·81	3·75	3·72	3·70	3·69	3·68	3·69	3·70	*5·3*
5·33	4·98	4·71	4·52	4·37	4·24	4·12	4·01	3·92	3·85	3·79	3·75	3·73	3·72	*5·4*
6·21	5·74	5·36	5·08	4·86	4·66	4·48	4·32	4·20	4·09	3·99	3·92	3·87	3·82	*5·5*
—	6·69	6·27	5·83	5·49	5·19	4·94	4·73	4·56	4·42	4·30	4·18	4·11	4·04	*5·6*
—	8·11	7·48	6·82	6·32	5·90	5·55	5·27	5·03	4·84	4·68	4·54	4·43	4·35	*5·7*
—	10·18	9·11	8·12	7·45	6·85	6·35	5·95	5·62	5·37	5·16	5·00	4·87	4·76	*5·8*
—	13·53	11·44	9·84	8·71	7·94	7·32	6·82	6·45	6·13	5·85	5·61	5·43	5·29	*5·9*
—	19·95	14·26	11·92	10·48	9·38	8·55	7·89	7·38	6·95	6·62	6·33	6·10	5·90	*6·0*
—	—	—	—	—	11·64	10·26	9·31	8·62	8·03	7·53	7·15	6·87	6·60	*6·1*
—	—	—	—	—	14·83	12·55	11·19	10·24	9·40	8·64	8·08	7·68	7·36	*6·2*
—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·65	8·22	*6·3*
—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·69	9·11	*6·4*
—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*
—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*
—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*
—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*
—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*
—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*

β_2

TABLE V.

Values of $\sqrt{N}\,\Sigma_{ak}$.

β_2 \ β_1	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	3·54	—	—	—	—	—	—	—	—	—	3·41	2·80	2·43	2·12	1·89	1·72
2·1	2·15	4·20	—	—	—	—	—	—	—	—	—	—	—	3·67	3·02	2·58
2·2	1·87	2·63	9·50	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10
2·3	1·64	1·78	2·88	—	—	—	—	—	—	—	—	—	—	—	—	—
2·4	1·46	1·49	2·02	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·41	1·62	2·02	2·80	4·06	5·08	—	—	—	—	—	—	—	—	—
2·6	1·28	1·30	1·43	1·75	2·18	2·82	3·65	4·96	—	—	—	—	—	—	—	—
2·7	1·25	1·25	1·31	1·52	1·84	2·29	2·85	3·68	4·84	—	—	—	—	—	—	—
2·8	1·23	1·22	1·24	1·36	1·59	1·89	2·21	2·70	3·36	4·36	—	—	—	—	—	—
2·9	1·22	1·20	1·20	1·28	1·43	1·63	1·86	2·19	2·60	3·33	4·30	—	—	—	—	—
3·0	1·23	1·21	1·20	1·25	1·34	1·48	1·62	1·84	2·12	2·49	3·00	3·76	4·62	6·02	8·12	—
3·1	1·25	1·22	1·21	1·23	1·27	1·36	1·50	1·67	1·88	2·16	2·50	3·06	3·76	4·72	6·10	8·08
3·2	1·27	1·23	1·22	1·22	1·23	1·29	1·40	1·53	1·70	1·90	2·17	2·58	3·17	3·97	4·87	5·83
3·3	1·29	1·25	1·23	1·21	1·21	1·24	1·33	1·44	1·58	1·74	1·94	2·27	2·73	3·28	3·88	4·53
3·4	1·30	1·27	1·24	1·21	1·20	1·23	1·29	1·38	1·49	1·61	1·78	2·04	2·36	2·75	3·18	3·63
3·5	1·31	1·29	1·25	1·21	1·20	1·22	1·28	1·35	1·43	1·54	1·68	1·87	2·09	2·37	2·68	2·98
3·6	1·32	1·30	1·26	1·22	1·20	1·22	1·26	1·32	1·40	1·50	1·61	1·75	1·91	2·13	2·39	2·65
3·7	1·31	1·31	1·28	1·24	1·20	1·23	1·27	1·32	1·39	1·47	1·56	1·67	1·80	1·98	2·19	2·40
3·8	1·30	1·32	1·30	1·27	1·23	1·25	1·28	1·33	1·38	1·46	1·54	1·63	1·75	1·88	2·04	2·22
3·9	1·29	1·34	1·33	1·30	1·28	1·28	1·30	1·34	1·39	1·45	1·53	1·61	1·71	1·82	1·96	2·12
4·0	1·27	1·36	1·38	1·36	1·35	1·35	1·36	1·39	1·43	1·48	1·56	1·63	1·72	1·81	1·92	2·04
4·1	—	—	1·46	1·45	1·45	1·44	1·44	1·46	1·50	1·54	1·60	1·67	1·74	1·82	1·91	2·03
4·2	—	—	1·58	1·57	1·57	1·56	1·55	1·56	1·59	1·62	1·67	1·72	1·78	1·85	1·93	2·02
4·3	—	—	1·75	1·74	1·73	1·71	1·70	1·69	1·70	1·72	1·76	1·81	1·86	1·92	1·98	2·06
4·4	—	—	1·95	1·94	1·92	1·90	1·88	1·85	1·86	1·87	1·90	1·93	1·97	2·01	2·06	2·12
4·5	—	—	2·26	2·19	2·13	2·09	2·07	2·06	2·05	2·06	2·07	2·08	2·11	2·15	2·19	2·23
4·6	—	—	—	—	—	—	2·48	2·42	2·37	2·35	2·33	2·31	2·31	2·32	2·33	2·35
4·7	—	—	—	—	—	—	3·11	2·93	2·82	2·73	2·65	2·60	2·57	2·54	2·52	2·50
4·8	—	—	—	—	—	—	3·78	3·53	3·35	3·21	3·08	2·97	2·89	2·82	2·76	2·71
4·9	—	—	—	—	—	—	5·48	4·66	4·17	3·87	3·63	3·44	3·30	3·17	3·07	3·01
5·0	—	—	—	—	—	—	11·12	6·96	5·52	4·82	4·36	4·02	3·77	3·58	3·45	3·37
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·45	4·16	4·00	3·87
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·02	4·72	4·49
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·18	5·66	5·26
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·76	6·80	6·22
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·80	10·67	8·87	7·71
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE V.—(*continued*).

Values of $\sqrt{N}\,\Sigma_{sk}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
1·59	1·48	1·39	1·30	1·24	1·19	1·14	1·10	1·06	1·02	·99	·95	·91	·87	·83	*2·0*
2·20	1·95	1·80	1·68	1·58	1·52	1·47	1·42	1·37	1·32	1·26	1·20	1·15	1·10	1·05	*2·1*
3·22	2·65	2·29	2·08	1·98	1·91	1·84	1·78	1·72	1·66	1·59	1·52	1·45	1·38	1·31	*2·2*
5·23	3·80	3·04	2·75	2·53	2·40	2·30	2·21	2·12	2·03	1·94	1·85	1·76	1·67	1·58	*2·3*
—	—	4·29	3·64	3·31	3·12	2·94	2·78	2·63	2·49	2·36	2·23	2·10	1·98	1·86	*2·4*
—	—	—	—	4·77	4·27	3·84	3·55	3·29	3·06	2·85	2·66	2·49	2·32	2·15	*2·5*
—	—	—	—	—	—	5·72	5·00	4·39	3·94	3·54	3·20	2·93	2·67	2·44	*2·6*
—	—	—	—	—	—	—	—	6·25	5·20	4·46	3·88	3·42	3·08	2·74	*2·7*
—	—	—	—	—	—	—	—	—	7·05	5·68	4·78	4·03	3·54	3·05	*2·8*
—	—	—	—	—	—	—	—	—	—	7·45	6·00	4·85	4·11	3·37	*2·9*
—	—	—	—	—	—	—	—	—	—	—	7·80	6·05	4·77	3·70	*3·0*
—	—	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10	*3·1*
7·00	—	—	—	—	—	—	—	—	—	—	—	—	—	4·58	*3·2*
5·30	6·24	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*
4·16	4·71	5·60	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*
3·38	3·89	4·59	5·58	6·85	8·38	11·48	—	—	—	—	—	—	—	—	*3·5*
2·95	3·38	3·99	4·66	5·50	6·48	7·55	9·92	—	—	—	—	—	—	—	*3·6*
2·67	3·05	3·49	3·97	4·52	5·22	6·00	7·36	9·42	—	—	—	—	—	—	*3·7*
2·44	2·75	3·08	3·43	3·80	4·25	4·78	5·64	7·08	9·02	—	—	—	—	—	*3·8*
2·29	2·53	2·79	3·07	3·38	3·70	4·15	4·77	5·62	6·89	8·76	—	—	—	—	*3·9*
2·20	2·38	2·58	2·82	3·06	3·33	3·69	4·14	4·77	5·50	6·42	7·40	8·57	10·12	—	*4·0*
2·16	2·31	2·47	2·65	2·85	3·09	3·34	3·72	4·15	4·61	5·14	5·84	6·80	8·44	11·00	*4·1*
2·14	2·26	2·40	2·55	2·71	2·88	3·10	3·37	3·67	4·03	4·44	5·04	5·94	7·12	8·67	*4·2*
2·16	2·27	2·38	2·50	2·65	2·80	2·96	3·14	3·37	3·65	4·01	4·55	5·28	6·18	7·21	*4·3*
2·20	2·30	2·41	2·52	2·64	2·76	2·89	3·04	3·23	3·48	3·78	4·22	4·78	5·44	6·26	*4·4*
2·29	2·35	2·44	2·53	2·63	2·75	2·88	3·02	3·20	3·40	3·65	3·97	4·40	4·91	5·56	*4·5*
2·39	2·43	2·48	2·56	2·66	2·77	2·89	3·01	3·16	3·34	3·55	3·79	4·16	4·55	5·10	*4·6*
2·52	2·54	2·58	2·63	2·71	2·80	2·90	3·01	3·14	3·29	3·47	3·67	3·96	4·28	4·72	*4·7*
2·70	2·72	2·74	2·78	2·83	2·89	2·97	3·06	3·16	3·28	3·41	3·58	3·80	4·06	4·42	*4·8*
2·98	2·97	2·96	2·97	3·00	3·04	3·08	3·14	3·21	3·30	3·40	3·53	3·68	3·88	4·16	*4·9*
3·31	3·25	3·21	3·20	3·21	3·22	3·24	3·26	3·31	3·36	3·43	3·51	3·60	3·73	3·96	*5·0*
3·75	3·64	3·55	3·49	3·45	3·42	3·41	3·41	3·43	3·44	3·46	3·51	3·57	3·65	3·78	*5·1*
4·28	4·10	3·96	3·85	3·76	3·68	3·63	3·60	3·57	3·55	3·53	3·53	3·55	3·60	3·67	*5·2*
4·93	4·69	4·48	4·29	4·13	4·02	3·92	3·84	3·76	3·68	3·63	3·60	3·58	3·59	3·62	*5·3*
5·78	5·42	5·09	4·80	4·56	4·40	4·26	4·13	4·00	3·89	3·80	3·73	3·68	3·65	3·63	*5·4*
6·94	6·32	5·82	5·40	5·07	4·84	4·64	4·46	4·30	4·17	4·06	3·95	3·87	3·80	3·75	*5·5*
—	—	6·75	6·22	5·79	5·46	5·19	4·97	4·77	4·60	4·44	4·30	4·19	4·10	4·01	*5·6*
—	—	8·15	7·30	6·73	6·26	5·91	5·61	5·34	5·10	4·90	4·73	4·59	4·47	4·38	*5·7*
—	—	10·20	8·76	7·98	7·26	6·76	6·34	5·99	5·68	5·44	5·24	5·06	4·91	4·78	*5·8*
—	—	13·53	10·83	9·66	8·71	7·90	7·28	6·78	6·40	6·10	5·84	5·62	5·44	5·28	*5·9*
—	—	19·96	14·30	12·02	10·51	9·39	8·56	7·90	7·39	6·96	6·61	6·33	6·10	5·89	*6·0*
—	—	—	—	—	—	11·64	10·26	9·31	8·56	8·03	7·53	7·15	6·82	6·54	*6·1*
—	—	—	—	—	—	14·83	12·55	11·19	10·13	9·30	8·64	8·08	7·63	7·24	*6·2*
—	—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·55	8·01	*6·3*
—	—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·60	8·92	*6·4*
—	—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*

β_2

TABLE VI (i).

Values of β_3.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	0	0	0	0	0						
0·1	0·48493	0·68971	0·94286	1·25688	1·64906	2·14375					
0·2	0·91585	1·32958	1·78182	2·37049	3·10000	4·01176					
0·3	1·29873	1·85270	2·53043	3·36094	4·38305	5·65000	7·2368				
0·4	1·63902	2·34286	3·20000	4·24478	5·52277	7·09474	9·0462				
0·5	1·94118	2·78126	3·80000	5·03582	6·53846	8·37500	10·6364				
0·6	2·20909	3·17350	4·33846	5·80178	7·44706	9·51429	12·0414	15·1585			
0·7	2·44615	3·52441	4·82222	6·38287	8·26202	10·53182	13·2885	16·6624			
0·8	2·65532	3·83820	5·25714	6·95698	8·99462	11·44375	14·4000	17·9932			
0·9	2·83917	4·12064	5·64828	7·47438	9·65454	12·26250	15·3940	19·1758	23·7791		
1·0	3·00000	4·36842	6·00000	7·94121	10·24999	13·00000	16·2857	20·2308	25·0000		
1·1	3·13980	4·59081	6·31613	8·36246	10·78796	13·66538	17·0877	21·1750	26·0857	32·0328	
1·2	3·26038	4·78812	6·60000	8·74286	11·27443	14·26667	17·8105	22·0225	27·0546	33·1082	
1·3	3·36330	4·96250	6·85454	9·08619	11·71461	14·81071	18·4633	22·7851	27·9217	34·0635	
1·4	3·45000	5·11589	7·08235	9·39582	12·11304	15·30345	19·0536	23·4727	28·7000	34·9164	42·3613
1·5	3·52174	5·25000	7·28571	9·67501	12·47368	15·75000	19·5864	24·0937	29·4000	35·6786	43·1538

TABLE VI (ii).

Values of β_4.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	5·00000	8·92856	15·0000	23·7288	31·0000						
0·1	5·27356	9·41054	15·7973	25·7430	41·7660	69·3682					
0·2	5·44361	9·75086	16·2648	26·4018	42·5000	69·4796					
0·3	5·53293	9·86224	16·4907	26·6520	42·5613	68·5776	114·4732				
0·4	5·55998	9·91072	16·5385	26·6144	42·1807	67·0888	109·4534				
0·5	5·53802	9·87751	16·4545	26·3742	41·5076	65·2679	104·4652				
0·6	5·47791	9·81734	16·2732	26·1077	40·6453	63·2707	99·6442	162·125			
0·7	5·38824	9·63895	16·0200	25·5026	39·6623	61·1946	95·0525	151·253			
0·8	5·27513	9·45991	15·7143	24·9478	38·6061	59·1016	90·7143	141·707			
0·9	5·14437	9·25645	15·3706	24·3462	37·5099	57·0279	86·6331	133·240	210·995		
1·0	5·00000	9·02746	15·0000	23·7495	36·3971	55·0000	82·8022	125·664	195·000		
1·1	4·84537	8·78075	14·6111	23·0744	35·2835	53·0316	79·2091	118·839	181·299	286·374	
1·2	4·68319	8·53522	14·2105	22·4107	34·1811	51·1309	75·8392	112·653	169·394	261·436	
1·3	4·51562	8·27700	13·8032	21·7535	33·0971	49·3447	72·6772	107·016	158·930	240·845	
1·4	4·34440	8·01454	13·3931	21·1002	32·0367	47·5471	69·7076	101·850	149·643	223·304	343·147
1·5	4·17097	7·75000	12·9832	20·4546	31·0037	45·9038	66·9117	97·112	141·333	208·129	313·704

TABLE VI (iii).

Values of β_3.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	0	0	0	0	0						
0·1	1·99086	4·39480	9·3207	19·9714	45·9387	128·529					
0·2	3·59438	8·03374	16·5960	34·7825	76·6000	193·361					
0·3	4·86677	10·68765	22·2196	45·7142	97·2263	228·104	668·284				
0·4	5·85929	12·85477	26·5187	53·7090	111·0237	246·506	650·398				
0·5	6·51704	14·51540	29·7545	59·4655	120·0543	255·295	614·633				
0·6	7·17383	15·7892	32·1362	63·9266	125·6629	258·147	581·205	1618·635			
0·7	7·56616	16·6546	33·8306	66·2045	128·8283	257·225	550·107	1368·373			
0·8	7·81963	17·2668	34·9714	67·8804	130·2010	253·872	521·257	1196·612			
0·9	7·95777	17·6667	35·6658	68·7533	130·2587	248·937	495·375	1068·877	2769·42		
1·0	8·00000	17·8291	36·0000	69·0644	129·3434	243·000	469·637	968·318	2280·00		
1·1	7·96281	17·8472	36·0437	68·7730	127·7158	236·441	446·547	886·541	1945·69	5313·80	
1·2	7·86015	17·7503	35·8535	68·1357	125·5684	229·524	425·062	818·040	1700·98	4135·56	
1·3	7·70375	17·5396	35·4754	67·2181	123·0362	222·562	405·663	759·486	1512·94	3388·18	
1·4	7·50358	17·2423	34·9467	66·0678	120·5142	214·828	386·347	708·620	1363·20	2870·08	7265·31
1·5	7·26808	16·8768	34·2983	64·7210	117·2460	207·227	368·843	663·926	1240·65	2488·62	5719·68

TABLE VI (iv).

Values of β_6.

β_1 \ β_2	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	14·0000	39·0649	105·000	290·678	868·015						
0·1	16·4616	45·7741	124·835	355·508	1243·832	10228·33					
0·2	17·7296	50·2472	132·998	369·894	1190·700	6204·69					
0·3	18·1764	51·0927	134·215	361·909	1089·739	4485·38	107697·95				
0·4	18·0667	50·2458	131·337	344·886	977·506	3471·87	25413·18				
0·5	17·5474	48·7896	126·107	323·447	877·884	2792·19	13737·63				
0·6	16·8560	46·8558	119·601	303·252	784·431	2303·07	9048·43	119230·33			
0·7	15·9787	44·3106	112·492	277·658	701·500	1934·79	6534·78	40994·77			
0·8	15·0148	41·7081	105·200	255·716	628·450	1648·52	5045·80	22660·09			
0·9	14·0113	39·0906	97·984	235·072	564·277	1420·51	4024·45	14836·90	137288·7		
1·0	13·0000	36·4119	91·000	216·137	507·894	1235·50	3286·65	10612·25	57584·9		
1·1	12·0030	33·7916	84·339	198·263	456·575	1083·04	2741·39	8135·91	33078·5	797653·2	
1·2	11·0354	31·3418	78·047	181·987	414·455	955·78	2322·13	6314·06	21891·8	155693·9	
1·3	10·1070	28·9775	72·146	167·142	375·834	848·97	1994·05	5108·55	15690·2	75009·7	
1·4	9·2240	26·7355	66·637	153·582	342·057	755·79	1726·18	4219·50	11846·9	44891·9	565740
1·5	8·3899	24·6268	61·512	141·477	310·976	676·32	1508·92	3544·82	9281·2	30280·3	180793

ON THE RELATION BETWEEN THE SYMMETRY OF THE EGG, THE SYMMETRY OF SEGMENTATION, AND THE SYMMETRY OF THE EMBRYO IN THE FROG*.

SECOND COMMUNICATION.

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PART I.

In a paper published in Volume v. of this *Journal* I gave an account of the relation between three meridional planes in the egg and embryo of the common frog, namely, the Plane of Symmetry of the unsegmented egg (as determined by the position of the grey crescent which appears after fertilization), the Plane of the First Furrow of segmentation, and the Sagittal Plane of the embryo.

The examination of the angles between these planes in a fairly large number of cases showed (1) that there was a fair tendency for the Sagittal Plane to coincide with the First Furrow since angles of small value considerably preponderated over those of large, but at the same time very little correlation between the two; (2) that there was a greater tendency for the Sagittal Plane to coincide with the Plane of Symmetry, and some correlation between the two, the value of the standard deviation from the mean (approximately 0°) being less, that of the correlation coefficient greater than in the former case; (3) that the First Furrow tended to lie either in or at right angles to the Plane of Symmetry, and that there was little correlation between the two.

* I should like to take this opportunity of expressing the great obligations I am under to Professor Pearson for the trouble he has taken in suggesting methods of dealing with the data. I have also to thank Mr Schuster for correcting some of the arithmetic. I may say, perhaps, that the delay in publication is due to the fact, first, that the experiments have extended over three years, and secondly, that the working out of the results—especially those of Part II.—has involved a good deal of labour. In cutting the necessary sections I had assistance, thanks to a grant of money from the British Association.

The results of this investigation are presented in tabular form herewith (Table I.).

TABLE I.

	First Furrow and Sagittal Plane	Plane of Symmetry and Sagittal Plane	Plane of Symmetry and First Furrow		
			All cases	About 0° only	About 90° only
σ	40·39° ± ·65	29·75° ± ·63	48·33° ± 1·14	18·70° ± ·60	23·29° ± ·86
ρ	·138 ± ·031	·372 ± ·025	·087 ± ·032	—	—

In this experiment the eggs were placed, haphazard with regard to the direction of their axes, in rows parallel to the length of long glass slides, and the angles were measured between the various planes and lines ruled across the slides. Under these circumstances the eggs in each row become pressed against one another by their jelly-membranes, the direction of the pressure being parallel to the length of the slide; and since the axis may make any direction with the vertical, it is possible that during the half-hour which elapses before the eggs turn over into their normal position with the axes vertical and the white (yolk) pole below, gravity may act upon, and cause, as in Pflüger's permanently inverted eggs, a redistribution of the contents (the heavy yolk granules sinking to the lower side, the lighter cytoplasm and pigment rising to the upper side) and so impress upon the eggs a gravitation plane of bilateral symmetry (the plane, including the egg-axis and the vertical, on each side of which the yolk descends, the cytoplasm ascends in a corresponding manner).

It is known of course that when the eggs are kept inverted this plane becomes the median plane of the embryo, that side on which the white pole is upturned being dorsal, and further that the First Furrow, while it may make any angle with this plane, tends usually to lie in, or at right angles to or at an angle of 45° to it. It is also well known that when eggs are strongly compressed at right angles to the axis the First Furrow lies in the direction of pressure, the median plane of the embryo at right angles to it (O. Hertwig). Thirdly, the direction of the incident light (day-light) may conceivably exert some influence upon the position of one or more of these planes (though Roux has indeed stated that this is not the case), and so upon the angles they make with one another.

In order to discover therefore whether any of these external agencies do affect the direction of the planes and so of the angles between them, the eggs have been examined under four principal different conditions.

I. The eggs were placed close together in longitudinal rows, with their axes horizontal, and the white pole facing the same side of the slide so that each egg had to turn through 90° to gain its normal position. The pressure therefore is parallel to, the gravitation symmetry plane at right angles to the length of the slide.

II. The eggs were closely placed as above, but with their axes vertical, and their white poles below. Since the eggs did not have to turn over the possible influence of gravity is eliminated, but the pressure remains.

III. While the eggs are placed as in I. with their axes horizontal, they are so spaced that their jellies do not touch. The pressure is eliminated, but gravity remains.

IV. By placing the eggs wide apart and with their axes vertical both these possibly disturbing factors are removed.

The direction of the incident light was also varied:—

In I. the majority of the slides were placed and kept with one end to the window, but a few were kept in darkness until segmentation had begun.

In II. the majority were placed with one end to the window, but in a few cases one side of the slide was directed towards the light.

In III. one end of the slide was certainly directed towards the light in some cases, in the remainder I am not certain of the position although I believe it was the same.

Lastly in IV. all the slides were placed with one end facing the light.

The results of the measurements of the angles between the Plane of Symmetry, the First Furrow and the Sagittal Plane under the four main conditions (the possible effect of light will be considered later) are given in the accompanying Table VI. in which are entered the values of the standard deviations and correlation coefficients for each angle under each set of conditions.

TABLES II.—V.

Frequency of Angles between Plane of Symmetry and Sagittal Plane.

	Negative												Positive												Totals	M	σ
Angle	180—165	165—150	150—135	135—120	120—105	105— 90	90— 75	75— 60	60— 45	45— 30	30— 15	15— 0	0— 15	15— 30	30— 45	45— 60	60— 75	75— 90	90—105	105—120	120—135	135—150	150—165	165—180			
II.	6	4	2	4	2	4	10	18	25	37	74	180	165	86	48	35	9	12	4	3	1	2	3	7	741	·54° ±1·12	45·29° ±·79
III.	5	1	—	2	2	4	8	22	38	56	98	173	198	92	42	30	9	3	2	3	1	—	1	2	792	−3·67° ±·88	36·84° ±·62
IV.	—	—	—	—	1	2	—	1	4	11	39	56	62	41	14	4	4	1	5	—	—	—	—	—	245	2·73° ±1·27	29·37° ±·89
V.	—	—	—	—	—	—	1	8	5	13	32	59	68	36	10	4	1	3	1	—	1	—	2	1	245	·92° ±1·36	31·50° ±·96

Plane of Symmetry and Sagittal Plane; the angle measured being between the "grey crescent" end of the former and the "dorsal lip" end of the latter.

II. Eggs close. Axes horizontal.

III. Eggs close. Axes vertical.

IV. Eggs spaced. Axes horizontal.

V. Eggs spaced. Axes vertical.

It may be pointed out that since the two ends of the First Furrow are indistinguishable from one another the range of the value of the angle between it and either the Plane of Symmetry or the Sagittal Plane does not extend beyond 90° in either direction. In the case of the other two planes however the two ends are externally unlike, the first having the grey crescent, the second the dorsal lip of the blastopore at one end. It is consequently possible to distinguish between any angle and its supplement, and the range of values may be extended on each side to 180°. This has been done in Tables II.—V. and the results are entered in brackets in the second column of standard deviations in Table VI.

TABLE VI.

	σ			ρ		
	First Furrow and Sagittal Plane (γ)	Plane of Symmetry and Sagittal Plane (β)	Plane of Symmetry and First Furrow (α)	First Furrow and Sagittal Plane ($\rho_{\alpha\beta}$)	Plane of Symmetry and Sagittal Plane ($\rho_{\alpha\gamma}$)	Plane of Symmetry and First Furrow ($\rho_{\beta\gamma}$)
I. Eggs close. Axes horizontal	38·42° ± ·70	31·86° ± ·56 (42·29° ± ·79)	41·59° ± ·84	·201 ± ·028	·263 ± ·027	·118 ± ·029
II. Eggs close. Axes vertical	33·44° ± ·44	30·17° ± ·51 (36·84° ± ·62)	39·72° ± ·61	·352 ± ·021	·276 ± ·022	·023 ± ·024
III. Eggs spaced. Axes horizontal	33·49° ± ·96	27·53° ± ·84 (29·37° ± ·89)	36·60° ± 1·11	·292 ± ·039	·399 ± ·036	·075 ± ·043
IV. Eggs spaced. Axes vertical	31·45° ± ·73	26·80° ± ·82 (31·50° ± ·96)	34·46° ± 1·07	·364 ± ·033	·451 ± ·035	·186 ± ·043

It will be clear from these tables that both "pressure" (as I will call it, though it is doubtful as we shall see, whether pressure is the real cause of the disturbance) and gravity affect the relations between the three planes, for in all three cases the standard deviation diminishes while the correlation coefficient increases when both factors are eliminated (IV.).

Taken separately however the two agencies do not modify the values of the angles between First Furrow and Sagittal Plane, and between Plane of Symmetry and Sagittal Plane in the same way. Thus it will be seen that in the first case the standard deviation is slightly less, the correlation coefficient greater when gravity alone is removed (II.) than when the pressure only is omitted (III.), while the agreement between the Plane of Symmetry and the Sagittal Plane is closer under the influence of gravity alone than of pressure alone, whether this agreement be measured by the standard deviation or by the correlation coefficient. The value of the standard deviation progressively declines in the case of the Plane of Symmetry and the First Furrow; the correlation coefficient is somewhat inexplicably large when both agencies are allowed to interfere (I.). If the values of standard deviation and correlation coefficient be considered for each of these angles

separately it will be evident that, with the one exception just noticed, the former declines when the latter increases, and conversely.

There is one other point that must be noticed. It is evident from Tables II. to V. that the dorsal lip appears in the great majority of cases on the same side of the egg as the grey crescent since in these observations the measurement was of the angle between the dorsal lip, or grey crescent, and the zero point. It may however occur that these two structures appear on opposite sides of the egg, though this does not occur at all, or very little, when the axes are horizontal but the eggs spaced (IV.), and not very much when the eggs are spaced and the axes vertical (V.). It is under these conditions that the correlation between these two planes is highest. It must however be remembered that in calculating the coefficient the angles made by each of these planes with the First Furrow are compared, angles in which it is impossible, owing to the indistinguishability of the ends of the Furrow, to discriminate between any value and its supplement.

Professor Pearson has however pointed out to me that since, any two of these angles being known, the third is directly obtainable; since in fact $\alpha - \beta = \gamma$ where α, β and γ are the three angles, the three correlation coefficients ought also to be definitely related, namely, they should be the cosines of three angles which are together equal to 180°, and further the standard deviation being known of two angles and the value of the coefficient of correlation between them, the remaining standard deviation and coefficients should be obtainable by the formulae

$$\sigma_\gamma^2 = \sigma_\alpha^2 + \sigma_\beta^2 - 2\sigma_\alpha \sigma_\beta \rho_{\alpha\beta},$$

$$\rho_{\alpha\gamma} = \frac{\sigma_\alpha - \sigma_\beta \rho_{\alpha\beta}}{\sigma_\gamma},$$

$$\rho_{\beta\gamma} = \frac{\sigma_\alpha \rho_{\alpha\beta} - \sigma_\beta}{\sigma_\gamma}.$$

In order however that γ may always $= \alpha - \beta$, it is necessary that the angles should always be measured in the same sense, and this gives in certain cases a value for α (Plane of Symmetry and First Furrow) or for γ (First Furrow and Sagittal Plane) which is greater than 90°. In working out the results already tabulated these cases had always been entered as less than 90°, that is, owing to the impossibility of distinguishing between the two ends of the First Furrow, the smaller of two supplementary values was always taken. For example:

Let β (Plane of Symmetry and Sagittal Plane) be 100°,

γ (First Furrow and Sagittal Plane) be $-30°$,

then $\alpha = \beta + \gamma = 130°$, but was entered always as $-50°$.

Hence the coefficients do not possess the magnitudes they should do to satisfy the equation. At the same time where $\beta + \gamma =$ nearly 180°, there is clearly a close relation between the First Furrow and the Symmetry Plane. After some discussion I have decided to adopt the following convention suggested by Professor Pearson.

The material is divided into two series, positive and negative. In the first the Sagittal Plane and the First Furrow (up to 90°) are on the *same* side of the Plane of Symmetry, either on the right or on the left. α is then taken as positive and less than 90°, β is positive and less than 180° (the angle being measured between "grey crescent" and "dorsal lip") while γ may be positive or negative and is less than 180°.

In the second, or negative series, the Sagittal Plane and First Furrow are on *opposite* sides of the Plane of Symmetry. β is taken as positive and less than 180°, α is taken as negative and is less than 90°, while γ is the sum of the two of them, and is negative.

α and β are then correlated, and the coefficients $\rho_{\alpha\gamma}$, $\rho_{\beta\gamma}$ found by the formula.

The results are given in the tables (Tables VII. to X., (*a*) positive series, and (*b*) negative series) and summarized in Table XI.

TABLE VII.

I. *Eggs Close. Axes Horizontal.*

a. Positive series. $\rho_{\alpha\beta} = \cdot 362$.

α \ β	180											o	Totals
90	2	—	1	—	1	1	1	5	3	1	3	6	24
	1	1	—	1	—	3	4	6	6	4	5	7	38
	—	—	—	1	—	1	4	4	7	8	8	9	42
	1	—	1	1	—	—	1	3	11	11	12	19	60
	—	—	—	—	1	1	2	—	3	10	19	30	66
o	1	—	—	2	1	—	—	—	4	8	23	43	82
Totals	5	1	2	5	3	6	12	18	34	42	70	114	312

b. Negative series. $\rho_{\alpha\beta} = -\cdot 195$.

α \ β	180											o	Totals
90	2	1	—	—	—	—	—	1	1	4	4	9	22
	—	1	—	—	—	—	5	1	1	1	1	7	17
	—	—	—	—	2	—	1	—	2	2	3	6	16
	—	2	2	—	—	—	1	2	2	3	6	10	28
	2	—	—	—	—	—	1	—	4	3	7	20	37
o	2	—	—	—	—	2	1	2	5	10	22	47	91
Totals	6	4	2	0	2	2	9	6	15	23	43	99	211

TABLE VIII.

II. *Eggs Close. Axes Vertical.*

a. Positive series. $\rho_{\alpha\beta} = \cdot 409$.

α \ β	180											o	Totals
90	—	—	—	1	—	1	5	5	7	7	9	5	40
	1	1	—	1	—	—	1	8	10	6	5	13	46
	—	—	—	—	—	1	1	4	13	14	20	11	64
	—	—	—	—	—	—	—	4	16	15	30	23	88
	—	—	—	—	—	—	—	—	5	16	32	39	92
o	2	—	—	—	—	—	—	4	5	10	35	106	162
Totals	3	1	0	2	0	2	7	25	56	68	131	197	492

b. Negative series. $\rho_{\alpha\beta} = -\cdot 252$.

α \ β	180											o	Totals
90	—	—	—	—	2	2	2	1	3	5	10	12	37
	1	—	—	—	—	1	1	2	2	3	2	7	19
	1	1	—	—	1	1	—	1	1	3	2	11	22
	1	—	—	—	—	—	1	—	—	2	8	13	25
	—	—	—	—	2	—	—	1	1	6	8	31	49
o	—	—	—	1	—	—	—	—	3	9	25	94	132
Totals	3	1	0	1	5	4	4	5	10	28	55	168	284

TABLE IX.

III. *Eggs Spaced. Axes Horizontal.*

a. Positive series. $\rho_{a\beta} = \cdot 359$.

α \ β	180											0	Totals
90	—	—	—	—	—	2	—	1	—	2	5	—	10
	—	—	—	—	—	1	—	1	2	1	2	5	12
	—	—	—	—	—	—	—	3	—	4	2	7	16
	—	—	—	—	—	—	—	—	1	3	12	7	23
	—	—	—	—	—	1	—	—	1	3	19	16	40
0	—	—	—	—	—	1	—	—	1	2	16	27	47
Totals	0	0	0	0	0	5	0	5	5	15	56	62	148

b. Negative series. $\rho_{a\beta} = -\cdot 237$.

α \ β	180											0	Totals
90	—	—	—	—	1	1	—	—	—	2	1	3	8
	—	—	—	—	—	—	—	—	—	—	—	4	4
	—	—	—	—	—	—	1	—	—	—	1	2	4
	—	—	—	—	—	—	—	—	1	1	1	6	9
	—	—	—	—	—	—	—	—	—	2	4	7	13
0	—	—	—	—	—	1	—	—	2	5	17	34	59
Totals	0	0	0	0	1	2	1	0	3	10	24	56	97

TABLE X.

IV. *Eggs Spaced. Axes Vertical.*

a. Positive series. $\rho_{a\beta} = \cdot 388$.

α \ β	180											0	Totals
90	—	—	—	—	—	—	—	—	1	—	2	1	4
	—	—	—	—	—	—	2	2	1	4	2	2	13
	—	—	—	—	—	—	—	1	1	—	6	7	15
	—	—	—	—	—	—	—	2	2	7	8	7	26
	—	—	—	—	—	—	—	—	—	5	8	17	30
0	—	—	—	—	—	—	—	—	2	3	17	31	53
Totals	0	0	0	0	0	0	2	5	7	19	43	65	141

b. Negative series. $\rho_{a\beta} = -\cdot 047$.

α \ β	180											0	Totals
90	—	—	—	—	—	—	—	—	1	—	2	2	5
	—	—	—	—	—	—	1	—	—	—	1	2	4
	—	—	—	—	—	—	—	—	—	—	4	3	7
	—	—	—	—	—	—	—	—	—	1	1	8	10
	—	—	—	—	—	—	—	1	—	—	3	8	12
0	—	—	—	1	—	1	—	1	1	3	13	36	56
Totals	0	0	0	1	0	1	1	2	2	4	24	59	94

TABLE XI.

Correlations	Positive Series			Negative Series		
	$\rho_{a\beta}$	$\rho_{a\gamma}$	$\rho_{\beta\gamma}$	$\rho_{a\beta}$	$\rho_{a\gamma}$	$\rho_{\beta\gamma}$
I. Eggs close. Axes horizontal ...	·362	·334	− ·757	− ·195	·657	− ·868
II. Eggs close. Axes vertical ...	·409	·544	− ·544	− ·252	·831	− ·747
III. Eggs spaced. Axes horizontal	·359	·648	− ·480	− ·237	·826	− ·743
IV. Eggs spaced. Axes vertical ...	·388	·684	− ·405	− ·047	·757	− ·688

The results in the positive series agree fairly well with those obtained by the other method. The positive correlation between Plane of Symmetry and Sagittal Plane ($\rho_{\alpha\gamma}$) and that between First Furrow and Sagittal Plane ($\rho_{\alpha\beta}$) rise as gravity and pressure are successively removed, while the negative correlation between Plane of Symmetry and First Furrow ($\rho_{\beta\gamma}$) diminishes.

Again, except when the eggs are close and their axes horizontal, the value of $\rho_{\alpha\gamma}$ is greater than that of $\rho_{\alpha\beta}$, the relation, that is, of the Sagittal Plane to the Plane of Symmetry is closer than to the First Furrow.

In the negative series, where First Furrow and Sagittal Plane are on opposite sides of the Plane of Symmetry, there is a negative correlation ($\rho_{\alpha\beta}$) between them, and this diminishes as the effects of pressure and gravity are removed: so also does the value of the negative correlation $\rho_{\beta\gamma}$. The third coefficient $\rho_{\alpha\gamma}$ (Plane of Symmetry and Sagittal Plane) is higher when both factors are removed than when both are present, but higher still when one is present and one absent.

There are yet other ways in which the relative positions of the three planes may be compared.

In Table XII. are given the equations to the regression lines, by which the mean values of any one of the three angles can be found when the value of one of the others is given.

TABLE XII.

	Positive Series			Negative Series		
Regression Lines ...	$\alpha =$	$\beta =$	$\gamma =$	$\alpha =$	$\beta =$	$\gamma =$
I. Eggs close. Axes horizontal	$\cdot 251\beta + 26{\cdot}97°$ $\cdot 235\gamma + 35{\cdot}54°$	$\cdot 522\alpha + 16{\cdot}25°$ $-\cdot 765\gamma + 35{\cdot}54°$	$\cdot 478\alpha - 16{\cdot}25°$ $-\cdot 749\beta + 26{\cdot}97°$	$-\cdot 128\beta - 26{\cdot}01°$ $\cdot 333\gamma - 9{\cdot}28°$	$-\cdot 297\alpha + 23{\cdot}63°$ $-\cdot 667\gamma - 9{\cdot}28°$	$1{\cdot}297\alpha - 23{\cdot}63°$ $-1{\cdot}128\beta - 26{\cdot}01°$
II. Eggs close. Axes vertical	$\cdot 409\beta + 22{\cdot}16°$ $\cdot 500\gamma + 30{\cdot}14°$	$\cdot 410\alpha + 13{\cdot}42°$ $-\cdot 500\gamma + 30{\cdot}14°$	$\cdot 590\alpha - 13{\cdot}42°$ $-\cdot 591\beta + 22{\cdot}16°$	$-\cdot 301\beta - 23{\cdot}12°$ $\cdot 570\gamma + 0{\cdot}14°$	$-\cdot 211\alpha + 16{\cdot}55°$ $-\cdot 430\gamma + 0{\cdot}14°$	$1{\cdot}211\alpha - 16{\cdot}55°$ $-1{\cdot}301\beta - 23{\cdot}12°$
III. Eggs spaced. Axes horizontal	$\cdot 413\beta + 21{\cdot}60°$ $\cdot 610\gamma + 26{\cdot}01°$	$\cdot 312\alpha + 13{\cdot}12°$ $-\cdot 390\gamma + 26{\cdot}01°$	$\cdot 688\alpha - 13{\cdot}12°$ $-\cdot 587\beta + 21{\cdot}60°$	$-\cdot 281\beta - 17{\cdot}36°$ $\cdot 568\gamma + 1{\cdot}17°$	$-\cdot 200\alpha + 14{\cdot}85°$ $-\cdot 432\gamma + 1{\cdot}17°$	$1{\cdot}200\alpha - 14{\cdot}85°$ $-1{\cdot}281\beta - 17{\cdot}36°$
IV. Eggs spaced. Axes vertical	$\cdot 487\beta + 18{\cdot}18°$ $\cdot 680\gamma + 23{\cdot}82°$	$\cdot 309\alpha + 12{\cdot}68°$ $-\cdot 320\gamma + 23{\cdot}82°$	$\cdot 691\alpha - 12{\cdot}68°$ $-\cdot 513\beta + 18{\cdot}18°$	$-\cdot 052\beta - 21{\cdot}57°$ $\cdot 550\gamma + 0{\cdot}30°$	$-\cdot 042\alpha + 16{\cdot}92°$ $-\cdot 450\gamma + 0{\cdot}30°$	$1{\cdot}042\alpha - 16{\cdot}92°$ $-1{\cdot}052\beta - 21{\cdot}57°$

In the positive series the least mean value of α compatible with β being positive, 26·97°, is greater than the least mean value of β compatible with α being positive, 16·25°. In the negative series the least mean value of α compatible with β being positive is greater than the least mean value of β compatible with α being negative.

All least mean values in the positive series diminish from I. to IV.: in the negative series they vary.

In the positive series the range of γ diminishes from I. to IV.; and the mean value of $\alpha = \beta$ when $\gamma = 0$ diminishes too. The rate of change of the mean value of β is greater than that of α as γ alters in I., equal to it in II., less than it in III. and still less in IV.

In the positive series the mean value of β is of course less than that of α only while γ is positive. In the negative series it is less than that of α as long as γ does not exceed a certain small value in II. III. and IV.: in I. it will be greater than that of α as long as γ does not fall below 55°.

Lastly, in Table XIII. are given the variabilities of each angle when one other is constant.

TABLE XIII.

Variabilities	Positive Series			Negative Series		
	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant
I. Eggs close. Axes horizontal	1·499	2·140	1·483	1·305	2·579	1·698
II. Eggs close. Axes vertical ...	1·368	1·490	1·489	1·224	1·863	2·226
III. Eggs spaced. Axes horizontal	1·179	1·254	1·443	0·746	1·308	1·553
IV. Eggs spaced. Axes vertical	1·057	1·066	1·337	0·976	1·344	1·494

When the eggs are close and their axes horizontal (I.) it is evident, from both the positive and negative series, that the variability of β when α is constant is greater than that of α when β is constant.

When the influence of gravity is removed (II.) the variability of β, α being constant, is equal to that of α, β being constant, in the positive series, but in the negative series that of β is less than that of α. Under other circumstances (III. and IV.) and in both positive and negative series, β is less variable than α (α or β being constant).

Again, comparing β and γ in the same way, β is slightly more variable than γ in I. (positive series) but less variable in all other cases. Lastly, α is less variable than γ.

It must not be forgotten, however, that in this method γ will have a much wider range of variability than in that first employed by me, in which the range was restricted to 90°. The range of α is, however, limited by our convention to 90°.

Comparisons in respect of variability may therefore be more fairly instituted between β and γ than between α and either β or γ.

It will also be noticed that the variability diminishes as the interfering agents are removed.

In whatever way we compare the relations between Plane of Symmetry, First Furrow and Sagittal Plane, whether by the standard deviation (first method), the correlation coefficient (first or second method), by comparing the mean values of two when the third is known, or by comparing the variabilities, it seems clear that there is a closer relation between the Plane of Symmetry and the Sagittal Plane than between either of these and the First Furrow, at least when the disturbing effects of either gravity or pressure are removed.

Let us consider now the relation between each of these planes and the direction of pressure, of the gravitation symmetry plane (including the egg-axis and the vertical) and of the light. These relations are immediately obtainable from the observed values of the angles between the planes and the lines ruled across the slides. It is only necessary to remember that in all cases the pressure is from 90° to 90°, the gravitation plane from 0° to 180°, the white pole of the egg being towards 0°. The direction of the light, as already mentioned, differs in the several experiments, and will be noted in each.

A. Let us take first the position of the First Furrow (Table XIV. and Fig. 1).

TABLE XIV.

First Furrow.

Angle	I. Eggs close Axes horizontal		II. Eggs close Axes vertical		III. Eggs spaced Axes horizontal		IV. Eggs spaced Axes vertical
	A All cases	B Darkness	A All cases	B Light from 0°	A All cases	B Light from +90°	Light from +90°
− *0—15*	23	2	58	4	18	6	40
15—30	23	2	57	5	14	2	37
30—45	19	2	81	6	29	7	36
45—60	50	5	76	7	23	4	43
60—75	88	5	95	15	24	13	40
75—90	114	2	133	27	23	14	34
+*90—75*	102	2	148	31	29	5	29
75—60	112	8	90	15	31	10	34
60—45	57	4	62	7	22	6	29
45—30	45	3	58	5	22	5	27
30—15	26	1	56	11	25	6	27
15—0	28	2	81	12	23	8	43
Totals	687		995		283		419
M	3·92° ± ·97		·12° ± ·99		3·36° ± 1·98		3·81° ± 1·65
σ	38·16° ± ·69		46·47° ± ·71		49·32° ± 1·40		52·76° ± 1·17

The direction of pressure is from 90° to 90°. The gravitation symmetry plane is at 0°.

I. When the eggs are close and their axes horizontal the First Furrow tends to lie parallel to the length of the slide. This is the position it ought to occupy when under the influence of pressure in that direction. It is however doubtful whether the pressure is in reality responsible for this very marked effect.

II. When the eggs are close, but the axes vertical, the First Furrow still lies in the direction of pressure, but to a less extent. It would appear then that this plane tends to some extent—a slight extent, truly—to place itself at right angles to the plane of gravitation, and this supposition is borne out by the persistence of this tendency when the eggs are spaced and the pressure therefore removed, but the axes horizontal (III.), and by its disappearance when the eggs are spaced and the axes vertical (IV.).

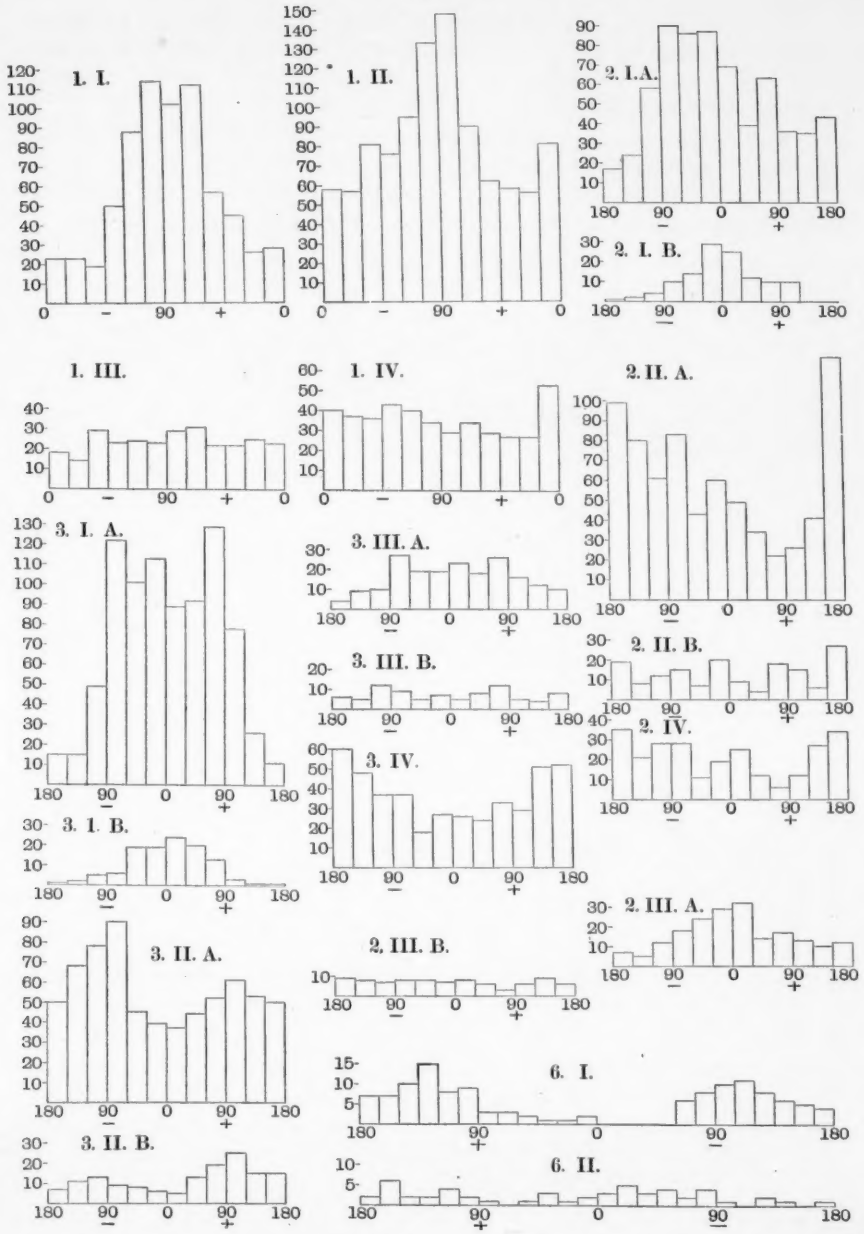
In fact the value of the standard deviation of the distribution about 90° steadily increases as first gravity only, then pressure only, and thirdly both factors are eliminated.

There seems, therefore, no escape from the conclusion that the First Furrow places itself in the direction of what I will still call the "pressure," but perpendicular to the gravitation plane. Gravity, therefore, even during the short interval that elapses between insemination and the turning over of the egg can apparently, though only to a slight extent, influence the position of the First Furrow.

In permanently inverted eggs the First Furrow, as has been mentioned, tends mainly to lie in or at right angles to or at 45° to the gravitation plane. Here only the second of these positions can be detected. It would not perhaps be going too far to attribute the difference to the continued action of gravity in the other case. The direction of division depends on the position in the egg of the fertilization spindle, since the Furrow passes in the equator of the mitotic figure. This depends in turn on the direction of elongation of the spindle, or on that of the initial division of the centrosome, and it is not difficult to understand how the centrosomes might preferably divide in, or the spindle be brought into the meridian of streaming set up by gravity in the cytoplasm. Should this streaming be more marked and lasting the two pronuclei, i.e. the equator of the spindle, not its axis, might be dragged into this plane and then division would be in the plane instead of at right angles to it.

In Table XIV. II. and Fig. 1 II. it will be seen that the frequency at 0° is high; the same peculiarity is observable in IV. I cannot find any explanation for this except by supposing that on some of the slides the rows and not merely the eggs in each row may have been in contact, and hence that there may have been a little "pressure" *across*, in addition to that along the slide.

It would seem that light exerts very little, if any, effect upon the position of the Furrow. In column I. B of Table XIV. are given the frequencies for the First Furrow when the eggs are kept in darkness. There is, as far as can be gathered from so small a number, the same tendency for the Furrow to lie in the direction of pressure as in the whole series (I. A) in which the majority were exposed to daylight falling upon them from one end of the slide.


1. I.
1. II.
2. I.A.
2. I. B.
1. III.
1. IV.
2. II. A.
3. I. A.
3. III. A.
2. II. B.
3. III. B.
2. IV.
3. IV.
3. I. B.
2. III. A.
3. II. A.
2. III. B.
6. I.
3. II. B.
6. II.

A comparison again of the frequencies in columns II. B and III. B, in the first of which the light is from 0°, in the second from 90°, shows that there is no greater tendency for the Furrow to lie either in or at right angles to the source of light in the one case than in the other.

We conclude, therefore, that light plays no appreciable part in determining the position of the First Furrow in the egg.

B. It is very different in the case of the Plane of Symmetry. In Table XV. are given the frequencies of the various values of the angle between the Plane of

TABLE XV. *Plane of Symmetry.*

Angle	I. Eggs close Axes horizontal		II. Eggs close Axes vertical		III. Eggs spaced Axes horizontal		IV. Eggs spaced Axes vertical
	A Light from +90°	B Darkness	A Light from +90°	B Light from 0°	A Light?	B Light from +90°	Light from +90°
−*180—165*	10	1	51	9	6	4	20
165—150	7	0	48	10	1	5	15
150—135	6	1	31	2	3	6	8
135—120	18	1	49	6	2	2	13
120—105	24	0	21	6	4	6	16
105—90	34	4	41	6	8	1	12
90—75	58	4	63	11	10	5	19
75—60	32	6	20	4	8	3	9
60—45	30	6	22	3	9	5	4
45—30	56	8	23	4	15	3	7
30—15	40	8	21	8	8	3	7
15—0	47	21	39	12	21	4	12
+ *0—15*	40	12	31	5	12	3	18
15—30	29	13	18	4	20	5	7
30—45	17	9	23	3	8	4	6
45—60	22	3	11	1	6	2	6
60—75	27	4	5	3	8	2	4
75—90	36	5	17	15	9	1	2
90—105	22	6	15	7	6	5	6
105—120	14	3	11	8	7	1	8
120—135	7	0	21	2	2	5	9
135—150	28	0	20	4	8	4	18
150—165	22	0	51	11	5	3	12
165—180	21	0	71	16	7	3	22
Total		115					
M		−·59° ± 1·81					
σ		*28·65° ± 1·27					

The direction of pressure is from 90° to 90°. The gravitation symmetry plane from 0° to 180°, the white pole being turned towards 0°.

* Calculated on groups of 30°.

Symmetry and the lines ruled across the slides, in groups of 15°. The ranges, for the reason already given, extend on each side from 0° to 180°, the angle being measured between the middle point of the grey crescent and the zero point. In Fig. 2 are given the corresponding frequency polygons, but the groups are here just twice as large, 30°.

I. Let us consider those eggs in which the axis is horizontal, while the eggs are closely pressed and kept in darkness (I. B).

There is a striking tendency for the Plane of Symmetry to lie in the gravitation symmetry plane; and for the grey crescent to appear on that side on which the white pole was turned up, namely 0°, hardly ever on the opposite. When, however, eggs under the same conditions of pressure and gravity are exposed to day-light coming from + 90° we find (I. A), in addition to the high frequency at 0°, another about − 90°, and a minor rise in the polygon at + 90°. The rise at 0° is mainly accounted for by gravity, but the large rise at − 90°, and the lesser rise at + 90° can only be explained, as far as I am able to see, by supposing that the Plane of Symmetry has a tendency to be developed in the direction of the incident light, the grey crescent appearing preferably, if I may so put it, on that side which is turned away from the light.

II. This explanation is strengthened by the position of the grey crescent when the eggs are close, and the axes vertical (II. A). Here there is the same rise at − 90°, the same drop at + 90°, as in the case just considered, but when the light is made to fall from 0° (II. B) the drop at + 90° is replaced by a rise which is as great as that on the opposite side. This effect is not caused by the pressure, for it reappears when the eggs are spaced and their axes have been vertical from the beginning (IV.), in which the direction of light is again from + 90°. In all these cases (II. A and B, and IV.) there are also high frequencies at 0° and 180°. In II. B these are in the direction of the light, but those in II. A and IV. and the corresponding ones at − 90° and + 90° in II. B are at right angles to the light.

An anomaly which I cannot, I fear, explain is the excessive frequency (in II. A and, though to a less extent, in IV.) about 180° as compared with 0°. One might, possibly, suppose that the light was not exactly from one end, but oblique, but I doubt if this would do*.

In II. B where the light comes from 0° it is interesting to observe that this anomaly does not occur. The frequency about + 90° is practically as great as that about − 90°; the frequency however at 180°—the side turned from the light—is decidedly greater than that at 0°, which is in consonance with the other results.

III. The results recorded in column III. A and B, and in the corresponding polygons are not so unmistakable. There is, indeed, in III. B, the same evident drop at + 90°, the side from which the light comes, and there is as great a frequency at − 90°, the side turned from the light, as at 0° and at 180°, but the rise at 0° which

* For the probable explanation of this, see p. 168.

ought to be produced by gravitation, as in I. A and B, does not appear. The number of instances (85) is perhaps too small. The effect of gravity is noticeable enough in III. A, but there is no excess in the frequency at $-90°$ compared with that at $+90°$, only a slight rise between $0°$ and $-90°$, which is not represented on the other side. I believe, though I am not certain, that the light came from $+90°$ in this case. Nevertheless we may conclude from the position of the Plane of Symmetry under all these conditions of pressure, gravity, and light in varying combination, that its direction in the egg may most certainly be affected by gravity and to some extent by light. It does not appear to be affected by pressure at all. It tends to lie in the gravitation symmetry plane, and the grey crescent appears on the side on which the white pole was upturned. It is clear, therefore, that the half-hour that passes before the egg assumes its normal position is time enough for gravity to impress a permanent effect upon the egg, and so affect the position of the Symmetry Plane. That gravity is not indispensable may, of course, be at once inferred from the appearance of the grey crescent in eggs which have never had to turn over at all.

C. We have now to enquire whether and how far the egg symmetry becomes the Symmetry of the Embryo.

The frequencies are given in Table XVI., the corresponding polygons in Fig. 3.

It will be seen immediately that in some cases at least the direction of the Sagittal Plane follows very closely on that of the Plane of Symmetry.

In column I. B and Fig. 3. I. B, are given the positions of this plane in eggs which were placed closely together, had their axes horizontal, and were kept in darkness until segmentation had begun, but thereafter exposed to day-light coming from $+90°$. The Sagittal Plane exhibits a very marked tendency to lie in the gravitation plane, the dorsal lip being turned towards $0°$, the side on which the yolk is upturned. Its direction follows, in fact, very closely upon that previously assumed (Table XV., I. B) by the Plane of Symmetry, the standard deviation for both having practically the same value, and, as we shall presently see, the correlation between the two is under these circumstances very high, though not complete.

Turning to column I. A and Fig. 3. I. A we again find a close correspondence between the position of the Sagittal Plane and that previously occupied by the Plane of Symmetry (Table XV., I. A, and Fig. 2. I. A). The correspondence is, however, not perfect, for there is here as high, or rather a higher frequency about $+90°$ as about $-90°$. This may be due to the light.

III. Again, when the eggs are spaced, but the axes still horizontal, there is the same high frequency at $0°$ and low frequency at $180°$ (III. A and B); in both these cases there is also a high frequency at $+90°$ and another at $-90°$. In III. B the direction of the light is known, from $+90$, and, as I have said, I believe that it had the same direction in the A series. Should this be so, the high frequency at $+90°$ of the Sagittal Plane as compared with the lower frequency at the same point of the Plane of Symmetry (Fig. 2. III. A) must, I think, be attributed to

the action of the light. The dorsal lip would then appear as readily on the side of the egg turned towards as on that turned away from the light. The dorsal lip is formed in a region of active cell-division, and cell-division may probably be promoted by light as well as it is known to be by heat. I had previously found, as noted in the publication referred to above, that the Sagittal Plane will place itself either in or at right angles to the direction of a strong artificial light.

TABLE XVI.

Sagittal Plane.

Angle	I. Eggs close Axes horizontal		II. Eggs close Axes vertical		III. Eggs spaced Axes horizontal		IV. Eggs spaced Axes vertical
	A Light from +90°	B Darkness	A Light from +90°	B Light from 0°	A Light?	B Light from +90°	Light from +90°
−180—165	6	0	29	4	1	4	32
165—150	9	1	21	3	3	2	28
150—135	4	2	33	4	2	2	25
135—120	11	0	38	7	7	3	23
120—105	14	2	30	7	4	6	17
105—90	35	3	48	6	6	6	20
90—75	55	1	50	8	12	6	26
75—60	67	5	40	1	15	3	11
60—45	53	12	17	5	5	1	6
45—30	48	7	28	3	14	4	12
30—15	57	9	16	2	9	3	7
15—0	56	10	23	4	10	4	20
+ 0—15	46	15	21	1	13	2	14
15—30	43	9	16	4	10	3	12
30—45	43	13	22	7	6	3	16
45—60	49	7	22	6	12	5	8
60—75	64	10	24	7	17	9	19
75—90	65	3	28	12	9	3	14
90—105	47	2	30	14	8	5	12
105—120	31	1	31	11	8	0	17
120—135	17	1	27	6	6	2	27
135—150	9	0	26	9	6	2	24
150—165	5	0	29	9	4	5	21
165—180	6	1	21	6	6	3	31
Total		114					
M		−1·32° ± 1·81					
σ		*28·64° ± 1·28					

The direction of pressure is from 90° to 90°. The gravitation symmetry plane from 0° to 180°, the white pole being turned towards 0°.

* Calculated on groups of 30°.

II. When the eggs are close and the axes vertical the high frequency at 0° does not appear (II. A and B). The frequency at +90°, when the eggs are illuminated from +90° (II. A), is less than that at −90° but not so very much less as it is in the case of the Plane of Symmetry (Table XV., II. A and Fig. 2. II. A). In the case of the eggs illuminated from 0° (II. B) no relation can be traced between the directions of the Sagittal and Symmetry Planes.

IV. Lastly, in those eggs which are freed from the influence of pressure and gravity, but exposed to light (coming from 90°) (IV.), there is a rough agreement in the position taken up by the two planes except that, as in the previous series, the frequency in the region of +90° is increased.

Taken as a whole this third set of observations appears to indicate that the dorsal lip of the blastopore shows some tendency to be formed on that side of the egg which is turned towards the light. There is, however, no evidence that its position is affected by the pressure.

Speaking generally then we may say, as the result of the whole series of measurements, that

(1) the position of the First Furrow depends upon the direction of pressure in a very marked degree; its tendency is to coincide with that direction. In a less degree, it is dependent on the gravitation plane, since it to some extent avoids that plane, and places itself perpendicular to it. The First Furrow is not, however, influenced by the direction of incident light,

(2) the Plane of Symmetry is independent of the direction of pressure, but is very markedly dependent on the gravitation plane, with which it exhibits a strong tendency to coincide. It is also apparently influenced by light, being placed in this plane with the grey crescent turned away from the source of light,

(3) the Sagittal Plane is as independent of the pressure as is the Plane of Symmetry. It indeed follows the latter very closely, and would appear to be largely determined by it, though not completely, for while the dorsal lip exhibits nearly as great a tendency to appear on that side of the egg on which the white hemisphere was turned up as does the grey crescent, it also displays an inclination, unlike the grey crescent, for that side of the egg which is turned towards the light.

More recently (this Spring, 1909) another series of experiments has been carried out with the view of ascertaining more exactly the effect of light upon the eggs when gravity and pressure were both removed.

The eggs were all spaced and their axes were vertical.

In one lot (A) they were exposed to light (from −90°) throughout—from the beginning of fertilization till the appearance of the dorsal lip of the blastopore.

In the second lot (B) they were kept in darkness till the beginning of segmentation. Some of these (B α) were then exposed to light coming from the same direction as before, while the remainder were kept permanently in darkness (B β).

The results of this experiment are tabulated in Tables XVII.—XXIII.

In Table XVII. the values of the standard deviations, as calculated by my method, are given. It appears that the relation between First Furrow and Sagittal Plane and between First Furrow and Plane of Symmetry is closer when the eggs

TABLE XVII.

Values of σ	First Furrow and Sagittal Plane (γ)	Plane of Symmetry and Sagittal Plane (β)	Plane of Symmetry and First Furrow (α)
A. In light throughout	38·55° ± 1·39°	30·24° ± 1·05° (38·77° ± 1·35°)	39·45° ± 1·36°
B α. In darkness till segmentation	32·30° ± ·88°	30·99° ± ·81° (41·55° ± 1·09°)	35·63° ± ·96°
B β. In darkness till dorsal lip ...	32·92° ± ·89°	32·88° ± ·88° (38·83° ± 1·03°)	37·65° ± 1·01°

are kept in darkness (B) than when they are exposed to light throughout (A). There is practically no difference, as estimated by this method, in either of these relations between the eggs exposed to light after segmentation has begun (B α) and those kept in darkness till the end (B β).

The action of light on the other hand appears to improve the relation between the Plane of Symmetry and the Sagittal Plane when the range is limited to 90° on each side*.

If, however, the range be extended to 180° on each side (the figures in brackets) then the tendency of the Sagittal Plane to lie in the Plane of Symmetry appears to be greater when the eggs are kept permanently in the light (A) or in the darkness (B β) than when the Symmetry Plane is developed in the darkness but the Sagittal Plane under the influence of light (B α).

It will be noticed that the value of σ is in each case smaller for β than for γ and for γ than for α in conformity with previous results (Tables I. and VI. IV.).

The absolute values are however larger. This must be due to some peculiarity of the eggs, or of the breeding season.

In Table XVIII. are shown the correlation coefficients (Pearson's method). The correlation between First Furrow and Sagittal Plane ($\rho_{\alpha\beta}$), and that between First Furrow and Symmetry Plane ($\rho_{\beta\gamma}$) are both improved when the eggs are kept in darkness, at least in the positive series. The correlation between Plane of Symmetry and Sagittal Plane ($\rho_{\alpha\gamma}$) hardly alters, but appears to be slightly less when the eggs are kept in the light throughout (A).

The correlations between the Plane of Symmetry and Sagittal Plane are again, always higher than those between First Furrow and Sagittal Plane.

* This is due to the fact that while the grey crescent turns away from the light, the dorsal lip turns towards it, but that this difference of 180° is ignored when the range is limited to 90°.

TABLE XVIII.

Correlations	Positive Series			Negative Series		
	$\rho_{\alpha\beta}$	$\rho_{\alpha\gamma}$	$\rho_{\beta\gamma}$	$\rho_{\alpha\beta}$	$\rho_{\alpha\gamma}$	$\rho_{\beta\gamma}$
A. In light throughout	·096	·562	−·769	−·140	·789	−·719
B *a*. In darkness till segmentation	·223	·586	−·659	−·328	·724	−·839
B *β*. In darkness till dorsal lip ...	·282	·588	−·610	−·504	·863	−·872

In the next two tables (XIX., XX.) are given the equations to the regression lines (mean values of each angle in terms of one other) and variabilities of each when one other is constant.

TABLE XIX.

Regression Lines ...	Positive Series			Negative Series		
	$\alpha =$	$\beta =$	$\gamma =$	$\alpha =$	$\beta =$	$\gamma =$
A. In light throughout	$\cdot074\beta + 26{\cdot}30°$ $\cdot361\gamma + 28{\cdot}77°$	$\cdot124\alpha + 25{\cdot}78°$ $-\cdot639\gamma + 28{\cdot}77°$	$\cdot876\alpha - 25{\cdot}78°$ $-\cdot926\beta + 26{\cdot}30°$	$-\cdot158\beta - 30{\cdot}50°$ $\cdot554\gamma - 2{\cdot}48°$	$-\cdot140\alpha + 20{\cdot}02°$ $-\cdot446\gamma - 2{\cdot}48°$	$-1{\cdot}140\alpha - 20{\cdot}02°$ $-1{\cdot}158\beta - 30{\cdot}50°$
B *a*. In darkness till segmentation	$\cdot307\beta + 18{\cdot}40°$ $\cdot452\gamma + 25{\cdot}94°$	$\cdot340\alpha + 16{\cdot}58°$ $-\cdot548\gamma + 25{\cdot}94°$	$\cdot660\alpha - 16{\cdot}58°$ $-\cdot693\beta + 18{\cdot}40°$	$-\cdot218\beta - 19{\cdot}25°$ $\cdot351\gamma - 6{\cdot}13°$	$-\cdot494\alpha + 17{\cdot}57°$ $-\cdot649\gamma - 6{\cdot}13°$	$1{\cdot}494\alpha - 17{\cdot}57°$ $-1{\cdot}218\beta - 19{\cdot}25°$
B *β*. In darkness till dorsal lip	$\cdot272\beta + 18{\cdot}80°$ $\cdot482\gamma + 27{\cdot}12°$	$\cdot292\alpha + 20{\cdot}20°$ $-\cdot518\gamma + 27{\cdot}12°$	$\cdot708\alpha - 20{\cdot}20°$ $-\cdot728\beta + 18{\cdot}80°$	$-\cdot489\beta - 12{\cdot}32°$ $\cdot490\gamma + 0{\cdot}02°$	$-\cdot520\alpha + 13{\cdot}13°$ $-\cdot510\gamma + 0{\cdot}02°$	$1{\cdot}520\alpha + 13{\cdot}13°$ $-1{\cdot}489\beta - 12{\cdot}32°$

TABLE XX.

Variabilities	Positive Series			Negative Series		
	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant
A. In light throughout	1·207	1·879	1·453	1·039	1·482	1·675
B *a*. In darkness till segmentation	1·326	1·649	1·532	1·141	2·352	1·561
B *β*. In darkness till dorsal lip ...	1·358	1·668	1·609	0·893	1·575	1·528

The range of γ in series B is equal to the range of γ in series III. and IV. (Table XII.): in A it is rather wider.

In all three (positive) series the mean value of $\alpha (= \beta)$ when $\gamma = 0$ is about the same. The least mean values of α and β compatible with β and α being positive (or β positive and α negative in the negative series) are about the same except in A, negative series, where that of β is less than that of α.

In the positive series β changes faster than α as γ alters.

In B α, β will be more often less than α than in B β.

The variability of β when γ is constant is always less than that of γ when β is constant (compare Table XIII. IV.), but, unexpectedly, β has a greater variability than α (except in A, negative series). α has always a less variability than γ.

It appears indeed that in this series of experiments, when the constants are calculated by Pearson's method, the relation between Plane of Symmetry and Sagittal Plane is not as close as one would have anticipated (compare the values of $\rho_{\alpha\gamma}$ in Tables XVIII. and XI. IV., and of σ_β in Tables XVII. and VI. IV.). There certainly are more instances of angles over 90° than were found in the series given in Table V.

Lastly, we have to consider the position of each of these planes separately with regard to the direction of the incident light (Tables XXI.—XXIII.).

From Table XXI., which gives the position of the First Furrow, it is clear that this plane is not affected by light. It may occupy any position, and the

TABLE XXI.

First Furrow.

Angle	A. Light (from −90°)	B. Darkness
−90—75	22	56
75—60	13	68
60—45	18	55
45—30	18	57
30—15	23	53
15—0	11	59
+ 0—15	14	57
15—30	21	48
30—45	16	45
45—60	11	54
60—75	17	53
75—90	16	54
Totals	200	659
M	−3·45° ± 2·49	−2·75° ± 1·38
σ	52·17° ± 1·05	52·55° ± ·98

standard deviation is the same in the two cases. This is the result already obtained (Table XIV. IV.).

In the case of the Plane of Symmetry (Table XXII.) there are high frequencies about 0° in both light (A) and darkness (B) and again about 180° in darkness. These high frequencies are probably due—at least I can think of no other explanation—to the manipulation. In placing the egg on the slide each one was taken up by its jelly in the forceps, being seized near the animal pole, placed first with its axis at right angles to the slide (i.e. in the direction 0° to 180°) and then turned into position with the axis vertical and the white pole below. It appears that the structure of the egg is unavoidably affected by this treatment, and a slight bilateral symmetry conferred upon it which is not without influence on the position of the Symmetry Plane. Similar high frequencies have been noticed already (Table XV.) and will be found to recur in the position of the Sagittal Plane. Obviously they cannot be attributed to the action of the light.

TABLE XXII.

Plane of Symmetry.

Angle	A. Light (from −90°)	B. Darkness
−*180—150*	10	56
150—120	13	23
120—90	14	40
90—60	12	55
60—30	30	57
30—0	28	103
\+ *0—30*	33	101
30—60	18	57
60—90	11	49
90—120	17	46
120—150	19	56
150—180	18	60
Totals	223	703

The influence of the light upon the position of the grey-crescent is indeed in this series very doubtful indeed.

The evidence that the Sagittal Plane is affected by this agency is, however, better (Table XXIII.) for when the eggs were exposed to light from the beginning of the segmentation until the appearance of the blastopore (B α) there is a slight rise in frequency between $-60°$ and $-90°$, a rise which does not occur when the eggs remain in darkness (B β). This is in agreement with the result already arrived at (Table XVI.), that the dorsal lip tends to turn towards the light.

The high frequencies about 0° must be referred, as already explained, to the manipulation of the eggs.

TABLE XXIII.

Sagittal Plane.

Angle	A. Light throughout from −90°	B α. In darkness till segmentation, then exposed to light from −90°	B β. In darkness till dorsal lip
−*180—150*	7	17	27
150—120	11	15	11
120—90	15	28	28
90—60	19	37	22
60—30	22	34	31
30—0	27	41	47
+ *0—30*	24	43	40
30—60	10	34	28
60—90	18	26	29
90—120	14	19	22
120—150	17	25	25
150—180	15	36	25
Totals	199	355	335

The very great interest attaching, I venture to think, to these results lies in the fact that it is experimentally possible to separate those factors in the constitution of the fertilized ovum upon which the direction of cell-division depends from those which determine first the Plane of Symmetry and, next, the position of the median plane of the embryo, or the direction, if I may so express it, of differentiation. The first depends clearly on the pressure, the second on gravitation (and light). When both pressure and gravitation act upon the egg, and, as in the conditions of the above experiment, at right angles to one another, the deviations of the Plane of Symmetry and of the Sagittal Plane from the First Furrow will be large, the result actually obtained; when, on the other hand, both agencies are eliminated, the agreement of these two planes with the First Furrow is closer. Light is also a disturbing factor, and were light, as well as those small interfering agencies—such as deformation of the eggs by natural pressure in the uterus, or by handling in the course of the experiment—to be also got rid of, we might surmise that the agreement between the First Furrow and the other two planes would be considerable, provided that there were any reasons for supposing that the internal factors on which the direction of cell-division and the direction of differentiation depended, did themselves, under these circumstances, coincide. What these internal causes may be we shall have in a moment to enquire.

Meanwhile there is still one point that demands some discussion.

We have seen that the Symmetry of the Embryo—the position of the Sagittal Plane—follows very closely upon that of the previous Plane of Symmetry. It does not, however, absolutely follow, and, as we know, the correlation between the two

is not complete. As far as we have seen at present the correlation is great when gravity alone acts upon the eggs, and impresses upon them a symmetry which affects the position of the grey-crescent, and then that of the Sagittal Plane. We also know that the effect of light upon the Sagittal Plane (the dorsal lip) is not the same as its effect upon the grey-crescent, and that when the eggs are grown in darkness the agreement between the positions occupied by these planes in the egg is considerable (Figs. 2 and 3, I B, Tables XV. and XVI. I B). Now it is of great interest to observe, that in these circumstances, the standard deviation of the angle between Plane of Symmetry and Sagittal Plane is low, namely, 17·64° ± 1·38 (Table XXIV.). If we except the solitary instance of an angle of 177° (which I feel sure is a mistake) there is no value greater than 60°, and only one greater than 45°. Of the 114 eggs in this series the position of the First Furrow was known, unfortunately, in only 37, but I have employed these in

TABLE XXIV.

Eggs Close. Axes Horizontal. Darkness. Plane of Symmetry and Sagittal Plane.

Negative						Positive						
90—75	*75—60*	*60—45*	*45—30*	*30—15*	*15—0*	*0—15*	*15—30*	*30—45*	*45—60*	*60—75*	*75—90*	Total
—	—	—	6	11	37+1*	38	12	8	1	—	—	114

* +177° M = 1·32° ± 1·95 σ = 17·64° ± 1·38.

making the correlation table (Table XXV.). The value of the coefficient is fairly high $\rho = ·642 \pm ·040$, and would have been higher still ($·736 \pm ·029$) had I not halved the frequency of the one instance in which the angle between First Furrow and Sagittal Plane was exactly 90°, though owing to the general trend of the correlation table it was perhaps hardly necessary to do this. Be that as it may, the correlation between Plane of Symmetry and Sagittal Plane is certainly greatest when gravity is allowed to act upon the eggs, but when light is eliminated. Under the same circumstances, the standard deviations (as calculated for this very small number) of the angles between First Furrow and Plane of Symmetry, and First Furrow and Sagittal Plane remain unaltered, or rather are slightly increased in comparison with the whole series (Table VI. I.).

Yet it may still be questioned whether, even so, the First Furrow does not exercise some independent influence upon the Embryonic Symmetry, and slight though this influence may be, it must, I think, be allowed for, at least as a possibility.

For even under these most favourable conditions, the correlation of egg-symmetry with embryonic-symmetry is not as great as might be desired. On the

TABLE XXV.

First Furrow and Plane of Symmetry.

First Furrow and Sagittal Plane.	90°			−			o			+			90°	Totals
		—	—	—	—	—	—	—	—	—	—	—	·5	·5
		—	3	—	—	—	—	—	—	—	—	—	—	3
	−	—	—	2	1	—	—	—	—	—	—	—	—	3
		—	—	—	1	—	—	—	—	—	—	—	—	1
		—	—	—	—	1	1	—	—	—	—	—	—	2
		—	—	1	1	1	1·5	1	1	—	—	—	—	6·5
	o													
		—	—	—	—	2	1·5	1	2	—	—	—	—	6·5
		—	—	—	—	1	—	2	2	—	—	1	—	6
	+	—	—	—	—	—	—	1	—	1	—	1	—	3
		—	—	—	—	—	—	1	—	—	—	—	—	1
		—	1	—	—	—	—	—	—	—	1	1	—	3
	90°	—	—	—	—	—	—	—	—	—	—	—	1·5	1·5
	Totals	0	4	3	3	5	4	6	5	1	1	3	2	37

$\sigma = 40{\cdot}88°$.

$\sigma = 42{\cdot}74°$.

$\rho = {\cdot}641 \pm {\cdot}051$ or when the 90° is not halved $= {\cdot}736 \pm {\cdot}037$.

other hand there is, even under the most unfavourable circumstances, some correlation between the First Furrow and the Sagittal Plane, while under the influence of pressure alone the correlation is very fair.

This is a point to which we shall return when we have discussed the internal causes which fix the positions of the Plane of Symmetry and the First Furrow.

PART II.

As will be well remembered, it has been asserted by Roux that the position of the First Furrow of the Frog's egg depends upon the point of entrance of the spermatozoon. Roux found either (1) that the sperm-path (the track of pigment left behind it in the egg by the entering spermatozoon) was either included in, or close to and parallel to the First Furrow, or (2) that if the path was inclined at an angle to the Furrow there was an inner portion which lay parallel to, or was included in it. The path was found to be in or parallel to the Furrow when the eggs had their axes vertical from the beginning, to be inclined to it when the eggs were (permanently) prevented from assuming their normal position. In the second the outer inclined portion was distinguished as the 'penetration' path, the direction of first entrance of the spermatozoon, from the inner parallel portion, or 'copulation' path, the line, that is, of union of the two pronuclei. In the first case the two portions were regarded as lying in one and the same meridian, since the 'penetration' path is directed towards some point in the axis, and the 'copulation' path

also towards the point in the axis occupied by the female pronucleus, and in all cases it was held that it was the line of union of the two pronuclei which determined the position of the (meridional) cell division. The pronuclei are situated in the equator of the fertilization spindle, and the spindle lies in a plane parallel to the equator of the egg. The direction of the axis of the spindle, and therefore of its equator, is however really determined, as many cytological studies have shown, by the direction of division of the sperm-centrosome. Since this, however, is at right angles to the line joining the two pronuclei and in a plane parallel to the equator of the egg the result is the same.

Roux further fertilized the eggs from an arbitrarily selected meridian, by applying the sperm containing water to one side of the egg by means of a fine cannulus, a camel's hair brush or a fine silk thread. The fertilization meridian so selected became the meridian of the First Furrow and, as Roux believed to be necessarily the case, the Median Plane of the embryo. Lastly, Roux believed that the grey-crescent appeared always on the side opposite to the entrance of the spermatozoon, and was in fact caused by that entrance, and that the Sagittal Plane lay in the Plane of Symmetry so produced in the egg. Since, moreover, the dorsal lip appeared upon the side of the grey-crescent, the entrance point of spermatozoon marked the future ventral side. With this plane the First Furrow—the plane of the predetermined or 'immanent' qualitative division of the segmentation nucleus —was also held to coincide. In short, this one cause was supposed to determine the Symmetry of the unsegmented egg, the Symmetry of Segmentation, and the Symmetry of the Embryo.

The subsequent statements of O. Schulze and Kopsch with regard to the sperm entrance, grey-crescent, First Furrow and Sagittal Plane are in substantial agreement with Roux's. Since, however, as we now know, the causes which determine the position of the First Furrow can be experimentally separated from those which determine the position of the other two planes, and are therefore presumably distinct, it has seemed to me to be worth while enquiring how far and in what sense it can be asserted that one cause, the entrance of the spermatozoon, is responsible for both.

I have accordingly examined by the help of sections the relation of the sperm-entrance point and sperm-path to the First Furrow in a number of eggs in which the angle between the Plane of Symmetry and the First Furrow had been previously ascertained.

As in the series of experiments already described the eggs were taken from the uterus and placed upon glass slides, fertilized, covered with water and left till the First Furrow appeared. The angles between the Plane of Symmetry and First Furrow and the lines ruled across the slide were then measured and recorded. Each egg was given a number, and killed by means of boiling water containing a little formol and preserved separately, being later cut into a series of sections. The eggs were placed on the slide under two different conditions. In one series they

were closely packed in rows parallel to the length of the slide, and their axes were horizontal, directed *across* the slide and with their white poles all facing one way. In the second series they were spaced and their axes were vertical. As I could only hope to have a limited number of eggs cut into sections I thought it best to obtain as many as possible under each of these two conditions, that is with both pressure and gravity, and without both, and not to attempt to investigate the effect of each factor separately.

In the first series (I)—eggs close, axes horizontal—I have 123 eggs each cut into a series of sections, in the second (II) 55 eggs.

The sections were cut equatorially, that is, at right angles to the axis—and therefore also to the First Furrow, since this is meridional—or as nearly as possible so. The direction of cutting was not, however, always exact, and in calculating the magnitudes of the several angles, an allowance had to be made for the obliquity of the sections to the axis or to both axis and furrow.

The obliquity of the plane of sectioning to the axis was determined by finding the number of sections—the thickness of which was known—intervening between the section in which the yolk first appeared and the section in which it appeared at a point diametrically opposite. The length of the radius of the egg being known, the angle made by the axis with the plane of section may be determined, on the assumption, of course, that the egg is a sphere and that the yolk is so uniformly distributed around the axis that any plane at right angles to the latter would cut equal areas of yolk in all directions from its centre.

These assumptions are unavoidable, and I think fair. The yolk is normally distributed in that way, and I do not think that the spherical eggs become much distorted in the processes of preserving and embedding. The same assumptions have to be made in calculating the obliquity of the sections to the furrow. This was done by counting the sections intervening between the one in which one blastomere appears and that in which the other is first seen. In a section at right angles to the furrow they would of course appear simultaneously.

Making these corrections the following angles have been, indirectly, measured.

(1) The angle between the meridian which includes the point of entrance of the spermatozoon and the meridian of the First Furrow.

(2) By subtraction, the angle between the sperm-entrance meridian and the Plane of Symmetry (a meridional plane).

(3) The angle between the meridian including the end of the first part of the sperm-path ('penetration' path of Roux) and the First Furrow. The former meridian I shall call the sperm-sphere meridian because this structure appears at the inner end of the path.

(4) The angle between the sperm-sphere meridian and the Plane of Symmetry.

(5) The angle between the sperm-entrance radius and the egg axis, or the angle subtended at the centre of the egg by the distance between the animal pole

and the entrance point. The sperm always enters somewhere in the animal hemisphere and the value of this angle therefore never exceeds 90°.

(6) The angle between the sperm-entrance meridian and the lines ruled across the slide, obtained from (1) and (2) and the previously determined positions of the First Furrow and Plane of Symmetry on the slide.

(7) The angle between Sperm-path and the First Furrow. By the Sperm-path I shall always mean the first part or 'penetration' path of Roux. This is a conical streak of pigment, the base outwardly, the apex inwardly directed. It is evidently what is termed in cytological descriptions of fertilization, the entrance funnel. It is formed when the sperm enters the egg by the aggregation of a clear hyaline substance (I describe what I have observed in another Amphibian—the Axolotl) about the anterior end—apical body or acrosome—of the spermatozoon. The substance streams into the interior of the egg—the phenomenon is probably

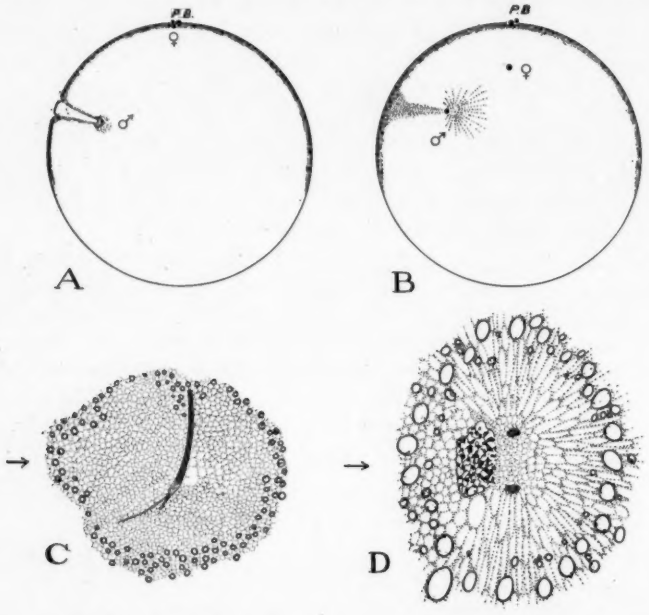


Fig. 4. A. Meridional section of Axolotl egg just after the entrance of the spermatozoon. On the left is seen the entrance-funnel and sperm-sphere, containing the spermatozoon (♂). ♀, female pronucleus. P. B. polar bodies.

B. Similar section a little later. In front of the Sperm-nucleus (♂) is the Sperm-aster. The entrance-funnel is marked by a streak of pigment ("penetration" path).

C. Enlarged view of the spermatozoon and sperm-sphere of A.

D. Enlarged view of the sperm-nucleus, and sperm-aster with the centrosome divided.

The direction of the sperm-path is shown by the arrows.

capillary—and carries with it the spermatozoon, and at the same time sweeps before it a good deal of the superficial pigment (Fig. 4 A). This pigment remains when the clear substance has disappeared as the visible sperm tract (Fig. 4 B and Fig. 5).

At the inner end of the entrance-funnel (if I may resume here very briefly what occurs in the Axolotl) a sperm-sphere—or yolk free area—appears round the middle piece (a modified centrosome), the sperm-head having meanwhile been rotated through 180° so that the middle piece is inwardly directed (Fig. 4 C). Radiations appear round the sperm-sphere which then becomes the sperm-aster, and the sperm-nucleus—developed from the sperm-head—preceded by its aster travels towards the egg-nucleus. As it does so its centrosome divides (Fig. 4 D) at right angles to the direction in which it is now moving. This direction (the 'copulation' path of Roux) may, but need not, be in the same meridional plane with the entrance funnel, and when it is, it may but need not be in the same straight line as the other, for the entrance funnel may be directed towards the

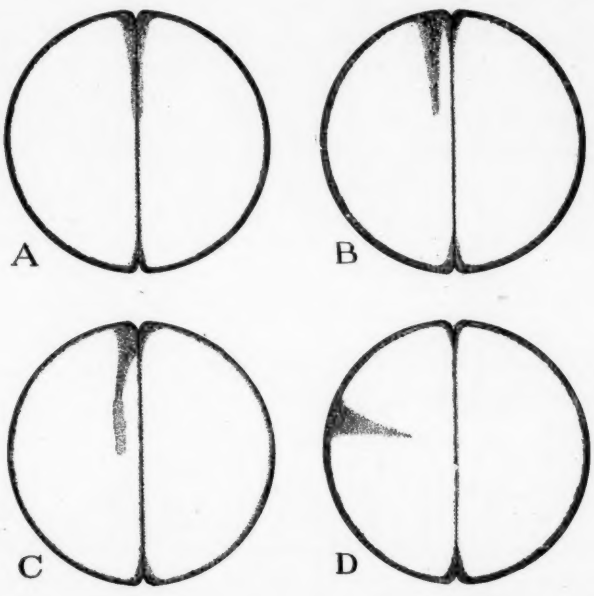


FIG. 5. Projections in an equatorial plane of the various relations of Sperm-path to First Furrow.

A. The whole path included in the Furrow.

B. The whole path parallel to the Furrow.

C. The "penetration" path turned away from, the "copulation" path parallel to the Furrow.

D. The Sperm-path inclined towards the Furrow at a large angle.

axis and yet not towards that point in the axis at which the fertilization spindle is to be developed (this point is pretty constant and about half-way between the animal pole and the centre of the egg). The funnel need not, however, even be directed towards the axis. Hence, the meridian of the 'copulation' path may make an angle with that of the 'penetration' path. As a matter of fact, I have not always in the Frog succeeded in detecting a 'copulation' path. It is generally most in evidence when, as in Fig. 5 C, the penetration path is turned away from the axis, and it is always, parallel to the furrow. In no case that I have observed did the penetration path under these circumstances make a greater angle than 30° with the First Furrow, and in all cases but one, the angle was less than 15°. We shall find that it will be necessary to divide the eggs into three classes according as the sperm-path is included in or parallel to the furrow (Fig. 5 A, B), inclined towards the furrow (Fig. 5 D) or away from it (Fig. 5 C): in the last case the path does not of course lie in a meridional plane at all.

The data necessary for the calculation of these angles are, the perpendicular distance, in the section in which it occurs, of the sperm-entrance point from the furrow, and the distance from the foot of this perpendicular to the middle point of the furrow in the same section; similar measurements for the inner end of the sperm-path ('penetration' path); the distances (the number of sections of known thickness) between the sections in which occur the entrance point, the inner end of the path and the centre of the egg; and lastly, the length of the egg radius. With the help of a little Trigonometry the value of the angles may then be found.

Let us now consider the measurements. The distributions and correlations for Plane of Symmetry and First Furrow, Sperm-entrance Meridian and First Furrow, Sperm-entrance Meridian and Plane of Symmetry, Sperm-sphere Meridian and First Furrow, and Sperm-sphere Meridian and Plane of Symmetry will be found in Tables XXVI. to XXXI. There is a summary of results in Tables XXXII. and XXXIII.

(1) The Plane of Symmetry and the First Furrow.

I tabulate these merely for the purpose of comparison with previous results.

I. When the eggs are close and the axes horizontal (XXVI. a, a', a'') the standard deviation is $\sigma = 45{\cdot}99 \pm 2{\cdot}00$ but there is clearly a tendency for the Furrow to be not only in but at right angles to the other plane. This is what occurred in the original series of eggs examined (described in the previous paper, and tabulated above in Table I.). The value of σ was there $48{\cdot}33 \pm 1{\cdot}14$. The values of the standard deviation have also been calculated about 0° and about 90° separately; they come to $\sigma = 18{\cdot}63° \pm 1{\cdot}39$ and $\sigma = 21{\cdot}48° \pm 1{\cdot}60$ respectively, as compared with $\sigma = 18{\cdot}70° \pm 0{\cdot}60$ and $\sigma = 23{\cdot}29° \pm 0{\cdot}86$ found before (Table I.). The agreement is, I think, sufficiently close.

II. When the eggs are spaced and the axes vertical, the standard deviation works out at $\sigma = 25{\cdot}34° \pm 1{\cdot}63$. This is not so near to the values obtained previ-

TABLE XXVI.

TABLE XXVII.

Angle	Eggs close. Axes horizontal			Eggs spaced. Axes vertical		
	Sperm-entrance Meridian and First Furrow XXVI. *b*	Plane of Symmetry and First Furrow XXVI. *a*	Sperm-entrance Meridian and Plane of Symmetry XXVI. *c*	Sperm-entrance Meridian and First Furrow XXVII. *b*	Plane of Symmetry and First Furrow XXVII. *a*	Sperm-entrance Meridian and Plane of Symmetry XXVII. *c*
−90°—75°	1	11	5	1	0	0
75 —60	3	3	1	1	1	0
60 —45	5	5	7	0	1	2
45 —30	5	9	12	1	2	3
30 —15	9	6	11	5	3	4
−15 —0	40	22	25	26	20	15
+ 0 —15	37	27	18	19	16	20
15 —30	5	9	12	0	4	4
30 —45	4	6	8	0	4	3
45 —60	6	5	7	1	2	2
60 —75	6	4	5	0	2	1
+75 —90	2	13	9	1	0	1
Totals	123	120	120	55	55	55
M	−1·53°±1·86	1·88°±2·83	3·63°±2·52	3·96°±1·91	3·41°±2·31	3·41°±2·33
σ	30·65°±1·32	45·99°±2·00	41·01°±1·79	21·02°±1·35	25·34°±1·63	25·67°±1·65

Plane of Symmetry and First Furrow. Eggs close. Axes horizontal.

XXVI. *a'*. About 0°		XXVI. *a''*. About 90°	
−45°—30°	9	+45°—60°	5
30 —15	6	60 —75	4
−15 —0	22	+75 —90	13
+ 0 —15	27	−90 —75	11
15 —30	9	75 —60	3
30 —45	6	60 —45	5
Total	79		41
M	−·11°±1·96		90·92°±3·36
σ	18·63°±1·39		21·48°±1·60

ously for eggs under the same conditions (Table VI., IV. α), when σ was $34{\cdot}46° \pm 1{\cdot}07$. The difference is evidently due to the absence in the present case of any angles over 75°, but I can suggest no reason for this.

(2) The Sperm-entrance Meridian and the First Furrow. (Tables XXVI. *b*, XXVII. *b*, XXIX. *b*, XXX. *b*.)

TABLE XXVIII.

Sperm-sphere Meridian.

Angle	I. Eggs close. Axes horizontal		II. Eggs spaced. Axes vertical	
	b. Sperm-sphere Meridian and First Furrow	*c*. Sperm-sphere Meridian and Plane of Symmetry	*b*. Sperm-sphere Meridian and First Furrow	*c*. Sperm-sphere Meridian and Plane of Symmetry
−90°—75°	3	3	0	2
75 —60	3	2	1	0
60 —45	5	5	1	2
45 —30	8	9	3	6
30 —15	12	12	15	5
15 —0	32	26	11	8
+ 0 —15	28	17	8	13
15 —30	10	12	9	9
30 —45	8	11	3	6
45 —60	2	7	0	3
60 —75	6	3	1	0
75 —90	3	13	3	1
Totals	120	120	55	55
M	$14{\cdot}00° \pm 2{\cdot}10$	$9{\cdot}37° \pm 2{\cdot}51$	$14{\cdot}33° \pm 2{\cdot}85$	$1{\cdot}77° \pm 2{\cdot}98$
σ	$34{\cdot}19° \pm 1{\cdot}48$	$40{\cdot}73° \pm 1{\cdot}77$	$31{\cdot}31° \pm 2{\cdot}02$	$32{\cdot}81° \pm 2{\cdot}11$

It is quite clear that the tendency for the First Furrow to include the point of entrance of the spermatozoon is very great, whether the eggs be under the influence of gravity and pressure or not. It is difficult indeed to determine whether these external conditions exert an unfavourable influence upon the value of the relation or not, since, where the standard deviation is less when both factors are omitted ($\sigma = 21{\cdot}02° \pm 1{\cdot}35$ as compared with $\sigma = 30{\cdot}65° \pm 1{\cdot}32$), the correlation is not greater as we should expect but less ($\rho = {\cdot}435° \pm {\cdot}074$ as compared with $\rho = {\cdot}613° \pm {\cdot}038$). In comparing these two correlation tables (XXIX. *b* and XXX. *b*) it will be seen at once that in the first case the coefficient is increased by the presence of high frequencies in the positive corners of the table (all examples of precisely 90°, I should say, have been halved), that is, of instances in which the Plane of Symmetry

deviates to a great *and an equal* extent from both First Furrow and Sperm-entrance Meridian. The second correlation is clearly thrown out by three abnormal values. There is no escaping these, though I may point out that in all three cases the sperm had entered near the equator of the egg (several instances of its entering at the same large inclination to the axis occur, however, in the rest of the table) and in all three of course the value of the angle between Sperm-entrance Meridian and First Furrow is great. Should these three cases be omitted the value of ρ would rise to $\cdot 598 \pm \cdot 060$, practically the same as when the eggs are in what one would have imagined to be unfavourable circumstances. That the coefficient is not greater still is due to the absence of high frequencies in the positive corners of the table, the absence, that is, of large deviations, and in the same sense, of both Sperm-entrance and First Furrow from the Plane of Symmetry. It must, indeed, be conceded that these two external agencies are not unfavourable to the relation between Sperm-entrance and First Furrow. This will become intelligible when we remember that the position of the latter is only so far affected by gravity that it tends, to some slight extent, to be at right angles to the gravitation plane, and when we see, as we shortly shall do, that 'pressure' affects the Sperm-entrance and First Furrow in precisely the same way.

TABLE XXIX.

I. Eggs close. Axes horizontal.

XXIX. *c.* *Correlation between Sperm-entrance Meridian and Plane of Symmetry.*

First Furrow and Sperm-entrance Meridian.

First Furrow and Plane of Symmetry.	− 90°						°					+ 90°	Totals
− 90°	—	1	1	—	1	2	1·5	—	·5	—	3	1	11
	—	—	—	1	—	1	—	—	—	—	—	1	3
−	—	—	1	—	1	2	1	—	—	—	—	—	5
	—	2	—	—	2	3	2	—	—	—	—	—	9
	—	—	—	1	—	4	—	—	—	—	1	—	6
	—	—	1	1	1	12	7	—	—	—	—	—	22
°	—	—	1	1	2	6	10	3	1	3	—	—	27
	—	—	—	—	—	3	2	2	—	1	1	—	9
	—	—	—	—	1	—	2	—	1	1	1	—	6
+	1	—	—	—	—	1	3	—	—	—	—	—	5
	—	—	—	—	1	2	1	—	—	—	—	—	4
	—	—	1	1	—	3	6·5	—	·5	1	—	—	13
+ 90°													
Totals	1	3	5	5	9	39	36	5	3	6	6	2	120

$\rho = \cdot 006 \pm \cdot 061.$

XXIX. *b.* *Correlation between Sperm-entrance Meridian and First Furrow.*

Plane of Symmetry and Sperm-entrance Meridian.

Plane of Symmetry and First Furrow.	90			−			o			+			90	Totals
−90	3·5	—	1·5	—	2	—	—	4	—	—	—	—		11
	—	1	—	—	1	—	—	—	1	—	—	—		3
−	—	—	3	—	1	1	—	—	—	—	—	—		5
	—	—	—	5	—	2	—	2	—	—	—	—		9
	1	—	—	—	3	1	—	1	—	—	—	—		6
	—	—	—	—	1	16	3	—	1	1	—	—		22
o														
	—	—	—	4	3	2	13	2	2	1	—	—		27
	—	—	1	1	—	—	3	3	1	—	—	—		9
+	—	—	—	1	—	2	—	—	2	—	1	—		6
	—	—	—	1	—	—	—	—	—	4	—	—		5
	—	—	—	—	—	—	—	—	—	—	3	1		4
90	·5	—	1·5	—	—	—	—	—	1	1	1	8		13
Totals	5	1	7	12	11	24	19	12	8	7	5	9		120

$\rho = \cdot 613 \pm \cdot 038.$

TABLE XXX.

II. Eggs spaced. Axes vertical.

XXX. *c.* *Correlation between Sperm-entrance Meridian and Plane of Symmetry.*

First Furrow and Sperm-entrance Meridian.

First Furrow and Plane of Symmetry.	90			−			o			+			90	Totals
−90	—	—	—	—	—	—	—	—	—	—	—	—		0
	·5	—	—	—	—	—	—	—	—	—	—	·5		1
−	—	1	—	—	—	—	—	—	—	—	—	—		1
	—	—	—	—	1	1	—	—	—	—	—	—		2
	—	—	—	—	—	1	2	—	—	—	—	—		3
	—	—	—	1	3	12	4	—	—	—	—	—		20
o														
	—	—	—	—	—	7	9	—	—	—	—	—		16
	—	—	—	—	—	2	2	—	—	—	—	—		4
+	·5	—	—	—	1	—	2	—	—	—	—	·5		4
	—	—	—	—	—	2	—	—	—	—	—	—		2
	—	—	—	—	—	1	—	—	—	1	—	—		2
90	—	—	—	—	—	—	—	—	—	—	—	—		0
Totals	1	1	0	1	5	26	19	0	0	1	0	1		55

$\rho = \cdot 302 \pm \cdot 083.$

XXX. *b.* *Correlation between Sperm-entrance Meridian and First Furrow.*

Plane of Symmetry and Sperm-entrance Meridian.

Plane of Symmetry and First Furrow.	90	−					o	+					90	Totals
	−90	—	—	—	—	—	—	—	—	—	—	—	—	0
		—	—	—	—	—	—	1	—	—	—	—	—	1
	−	—	—	1	—	—	—	—	—	—	—	—	—	1
		—	—	—	1	1	—	—	—	—	—	—	—	2
		—	—	—	1	2	—	—	—	—	—	—	—	3
		—	—	—	1	1	10	7	1	—	—	—	—	20
	o													
		—	—	—	—	—	5	11	—	—	—	—	—	16
		—	—	—	—	—	—	—	3	1	—	—	—	4
	+	—	—	1	—	—	—	—	—	2	1	—	—	4
		—	—	—	—	—	—	—	—	—	1	1	—	2
		—	—	—	—	—	—	1	—	—	—	—	1	2
	90	—	—	—	—	—	—	—	—	—	—	—	—	0
	Totals	0	0	2	3	4	15	20	4	3	2	1	1	55

$\rho = \cdot 435 \pm \cdot 074$ ($\cdot 598 \pm \cdot 060$).

TABLE XXXI.

XXXI. *b.* II. Eggs spaced. Axes vertical.

Correlation between Sperm-sphere Meridian and Plane of Symmetry.

First Furrow and Sperm-sphere Meridian.

First Furrow and Plane of Symmetry.	90	−					o	+					90	Totals
	−90	—	—	—	—	—	—	—	—	—	—	—	—	0
		—	—	—	—	—	—	—	—	—	—	1	—	1
	−	—	1	—	—	—	—	—	—	—	—	—	—	1
		—	—	—	—	1	1	—	—	—	—	—	—	2
		—	—	—	—	—	1	1	1	—	—	—	—	3
		—	—	1	2	7	6	1	2	—	—	—	1	20
	o													
		—	—	—	1	3	2	4	4	2	—	—	—	16
		—	—	—	—	2	—	1	1	—	—	—	—	4
	+	—	—	—	—	—	—	1	1	—	—	—	2	4
		—	—	—	—	1	1	—	—	—	—	—	—	2
		—	—	—	—	1	—	—	—	1	—	—	—	2
	90	—	—	—	—	—	—	—	—	—	—	—	—	0
	Totals	0	1	1	3	15	11	8	9	3	0	1	3	55

$\rho = \cdot 188 \pm \cdot 088$.

XXXI. *a.* I. Eggs close. Axes horizontal.

First Furrow and Sperm-sphere Meridian.

First Furrow and Plane of Symmetry.	− 90°			−			o			+		90°	Totals
− 90°	·5	1	—	—	1	1	2	1	—	·5	3	1	11
	—	—	—	1	—	1	—	—	—	—	—	1	3
−	—	—	—	3	—	1	1	—	—	—	—	—	5
	1	1	—	—	3	2	1	—	1	—	—	—	9
	—	—	1	—	—	2	2	—	—	—	—	1	6
	—	—	2	2	2	9	6	1	—	—	—	—	22
o	—	—	—	2	4	4	8	5	3	1	—	—	27
	—	—	1	—	—	2	1	3	—	—	2	—	9
+	—	—	—	—	1	2	—	—	2	—	1	—	6
	1	—	—	—	—	3	1	—	—	—	—	—	5
	—	—	—	—	1	2	—	—	1	—	—	—	4
90°	·5	1	1	—	—	3	6	—	1	·5	—	—	13
Totals	3	3	5	8	12	32	28	10	8	2	6	3	120

$\rho = -\cdot 048 \pm \cdot 061.$

TABLE XXXII.

Standard Deviations.

Eggs Close. Axes Horizontal.

	Plane of Symmetry and First Furrow	Sperm-entrance and First Furrow	Sperm-entrance and Plane of Symmetry	Sperm-sphere and First Furrow	Sperm-sphere and Plane of Symmetry	Sperm-path and First Furrow
All cases ...	45·99° ± 2·00	30·65° ± 1·32	41·01° ± 1·78	34·19° ± 1·48	40·73° ± 1·77	21·47° ± ·93
Towards furrow	50·87° ± 3·39	45·15° ± 2·96	40·53° ± 2·71	—	36·66° ± 2·45	29·08° ± 1·90
In or parallel to furrow ...	43·91° ± 3·20	—	42·90° ± 3·12	—	—	—
Away from furrow	30·77° ± 2·88	8·51° ± ·78	31·05° ± 2·91	—	35·67° ± 3·34	17·66° ± 1·62

Eggs Spaced. Axes Vertical.

	Plane of Symmetry and First Furrow	Sperm-entrance and First Furrow	Sperm-entrance and Plane of Symmetry	Sperm-sphere and First Furrow	Sperm-sphere and Plane of Symmetry	Sperm-path and First Furrow
All cases ...	25·34° ± 1·63	21·02° ± 1·35	25·67° ± 1·65	31·31° ± 2·02	32·81° ± 2·11	17·94° ± 1·15
Towards furrow	32·67° ± 4·18	40·26° ± 5·13	33·33° ± 4·25	—	37·19° ± 4·74	28·90° ± 3·68
In or parallel to furrow ...	20·01° ± 2·88	—	20·01° ± 2·88	—	—	—
Away from furrow	18·12° ± 1·58	8·31° ± ·72	23·22° ± 2·02	—	33·32° ± 2·90	12·09° ± 1·05

TABLE XXXIII.

Correlations.

		Sperm-entrance and First Furrow	Sperm-entrance and Plane of Symmetry	Sperm-sphere and Plane of Symmetry	Sperm-path and Plane of Symmetry
All cases	I	·613 ± ·038	·006 ± ·061	− ·048 ± ·061	·030 ± ·061
„ ...	II	·435 ± ·074	·302 ± ·083	·188 ± ·088	·479 ± ·070
Towards and in or parallel to furrow	I	—	—	—	—
	II	—	—	—	·725 ± ·064
Towards furrow only	I	—	—	—	·086 ± ·094
„ „	II	·502 ± ·135	·411 ± ·148	—	·880 ± ·040

I = Eggs close. Axes horizontal. II = Eggs spaced. Axes vertical.

The Plane of Symmetry is, however, affected by gravity, and since gravity and pressure are here acting at right angles to one another, there is a marked tendency for the Plane of Symmetry to lie at 90° as well as to coincide with the Furrow. This of course accounts for the high frequencies in the positive corners of Table XXIX. *b* and the high value of ρ.

(3) The Sperm-entrance Meridian and the Plane of Symmetry.

When the eggs are close and the axes horizontal the standard deviation is high, $\sigma = 41{\cdot}01° \pm 1{\cdot}79$ (Table XXVI. *c*), the correlation negligible $\rho = {\cdot}006 \pm {\cdot}061$ (Table XXIX. *c*).

We know that gravity affects the position of the Plane of Symmetry. We should expect therefore, that the relation between the Sperm-entrance and this plane would be closer if the influence of gravity were removed. This is, as a matter of fact, the case; the standard deviation sinks to $25{\cdot}67° \pm 1{\cdot}65$, while the correlation coefficient rises to ·302 ± ·083 (Tables XXVII. *c*, XXX. *c*).

(4) The Sperm-sphere Meridian and the First Furrow (Table XXVIII. I. *b*, II. *b*).

I give the values of the standard deviations since they may have some significance, though I think this is doubtful. It will be seen that in each series (I. and II.) they are greater than is the case with the Sperm-entrance Meridian and the First Furrow. Nor is the reason for this far to seek. The Sperm-path is often parallel to the First Furrow. In such cases the angle between the furrow and the meridian including the inner end of the path must needs be greater than that between the furrow and the meridian of the outer end of the path, or entrance point.

(5) The Sperm-sphere Meridian and the Plane of Symmetry.

In view of the known deviation, in many cases, of the First Furrow from the Plane of Symmetry, there might possibly be some significant relation between the latter and the Sperm-sphere.

In Table XXVIII. I. *c* and II. *c* will be found the distributions, and the correlations in Table XXXI. *a* and *b*.

When the eggs are close and their axes are horizontal the standard deviation is high, $\sigma = 40{\cdot}73° \pm 1{\cdot}77$, and practically as high as that found in the case of Sperm-entrance and Plane of Symmetry (Table XXVI. *c*). The correlation is negligible.

When, however, the eggs are spaced and their axes vertical the standard deviation is lower, the correlation coefficient higher, $\sigma = 32{\cdot}81° \pm 2{\cdot}11$, $\rho = {\cdot}188 \pm {\cdot}088$. Judged by both constants the relation of the Sperm-sphere Meridian to the Plane of Symmetry is less than that between the latter and the Sperm-entrance. There is however some slight connection between the two.

(6) The position, on the slide, of the point of entrance of the spermatozoon, or the relation of this point to (α) the direction of 'pressure,' parallel to the length of the slide, and (β) the direction, *across* the slide, of the gravitation plane of symmetry.

When the eggs are spaced and the axes horizontal, the spermatozoon enters at any point at random, as may be gathered from the frequency polygon in Fig. 6 II. (p. 159, above.)

When however the eggs compress one another in the longitudinal rows, the axes being horizontal, the spermatozoon enters mainly on those sides of the eggs which are in contact, as is evident from Table XXXIV. and Fig. 6 I. The frequencies are much higher about 90° than elsewhere. Further, while a few eggs may be found in which the sperm has entered about 180°, hardly a single one has entered about 0°.

It was towards this side that the white pole was turned up, and, as is known, the sperm does not enter in the vegetative hemisphere.

These values have been determined from the known value, in each egg, of the angle between the entrance point and the First Furrow, and by subtraction of the angle between Sperm-entrance and Plane of Symmetry. The position of the latter on the slide is known, and hence the point of entrance of the sperm. On the assumption that this point is opposite to the grey-crescent, angles may be distinguished from their supplements, a distinction which could not otherwise be made since what is actually measured is the angle between Sperm-entrance and First Furrow after the eggs have been removed from the slide. This assumption may or may not be justified. It might be better therefore to ignore the distinction and tabulate the frequencies as in Table XXXIV. *b*, where the distribution is about 90°. It may be worth while to point out that the standard deviation of this distribution ($\sigma = 44{\cdot}13° \pm 1{\cdot}87$) is nearly the same as those of the distributions of the First Furrow given in Table XIV., I. A and II. A.

TABLE XXXIV.

Sperm Entrance Meridian and Direction of "Pressure."
Eggs close. Axes horizontal.

Difference between angles and their Supplements.

	a. Regarded.			*b.* Disregarded.	
+*180—165*	7	− *0— 15*	0	− *0—15*	7
165—150	7	*15— 30*	0	*15—30*	7
150—135	10	*30— 45*	0	*30—45*	10
135—120	15	*45— 60*	0	*45—60*	15
120—105	8	*60— 75*	6	*60—75*	14
105— 90	9	*75— 90*	8	*75—90*	17
90— 75	3	*90—105*	10	+*90—75*	13
75— 60	3	*105—120*	11	*75—60*	14
60— 45	2	*120—135*	8	*60—45*	10
45— 30	1	*135—150*	6	*45—30*	7
30— 15	1	*150—165*	5	*30—15*	6
15— 0	2	*165—180*	4	*15—0*	6
Total	68	Total	58	Total	126
M = +118·46° ± 3·18		−114·32° ± 2·68		−85·83° ± 2·65	
σ = 38·94° ± 2·25		30·27° ± 1·90		44·13° ± 1·87	

But however that may be, it is clear that spermatozoa enter mainly where the eggs in the rows are in contact. This must be attributed, I think, to their congregating in greater numbers in these places, and this in turn to either capillarity, or to some chemotactic stimulus exerted in excess at these points by the jelly. It cannot, I believe, be assigned to any phototactic stimulus, since it is not observable in the case of the spaced eggs.

We have already established the facts that the First Furrow tends to place itself parallel to the length of the slide, and that it very frequently includes the point of entrance of the sperm. The position of the First Furrow in compressed eggs would then appear not to be determined by the pressure, which is slight, (certainly less than that used in ordinary pressure experiments), but to be due simply to the fact that the sperm prefers to enter in between the eggs in the rows.

There is no reason that I can see for supposing that gravity, which is not eliminated in this experiment, has any share in the result.

(7) The inclination of the Sperm-radius to the Egg-axis, or the angle subtended at the centre of the circle by the distance of the point of entrance from the animal pole. This angle is never greater than 90°.

There is naturally no distinction between positive and negative angles.

I. When the eggs are close, their axes horizontal (Table XXXV. I.), the mean value of this angle is 40·65°, that is, the sperm under these conditions usually enters about half-way between the animal pole and the equator. These eggs it

TABLE XXXV.

Inclination of Sperm-radius and Egg-axis, or angle subtended at the centre of the egg by the distance of the sperm-entrance-point from the animal pole.

I. Eggs close. Axes horizontal.		II. Eggs spaced. Axes vertical.	
90—75	5	*90—75*	27
75—60	9	*75—60*	23
60—45	42	*60—45*	5
45—30	35	*45—30*	0
30—15	15	*30—15*	0
15— 0	15	*15— 0*	0
Total	123	Total	55
M=40·65°		M=73·50°	

must be remembered are laid with the white hemisphere, into which the sperm cannot enter, on one side. While the axis lies across the slide the equator of the egg is parallel to its length. The mean point of entry is (Table XXXIV. *a*) 118° on one side and 114° on the other, or 28° and 24° from the equator respectively. This may account in part for the point of entry selected by the sperm.

II. When the eggs are spaced and their axes vertical, the point selected by the spermatozoon for entrance is near the equator (Table XXXV. II.), the mean value is 73·50°. The eggs were white pole downwards, and the sperm-containing water with which they were inseminated was placed all round the eggs but did not cover them. This affords a sufficient explanation of what occurs.

As we have already seen, the correlations between Sperm-entrance Meridian and Plane of Symmetry, between Plane of Symmetry and First Furrow, and between Sperm-sphere Meridian and Plane of Symmetry are all higher when "pressure" and "gravity" are not allowed to interfere, while that between the Sperm-entrance Meridian and the First Furrow is not quite as high in the absence as in the presence of these factors (Table XXXIII.). We might almost infer from this that the relations between these planes are independent of the distance of the Sperm-entrance from the animal pole.

An inspection of the correlation tables (Table XXXVI.) will show how far this inference is justified.

I have not calculated the coefficients, but the mean values of this angle (Inclination of Sperm-radius to Egg-axis) for each class of the several planes (the means are inserted at the bottom of the tables) will show at once that the correlation is to all intents and purposes nil (I. *a*, I. *b*, I. *c*), at least, when the eggs are close and their axes horizontal.

The correlation is just as poor in the case of those eggs that were spaced and had their axes vertically placed, II. *a*, II. *b*, II. *c*. Where however the angle between the Plane of Symmetry and the First Furrow is small there is a *slight* lowering in the value of the means of the other angle (II. *c*).

TABLE XXXVI.

Correlations between Inclination of Sperm-radius to Egg-axis and the several planes.

Inclination of Sperm-radius to Egg-axis.

II. *a.*

Sperm-entrance and First Furrow.

	0°					90°	Totals
0°	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	3	2	—	—	—	—	5
	19	3	1	—	—	—	23
90°	23	—	—	1	1	2	27
Totals	45	5	1	1	1	2	55

Means 74·0° 61·5° 67·5° 82·5° 82·5° 82·5°

II. *b.*

Sperm-entrance and Plane of Symmetry.

	0°					90°	Totals
0°	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	4	—	—	1	—	—	5
	14	4	3	—	1	1	23
90°	17	5	3	2	—	—	27
Totals	35	9	6	3	1	1	55

Means 73·0° 76·5° 75·0° 72·0° 67·5° 67·5°

II. *c.*

Plane of Symmetry and First Furrow.

	0°					90°	Totals
0°	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	4	—	1	—	—	—	5
	16	4	1	1	1	—	23
90°	16	3	4	2	2	—	27
Totals	36	7	6	3	3	0	55

Means 71·0° 73·5° 75·0° 78·0° 78·0° 0

Inclination of Sperm-radius to Egg-axis.

I. *a.*

Sperm-entrance and First Furrow.

	0°					90°	Totals
0°	14	—	—	—	1	—	15
	12	—	—	1	2	—	15
	22	1	3	5	3	1	35
	22	8	5	4	2	1	42
	3	4	—	—	1	1	9
90°	2	1	1	1	—	—	5
Totals	75	14	9	11	9	3	121

Means 36·0° 58·5° 51·0° 46·5° 37·5° 52·5°

I. *b.*

Sperm-entrance and Plane of Symmetry.

	0°					90°	Totals
0°	5	2	2	2	1	2	14
	8	2	—	1	1	3	15
	14	6	5	4	1	5	35
	14	6	9	5	3	3	40
	1	5	1	2	—	—	9
90°	1	1	3	—	—	—	5
Totals	43	22	20	14	6	13	118

Means 37·5° 46·5° 42·0° 42·0° 37·5° 33·0°

I. *c.*

Plane of Symmetry and First Furrow.

	0°					90°	Totals
0°	5	1	2	2	1	3	14
	7	2	2	1	—	3	15
	15	5	2	3	2	8	35
	18	4	6	3	3	6	40
	4	1	1	1	—	2	9
90°	—	1	2	—	1	1	5
Totals	49	14	15	10	7	23	118

Means 40·5° 43·5° 45·0° 37·5° 46·5° 37·5°

(8) The Sperm-path and the First Furrow.

As explained above the Sperm-path (outer or "penetration" path) may be parallel to the First Furrow when it is not actually included in the latter, or when it makes an angle with the Furrow it may be turned towards or away from it.

In Table XXXVII. will be found the distribution of this angle in all cases, and in each of these three variations of position, both when the eggs are affected by gravity and pressure (I.) and when they are not (II.).

TABLE XXXVII.

Sperm-path and First Furrow.

Angle	I. Eggs close. Axes horizontal.				II. Eggs spaced. Axes vertical.			
	a. All cases	*b.* Towards furrow	*c.* Away from furrow	In or parallel to furrow	*a.* All cases	*b.* Towards furrow	*c.* Away from furrow	In or parallel to furrow
−*90—75*	0	0	0	—	0	0	0	—
75—60	1	1	0	—	0	0	0	—
60—45	0	0	0	—	2	2	0	—
45—30	4	4	0	—	0	0	0	—
30—15	11	9	2	—	5	2	3	—
15—0	42	11	9	43	26	7	13	11
+ *0—15*	41	11	9		17	1	11	
15—30	8	4	4	—	3	0	3	—
30—45	9	7	2	—	1	1	0	—
45—60	6	5	1	—	0	0	0	—
60—75	1	1	0	—	1	1	0	—
75—90	0	0	0	—	0	0	0	—
Totals	123	53	27	43	55	14	30	11
M	$3{\cdot}47° \pm 1{\cdot}31$	$4{\cdot}85° \pm 2{\cdot}69$	$6{\cdot}35° \pm 2{\cdot}29$	—	$-2{\cdot}04° \pm 1{\cdot}63$	$6{\cdot}44° \pm 5{\cdot}20$	$0{\cdot}62° \pm 1{\cdot}49$	—
σ	$21{\cdot}47° \pm 0{\cdot}93$	$29{\cdot}08° \pm 1{\cdot}90$	$17{\cdot}66° \pm 1{\cdot}62$	—	$17{\cdot}94° \pm 1{\cdot}15$	$28{\cdot}89° \pm 3{\cdot}68$	$12{\cdot}09° \pm 1{\cdot}05$	—

It will be seen that the tendency of the Sperm-path to lie in or parallel to the First Furrow is great, and greater when the external disturbing influences are removed, $\sigma = 17{\cdot}94° \pm 1{\cdot}15$ as opposed to $\sigma = 21{\cdot}47° \pm 0{\cdot}93$, when gravity and pressure act upon the egg. It will be noticed however that in any case the value of the standard deviation is less for this angle than for that between Sperm-entrance Meridian and First Furrow (Table XXXII.). This is due to the fact that while the angle between Sperm-path and First Furrow cannot be greater than that between Sperm-entrance Meridian and First Furrow, it may, of course, be less. Also the path may be parallel to the Furrow when the Entrance Meridian makes a considerable angle with it (Table XL.).

Further, when the path is turned away from the Furrow the standard deviation diminishes, when it is turned towards the Furrow its value increases. It is in the former cases that the inner or "copulation" path is generally seen, parallel to the Furrow, but inclined to the "penetration" path (Fig. 5 C).

It might be supposed that when the Sperm-path is turned away from the Furrow, the relations between Sperm-entrance and Sperm-sphere Meridian, on the one hand, and Plane of Symmetry and First Furrow on the other would be less close than when it is turned towards it. This however (to accept the standard deviation as a criterion) is not the case, as may be gathered at once from Tables XXXVIII. and XXXIX. Whether the eggs are close and under the influence of gravity, or not, the standard deviation, in the case of each of the angles considered, except that between the Sperm-sphere Meridian and the Plane of Symmetry, is considerably less in the former than in the latter contingency. The numbers of course are very small, but all the results point in the same direction.

When the path is parallel to or in the Furrow (Table XL.) the angles made by Sperm-sphere and Sperm-entrance Meridians with the Furrow are of course nil. The standard deviations of the other two are greater than when the path turns away from the First Furrow when the eggs are under the influence of gravity and pressure, about the same when they are not.

Nevertheless I am bound to point out that if the relation between Sperm-entrance and Symmetry Plane and First Furrow be measured by the correlation coefficient, instead of by the standard deviation (the results are given in Tables XLI. and XLII.), it is closer in those eggs in which the Sperm-path is turned towards the Furrow than it is in the whole series.

This may be readily seen from the values of ρ in Table XXXIII., first and second columns. Yet the corresponding standard deviations (Table XXXII., second and third columns, first and third rows) are also greater, instead of less, when the path is inclined towards the Furrow than in the whole series. This shows plainly enough the difficulty of interpreting results based on such small numbers.

(9) Sperm-path and Plane of Symmetry.

At present we have discovered no close correlation between the Plane of Symmetry and any one point of the Sperm-path. There is however a correlation—if calculations based on such small numbers can be trusted—between the angle made by the Sperm-path with the First Furrow and the deviation of the latter from the Plane of Symmetry, at least when the eggs are not subjected to the influence of gravity and pressure.

When the eggs are close and their axes horizontal there is no correlation (Table XLIII. *a*).

TABLE XXXVIII.

When the Sperm-path is turned towards the First Furrow.

I. Eggs close. Axes horizontal.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
−*90—75*	1	1	7	1
75—60	3	1	3	0
60—45	5	5	3	5
45—30	4	8	4	6
30—15	7	7	2	7
15—0	7	7	5	9
+ *0—15*	4	2	11	4
15—30	4	8	3	8
30—45	4	4	4	5
45—60	6	4	2	3
60—75	6	2	2	2
75—90	2	2	5	1
Totals	53	51	51	51
M	+4·67° ±4·18	− 2·82° ±3·83	− 4·23±4·80	− 0·44° ±3·46
σ	45·15° ±2·96	40·53° ±2·71	50·87±3·39	36·66° ±2·45

II. Eggs spaced. Axes vertical.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
−*90—75*	1	0	0	1
75—60	1	0	1	0
60—45	0	1	1	1
45—30	1	1	1	3
30—15	4	2	1	1
15—0	4	3	5	2
+ *0—15*	1	4	1	1
15—30	0	1	0	3
30—45	0	0	2	1
45—60	1	1	0	1
60—75	0	0	2	0
75—90	1	1	0	0
Totals	14	14	14	14
M	−11·35° ±7·26	− 0·36° ±6·01	− 1·07±5·90	− 8·57° ±6·70
σ	40·26° ±5·13	33·33° ±4·25	32·67±4·18	37·19° ±4·74

TABLE XXXIX.

When the Sperm-path is turned away from the First Furrow.

I. Eggs close. Axes horizontal.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
−90—75	0	2	2	1
75—60	0	0	0	0
60—45	0	1	1	0
45—30	0	3	3	2
30—15	0	2	2	3
15—0	13	8	8	7
+ 0—15	13	6	5	4
15—30	1	2	4	3
30—45	0	1	0	3
45—60	0	1	1	1
60—75	0	0	0	0
75—90	0	0	0	2
Totals	27	26	26	26
M	0·84° ± 1·11	− 9·81° ± 4·11	− 2·31° ± 4·07	− 1·73° ± 4·72
σ	8·51° ± 0·78	31·05° ± 2·91	30·77° ± 2·88	35·67° ± 3·34

II. Eggs spaced. Axes vertical.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
−90—75	0	0	0	1
75—60	0	0	0	0
60—45	0	1	0	1
45—30	0	1	0	3
30—15	1	2	2	1
15—0	16	9	12	5
+ 0—15	13	11	10	8
15—30	0	3	4	4
30—45	0	1	0	4
45—60	0	1	2	2
60—75	0	1	0	0
75—90	0	0	0	1
Totals	30	30	30	30
M	−1·50° ± 1·02	− 3·49° ± 2·86	4·50° ± 2·23	− 6·00° ± 4·10
σ	8·31° ± 0·72	23·22° ± 2·02	18·12° ± 1·58	33·32° ± 2·90

TABLE XL. *When the Sperm-path is either included in or is parallel to the First Furrow.*

Angle	Eggs close. Axes horizontal		Eggs spaced. Axes vertical	
	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow
− *90—75*	2	2	0	0
75—60	0	0	0	0
60—45	1	1	0	0
45—30	1	2	1	1
30—15	2	2	0	0
15—0	10	9	3	3
+ *0—15*	10	11	5	5
15—30	2	2	0	0
30—45	3	2	2	2
45—60	2	2	0	0
60—75	3	2	0	0
75—90	7	8	0	0
Totals	43	43	11	11
M	17·27° ± 4·41	16·22° ± 4·52	4·77 ± 4·07	4·77 ± 4·07
σ	42·90° ± 3·12	43·91° ± 3·20	20·01 ± 2·88	20·01 ± 2·88

TABLE XLI. *Correlation between Sperm-entrance Meridian and First Furrow when the Sperm-path is directed towards the First Furrow.*

Eggs spaced. Axes vertical.

Sperm-entrance Meridian and Plane of Symmetry.

First Furrow and Plane of Symmetry.	90			−			0			+			90	Totals
90	—	—	—	—	—	—	—	—	—	—	—	—		0
	—	—	—	—	—	—	1	—	—	—	—	—		1
−	—	—	—	—	—	—	1	—	—	—	—	—		1
	—	—	—	—	1	—	—	—	—	—	—	—		1
	—	—	—	—	1	—	—	—	—	—	—	—		1
	—	—	—	1	—	2	1	1	—	—	—	—		5
0	—	—	—	—	—	1	—	—	—	—	—	—		1
	—	—	—	—	—	—	—	—	—	—	—	—		0
+	—	—	1	—	—	—	—	—	—	1	—	—		2
	—	—	—	—	—	—	—	—	—	—	—	—		0
	—	—	—	—	—	—	1	—	—	—	—	1		2
90	—	—	—	—	—	—	—	—	—	—	—	—		0
Totals	0	0	1	1	2	3	4	1	0	1	0	1		14

$\rho = \cdot 502 \pm \cdot 135.$

TABLE XLII.

Correlation between Sperm-entrance Meridian and Plane of Symmetry when the Sperm-path is directed towards the First Furrow.

Eggs spaced. Axes vertical.

Sperm-entrance Meridian and First Furrow.

First Furrow and Plane of Symmetry.	− 90°					− 0°	+ 0°				+ 90°	Totals
− 90°	—	—	—	—	—	—	—	—	—	—	—	0
	·5	—	—	—	—	—	—	—	—	—	·5	1
	—	1	—	—	—	—	—	—	—	—	—	1
	—	—	—	—	1	—	—	—	—	—	—	1
	—	—	—	—	—	1	—	—	—	—	—	1
	—	—	—	1	2	2	—	—	—	—	—	5
0°	—	—	—	—	—	—	1	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	0
+ 90°	·5	—	—	—	1	—	—	—	—	—	·5	2
	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	1	—	—	—	1	—	2
	—	—	—	—	—	—	—	—	—	—	—	0
Totals	1	1	0	1	4	4	1	0	0	1	0	1	14

$\rho = \cdot 411 \pm \cdot 148.$

TABLE XLIII.

Correlation between Sperm-path and Plane of Symmetry.

XLIII. *a. All cases.*

Eggs close. Axes horizontal.

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	− 90°					− 0°	+ 0°				+ 90°	Totals
− 90°	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	1	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	0
	2	—	—	—	—	—	—	—	1	—	—	1	4
	1	1	3	2	—	2	—	—	—	1	—	1	11
	1	1	1	4	4	9	11	2	1	2	2	4	42
0°	2	—	1	2	2	8	9	4	1	1	2	6	38
	3	—	—	—	—	1	3	1	—	—	—	—	8
+ 90°	—	—	—	1	—	1	3	1	2	—	—	1	9
	2	—	—	—	—	—	1	1	1	1	—	—	6
	—	1	—	—	—	—	—	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	11	3	5	9	6	22	27	9	6	5	4	13	120

$\rho = \cdot 030 \pm \cdot 061.$

XLIII. *b.* *When only those cases are considered in which the Sperm-path is directed towards the Egg-axis and First Furrow.*

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	− 90°					− 0°	+ 0°					+ 90°	Totals
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	1	—	—	—	—	—	—	1
−	—	—	—	—	—	—	—	—	—	—	—	—	0
	2	—	—	—	—	—	—	—	1	—	—	1	4
	1	1	3	2	—	—	—	—	—	1	—	1	9
	—	1	—	2	1	2	3	—	—	1	1	—	11
0°	—	—	—	—	1	2	2	1	—	—	1	2	9
	2	—	—	—	—	—	2	—	—	—	—	—	4
+	—	—	—	—	—	—	3	1	2	—	—	1	7
	2	—	—	—	—	—	1	1	1	—	—	—	5
	—	1	—	—	—	—	—	—	—	—	—	—	1
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	7	3	3	4	2	5	11	3	4	2	2	5	51

$\rho = \cdot 086 \pm \cdot 094.$

TABLE XLIV.

Correlation between Sperm-path and Plane of Symmetry.

XLIV. *a.* *All cases.*

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	− 90°					− 0°	+ 0°					+ 90°	Totals
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	—	—	—	—	0
−	—	1	1	—	—	—	—	—	—	—	—	—	2
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	4	1	—	—	—	—	—	5
	—	—	—	2	1	11	6	2	1	2	1	—	26
0°	—	—	—	—	1	3	9	2	2	—	—	—	17
	—	—	—	—	1	2	—	—	—	—	—	—	3
+	—	—	—	—	—	—	—	—	—	—	1	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	1	—	—	—	1
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	1	1	2	3	20	16	4	4	2	2	0	55

$\rho = \cdot 479 \pm \cdot 070.$

XLIV. *b.* *When only those cases are considered in which the Sperm-path is either directed towards or is parallel to the First Furrow.*

Eggs spaced. Axes vertical.

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	− 90					− 0	+ 0				+ 90		Totals
90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	—	—	—	—	0
−	—	1	1	—	—	—	—	—	—	—	—	—	2
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	2	—	—	—	—	—	—	2
	—	—	—	2	1	6	2	—	1	—	1	—	13
0	—	—	—	—	—	—	4	—	2	—	—	—	6
	—	—	—	—	—	—	—	—	—	—	—	—	0
+	—	—	—	—	—	—	—	—	—	—	1	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	1	—	—	—	1
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	1	1	2	1	8	6	0	4	0	2	0	25

$\rho = \cdot 725 \pm \cdot 064.$

XLIV. *c.* *Only those cases in which the path is turned towards the Furrow.*

Eggs spaced. Axes vertical.

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	− 90					− 0	+ 0				+ 90		Totals
90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	—	—	—	—	0
−	—	1	1	—	—	—	—	—	—	—	—	—	2
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	2	—	—	—	—	—	—	2
	—	—	—	1	1	3	—	—	1	—	1	—	7
0	—	—	—	—	—	—	1	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
+	—	—	—	—	—	—	—	—	—	—	1	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	1	—	—	—	1
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	1	1	1	1	5	1	0	2	0	2	0	14

$\rho = \cdot 880 \pm \cdot 040.$

Under the contrary conditions the value of the coefficient is $\rho = \cdot 479 \pm \cdot 070$ (Table XLIV. *a*) which rises to $\cdot 725 \pm \cdot 064$, and $\cdot 880 \pm \cdot 040$, as first those instances are excluded in which the path turns away from the Furrow (Table XLIV. *b*) and then those also in which the path is parallel to the Furrow or in it (Table XLIV. *c*).

It may be noted that, when the eggs are close and their axes horizontal, there is still no correlation even under what appear to be the most favourable circumstances (Table XLIII. *b*).

I am well aware of the doubtful value of a correlation determined on so very few examples, but, if the results can be accepted, it is evident that when the path makes a wide angle with the Furrow, the deviation of the grey crescent from the Furrow is large too.

More direct evidence can, however, also be brought forward in support of this conclusion, for from the known values of the angle between Sperm-path and First Furrow, and First Furrow and Plane of Symmetry, the values of the angles between Sperm-path and Plane of Symmetry may be obtained. The results are given in Table XLV. in which the standard deviations are shown (the difference between negative and positive angles being neglected). In the same table the values of σ, calculated in the same way, for the angle between Sperm-path and First Furrow are given for comparison.

It is evident at once that when all cases are considered the relation of Sperm-path to First Furrow is as close as its relation to Symmetry Plane, if not closer; that when the Sperm-path is included in or parallel to the First Furrow its relation to the Symmetry Plane is as in all the cases taken together; that when the path is turned away from the Furrow its relation to the Furrow is much closer than to the Plane of Symmetry, but that when it is turned towards the Furrow the reverse is the case.

If, however, the numbers are considered to be so small that this result must be rejected, then it must be confessed that no internal factor has yet been discovered by which the direction occupied by the plane of the grey crescent is conditioned; nor am I able to see by what known internal factor it could be conditioned since, as far as I am aware, every possible cause has been tried. Provisionally, therefore, we must regard the direction of the whole ("penetration") Sperm-path as the cause which determines, in the absence of all outside interference, the Plane of Symmetry of the unsegmented egg, and so the median plane of the embryo.

The foregoing investigations have established a very close relation between the point of entrance of the spermatozoon, or the path taken by the spermatozoon in the egg, and the position of the First Furrow of segmentation. The relation is closest when the influence of gravity is removed by placing the eggs upon the slides with their axes vertical and their white poles below, but it still exists in a degree which cannot be ignored when the axes are horizontal; indeed, if it be measured by the correlation coefficient instead of by the standard deviation it is as

TABLE XLV.

Eggs spaced. Axes vertical.

All cases.

	Angle between Sperm-path and Plane of Symmetry	Angle between Sperm-path and First Furrow
0—15	33	43
15—30	13	8
30—45	7	1
45—60	1	2
60—75	1	1
Totals	55	55
	$\sigma = 13{\cdot}56° \pm 0{\cdot}87°$.	$\sigma = 12{\cdot}58° \pm 0{\cdot}81°$.

When Sperm-path turns away from Furrow.

0—15	17	24
15—30	9	6
30—45	4	0
Totals	30	30
	$\sigma = 10{\cdot}72° \pm 0{\cdot}94$.	$\sigma = 6{\cdot}00° \pm 0{\cdot}53°$.

When Sperm-path is included in or parallel to First Furrow.

0—15	8
15—30	0
30—45	3
Total	11
	$\sigma = 13{\cdot}35° \pm 1{\cdot}92$. $\sigma = 0°$.

When Sperm-path is turned towards Furrow.

0—15	8	8
15—30	4	2
30—45	0	1
45—60	1	2
60—75	1	1
Totals	14	14
	$\sigma = 10{\cdot}16° \pm 1{\cdot}29$.	$\sigma = 19{\cdot}65° \pm 2{\cdot}50$.

close in the latter as in the former case, if not closer. This is due to the fact that there is a slight tendency for the First Furrow to place itself at right angles to the gravitation plane, that is, parallel to the length of the slide. Since, moreover, the spermatozoa enter the egg mainly from the compressed sides, there is already a considerable tendency for the First Furrow to lie in that direction, and under the conditions of the experiment the two are summed. The pressure, as such, seems to have little influence.

In certain cases the Sperm-path is inclined away from the Furrow, none the less its inner end bends upon its outer end, and is found to be in, or parallel to the cell division.

There is on the other hand but a very slight relation between the meridian occupied by either the entrance point or the inner end of the Sperm-path and the Plane of Symmetry, even under the most favourable conditions, and when the position of the Symmetry Plane is altered, as we know it can be altered, by gravitation, the correlation is negligible. A correlation of considerable magnitude has, however, been found between the whole ("penetration") path of the sperm and the position of the Plane of Symmetry, and since the inclination of the Sperm-radius and Sperm-path have, by themselves, but little effect, the only possible conclusion is that if the position of the grey crescent is dependent upon the spermatozoon at all, it must be determined by all the meridians successively occupied by the sperm as it travels from the surface into the interior.

In this case it is evident that while gravity improves the relation between Sperm-path and First Furrow it utterly destroys the relation between Sperm-path and Plane of Symmetry. This is in exact accord with the results previously obtained. For it was shown that while gravity, during the short interval in which it operated, could very largely determine the position of the Plane of Symmetry, it had but little influence on the direction of the Furrow; while there was a considerable tendency for the Plane of Symmetry to lie in the Gravitation Plane, there was but a slight tendency for the Furrow to be situated not in but at right angles to that plane.

The internal factors which determine the direction of the deviation of cell division must be therefore distinct from those which determine the symmetry of the egg, and later, of the embryo.

Let us briefly enquire what the mechanisms may be by which these two different processes are brought about.

As far as the division of the fertilized ovum is concerned, it is not hard to understand why the Furrow should include the Sperm-entrance and path, or at least be parallel to the inner portion of the latter. The cell-division takes place in the equator of the fertilization spindle, where the spindle fibres thicken to form a cell-plate after the chromosomes have passed to opposite poles; this equatorial plane is of course at right angles to the spindle axis—the line joining the two centrosomes, or, at an earlier stage, the direction of separation of the two halves of the divided sperm-centrosome (Fig. 4 D).

This line lies in a plane parallel to the egg equator (probably owing to the manner of distribution of the yolk about the axis) and is at right angles to that uniting the male and female pronuclei, or "copulation" path of Roux.

The first part of the path ("penetration" path of Roux) is simply the entrance funnel; its formation is probably a capillary phenomenon, attributable to the aggregation about the acrosome of a watery substance withdrawn from the egg cytoplasm, and to the streaming of this substance into the interior of the egg (Fig. 4 A). The spermatozoon or sperm nucleus previously carried in by this movement then turns towards the female pronucleus, and this second portion of

the path makes an angle with the first. Should the first portion have been directed towards the axis, and should the female pronucleus, as is usually the case, also lie in the axis, then both parts will lie in one and the same meridional plane; this becomes the plane of the First Furrow. But should the first part be inclined away from the axis, only the second will lie in a meridional plane, which becomes the plane of division for the same reasons. Should the divergence of the first part from the axis not have been great (in only three cases out of fifty-seven has it been found to exceed 30°, and in only one of these to be greater than 45°) the First Furrow may still practically include the point of entrance of the sperm. But there are cases in which the First Furrow deviates widely from the Sperm-path, even when the latter is directed towards it. The reason for this may very possibly be that the female pronucleus has not moved axially, but exaxially in its return from the animal pole; the meridians of Sperm-entrance and "copulation" path, or in other words the meridians of Sperm-entrance and First Furrow, would then be more or less widely separate from one another. I have, however, no direct evidence that this is what has occurred; but Roux states that in eggs taken at the end of the breeding season—after a long stay in the uterus—the female pronucleus is exaxial. The dates on which my eggs were taken were, for Series I. (close, horizontal), March 30th, for Series II. (spaced, vertical), April 4th. In Oxford the breeding-season ends usually in the third week of April.

And now let us turn to the Plane of Symmetry. This is brought about by the formation of the grey crescent, and this is due, according to Roux, to the disappearance into the interior of some of the superficial pigment of the egg. The pigment disappears over a crescentic region at the border of the pigmented area on one side of the egg. When all the pigment has gone, the crescent is white and indistinguishable in colour from the original circular white area, though its position may of course be readily detected (Fig. 7). I do not, of course, know, but I cannot help supposing that this retreat of the pigment into the inside is directly due to the streaming movement set up in the egg-cytoplasm by the sperm.

It is known from cytological descriptions that the entrance-funnel is an accumulation of a clear substance; this takes place about the apical body or acrosome of the spermatozoon as soon as the latter gets inside the surface membrane of the egg; it looks as though the clear substance were more watery than the surrounding cytoplasm, as though it were due to a local abstraction of water. Again, at the bottom of the entrance-funnel the Sperm-sphere, surrounded later by the Sperm-aster, is formed, and this consists of a yolk-free area of cytoplasm (Fig. 5 C) in which later on large vacuoles of a clear, watery looking substance, become evident. Some of these vacuoles can be seen in the Sperm-sphere of the Axolotl figured in Fig. 5 D, and I have observed a precisely similar appearance on the inner side of the Sperm-nucleus in the Frog's egg. It may be supposed in like manner that these vacuoles are due to a local abstraction of water from the cytoplasm.

On this view the acrosome and the middle piece are both hygroscopic particles, and it is interesting to remember that they are both of centrosomal origin, the one being derived from the centrosphere, the other from the centrosome of the spermatid.

This hypothesis is, it seems to me, very much strengthened by the result of a simple experiment. By placing a hygroscopic particle—a small crystal of salt or sugar for example—in a drop of the yolk of a Hen's egg, an imitation Sperm-sphere may very easily be made. A clear area from which the yolk granules have been driven away appears round the particle, and as the latter begins to dissolve it soon comes to lie in a little pool or vacuole of its own solution.

Should this interpretation of the processes observed in the egg during fertilization be correct, it would seem obvious to connect the immigration of the pigment with the streaming movements of watery substance so set up, the focus towards which the streams are directed being in that case the whole (" penetration ")

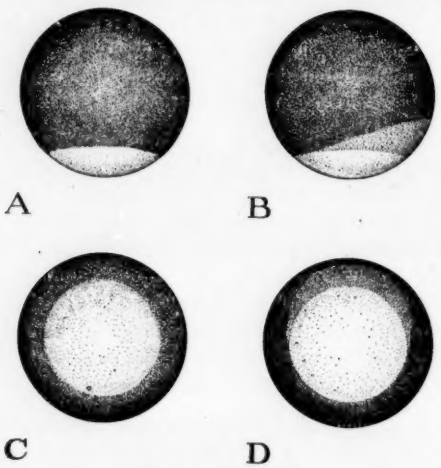


Fig. 7. The formation of the grey crescent.
In A and C it has not yet appeared.
A, B, the egg seen from the side. C, D, from the vegetative pole (centre of the white area).

Sperm-path—the entrance-funnel and Sperm-sphere. It is with this that the direction of the Symmetry Plane appears to be correlated when gravity is not permitted to exert its influence. When gravity does operate the correlation disappears, and intelligibly so. For the movements due to the sinking of the yolk, the rising of the cytoplasm and pigment which we have known to be produced ever since Born described them in the permanently inverted eggs of Pflüger's experiment, will evidently alter the direction of the other movement directed

towards the Sperm-path, while from what has been ascertained of the behaviour of chromatophores in the presence of light, we may also understand why this agent should modify the position of the grey crescent.

The union of the pronuclei is, however, a process independent of the direction of first entrance of the sperm, and of the formation of the Sperm-sphere, in which the vacuoles are developed before the two nuclei meet. Here then are two perfectly distinct processes, on one of which the direction of cell division depends, while the other determines the symmetry of the egg, with which is correlated the symmetry of the embryo. One of these can be readily affected by gravity and by light, the other cannot. But when all external disturbing influences are removed, when the Sperm-path is radial and the egg-nucleus is axial, then the causes which determine the symmetry of segmentation and those on which the symmetry of egg and embryo depend will be coincident.

Experimentally however they may be separated from one another. This is, indeed, no new discovery for the Frog's egg, for Pflüger showed that in forcibly inverted ova, while the First Furrow might make any angle with the plane including the egg-axis and the vertical (the "streaming meridian" of Born, or, as I have called it, the gravitation symmetry plane), the dorsal lip of the blastopore always appeared in this plane and on the side on which the white pole was turned up.

What is, I believe, new, is that the thirty minutes that pass before the egg turns over are sufficient for gravity to do its work. And this, I may remark, will explain the controversy that has been carried on for so long between Roux, Morgan and Kathariner on the one side, and Schulze and Moszcowski on the other. Roux maintained that the position of the First Furrow was determined not by gravity, but by causes residing in the structure of the egg. Schulze replied that gravity was a necessity for normal development, while Moszcowski claimed that the grey crescent, on which, as admitted and asserted by Roux, the position of the First Furrow, and later of the Sagittal Plane, depended, was brought about by gravity, quite a short time being amply sufficient for the purpose. It may be regarded as quite certain that the eggs kept in any constant state of slow rotatory motion (by being placed on a vertical wheel or in any other way) will produce normal tadpoles; and with regard to the second point, Morgan and Kathariner have shown that eggs kept in motion during insemination and later, still develope the grey crescent. This experiment is paralleled by placing the eggs on the slide with the axis vertical; in them the crescent appears. But it is interesting to notice that though gravity is certainly not indispensable, it may still modify the position of the Symmetry Plane.

The grey crescent of the Frog's egg is no solitary example of the change of structure that may be brought about by fertilization. The same conversion of a radial into a bilateral symmetry may be observed in other cases, and the same determination of the First Furrow by the point of entrance of the spermatozoon.

In the Sea-urchin *Toxopneustes* Wilson and Matthews were able to show that after the extrusion of the polar bodies at the animal pole of what may be called the primary egg-axis, the female pronucleus wandered in the cytoplasm to any position till met by the male pronucleus. The segmentation nucleus formed of their union then took up an excentric position near the centre, so determining a new egg-axis and a new animal pole. This axis became the axis of cleavage, the meridian of the First Furrow being determined by the point of entrance of the spermatozoon.

In the Ascidian *Cynthia* according to Conklin, the immature oocyte has a radial symmetry about an axis determined by the excentric position of the germinal vesicle; there is a peripheral layer of yellow cytoplasm surrounding a central grey yolk. Upon the entrance of the spermatozoon near the vegetative pole the nucleus breaks down and from it a clear substance is formed. Some of this remains at the animal pole as a small patch containing the female pronucleus; the rest flows, with the peripheral yellow substance, to the vegetative pole, where it lies in a shallow layer over the latter.

The symmetry is still radial. It soon, however, becomes bilateral, for the yellow and clear substances stream up on one side, the future posterior side, to the equator of the egg, the sperm being carried along in the latter to meet the female nucleus, which has meanwhile descended from the animal pole. The two together pass into the centre, along with the clear substance, and the fertilization spindle is formed at right angles to their path. The First Furrow, and later the Sagittal Plane, lie therefore in the plane of egg symmetry which is established during fertilization.

Again, in the Mollusc *Dentalium* we know from Wilson's account that when the sperm enters, the centrally placed germinal vesicle breaks down and its substance becomes confluent with a small clear area at the animal pole, a larger clear area at the vegetative pole. Later the brick-red yolk is interposed between animal and vegetative hyaline areas, the latter being soon extruded as the polar lobe.

A few other instances may be cited. In the Nematode *Diplogaster* it is the point of union of the two pronuclei which determines which end of the elliptical egg—the anterior ovarian end at which the polar bodies are formed, or the posterior oviducal end at which the sperm enters—shall become the animal and ectodermal end (Ziegler). In another Nematode *Strongylus* Wandolleck has shown that the yolk, originally uniformly distributed, becomes placed at one end, and the egg therefore telolecithal when fertilization occurs. Similarly in Cirrhipedes (Groom), in the Snail *Physa* (Kostanecki and Wierzejski), and in Turbellarians (Lang). In Ctenophora in the same way the micromere-forming substance becomes aggregated at one pole (Agassiz), and in Teleostei the blastodisc is formed by the local accumulation of the periblast. Lastly may be mentioned the polar rings of Leeches and Oligochaets (Whitman, Vejdovský, Foot).

The phenomenon is therefore of *fairly* widespread occurrence, though it must be remembered that there are cases in which the bilateral symmetry of the egg is fixed, apparently unalterably, in the unfertilized egg, for instance, in Cephalopods and many Insects, and secondly, that in parthenogenesis, the determination of symmetry, if not pre-existent, must be due to other causes.

But however the definite arrangement of cytoplasmic substances, on which the bilateral symmetry of the egg depends, may be brought about, modern experimental research has made it increasingly clear that it is upon these substances that at any rate the first differentiation of the embryo depends. Thus, to select only one or two examples, the Plane of Symmetry in the Frog becomes the Sagittal Plane of the embryo, in the absence of external interferences, the animal pole being approximately at the anterior end, the grey crescent on the dorsal side.

In *Dentalium* the polar lobe contains the material for the apical organ and trunk region of the larva; removal of the lobe entails the absence of both these structures.

Derangement of the micromeres brings about a multiplication of the sense organ of the Ctenophor embryo (Fischel); an isolated vegetative blastomere of a Nemertine will give rise to a larva with an archenteron but without an apical organ, an animal blastomere to one with an apical organ but no archenteron (Zeleny), and in Sea-urchins vegetative blastomeres gastrulate more readily than do animal cells (Driesch).

By the removal of one of the two posterior cells—containing the yellow stuff—of *Cynthia*, an embryo is produced which is complete anteriorly, with nerve plate, notochord and anterior mesenchyme, but possesses posterior mesenchyme and muscles upon one side only.

The substance which in these cases is necessarily associated with the formation of some particular organ, need not be, however, originally present in what will become its ultimate situation. Thus the stuff for the apical organ of *Dentalium* is at first situated in the polar lobe, subsequently migrating, between the first and second divisions, into the animal hemisphere. Though the organ-forming substances may be said to be in a sense preformed, they are not therefore necessarily prelocalized.

Speaking generally, a very definite relation may be noticed between the symmetry of the egg and that of the embryo, even where experiment has not, as yet, shown the necessary connection between this or that substance, and this or that organ. In a very large number of cases the animal pole is anterior, while the blastopore closes at the vegetative pole (Turbellarians, Annelids, Mollusca other than Cephalopods). In Cephalopods the animal pole is dorsal, the more convex side of the bilateral egg anterior, and a similar relation obtains in many Insect ova. In *Cynthia* the animal pole is ventral, the yellow side posterior. In *Ascaris megalocephala*, the plane in which the first four cells all lie is sagittal, while the vegetative germ-cell slips round to what will be the posterior side. The relation between

the cytoplasmic organization of the egg and the symmetry of the future embryo may be taken then to be a well established fact. It does not, however, at all follow that the symmetry of segmentation will necessarily coincide with these other two symmetries.

According to the terms of Roux's original "Mosaik-Theorie" the process of cell division in a segmenting ovum was qualitative, not, however, entirely because the parts of the cytoplasm were unlike, for the cytoplasm was imagined to be practically "isotropic" or equipotential in all its parts, but because the dissimilar units or determinants of the nucleus were supposed to be gradually sundered from one another by successive divisions. Later on, of course, these nuclear determinants were held to incite in the cytoplasm to which they were distributed the various processes of differentiation.

This hypothesis of qualitative nuclear division has, on a number of grounds which need not now be recalled, been shown to be untenable, and has been given up by its author himself. It may, however, well be urged that since the "anisotropy" of the cytoplasm has now been demonstrated, cell, though not nuclear division is still the qualitative process of the "Mosaik-Theorie," that it is still an essential factor in differentiation.

But even this view must be abandoned.

In the Frog the First Furrow bears no necessary relation either to the Plane of Symmetry or to the Sagittal Plane. When outside interference is removed it is true that the tendency of all three towards coincidence increases, but by means of these same external agents it is possible experimentally to separate those internal factors which determine cell division from those which decide which meridian of the egg shall be occupied by the embryo. Further, by means of pressure (O. Hertwig, Born) the character of segmentation may be altered, but the embryos are nevertheless completely normal.

In the Newt (Spemann) the First Furrow is sometimes in the Sagittal, sometimes in the Transverse Plane.

In Teleostei (Morgan) the First Furrow may make any angle with the median plane of the embryo.

In Sea-urchins it is not easy to say what the exact relation of embryonic and egg-axes may be, since the egg is colourless and almost isolecithal, and for that reason impossible to orient except either by the position of the germinal vesicle and polar bodies or by that of the segmentation axis (intersection of the first two divisions). As we have seen, these two axes do not coincide, and it is not certainly known which determines the embryonic axis, though it has been stated that the First Furrow lies in the Sagittal Plane.

In *Strongylocentrotus lividus*, however, the egg has a ring of pigment sub-equatorial in position (the axis as determined by the point of extrusion of the polar bodies coincides with the axis of segmentation). In ordinary development

this pigment passes first into the macromeres and then into the archenteron, and Boveri further states that if the order of segmentation be deranged by pressure or by stretching this pigment ring plays the same part as in normal development. Garbowski has, however, asserted that the ring need not occupy the position assigned to it by Boveri: it may be wholly in the animal hemisphere, or wholly in the vegetative hemisphere, or oblique to the axis. This being so we are compelled to suppose either that the processes of differentiation are indifferent to the previous order of segmentation, or that the pigment is not the organ-forming (archenteric) substance which Boveri imagined it to be.

There are other experiments on Echinoderms, however (due to Driesch), which prove that abnormalities in segmentation do not prejudice the complete normality of differentiation. For Driesch showed that eggs in which the character of segmentation had been altered by heat, pressure, shaking, dilution of the sea-water or the calcium-free sea-water introduced by Herbst, could still give rise to normal embryos. The segmentation of artificially parthenogenetic eggs may also be absolutely irregular (Loeb), but these eggs still produce normal larvae.

There is also the significant fact that the isolated blastomeres of these animals —at least as far as the four-celled stage—(or the vegetative blastomeres after the next division or the macromeres after the fourth division) will give rise to normal larvae, although they segment as though the missing blastomeres were still there, and precisely the same thing is seen in Nemertines (Wilson, Zeleny). The converse of this behaviour is exhibited by the Ctenophore egg, which segments normally when a portion of the vegetative hemisphere is removed, yet gives rise to an embryo devoid of costae and stomodaeum (Driesch and Morgan).

In the eggs of Turbellaria, Annelids and Mollusca (except Cephalopods) it is possible to trace the "lineage" of all the important organs of the larval body back to individual blastomeres or groups of blastomeres, and here, if anywhere, we should expect to find a "mosaic" segmentation. Even in this case, however, the factors which determine the order of division appear to be as distinct from those which determine differentiation as they are elsewhere. An isolated cell, as we know from the researches of Wilson and Crampton, will continue to segment as though its fellows were still present, but provided it contains the indispensable specific material will still produce a whole larva. Thus the CD cell of the two-cell stage or the D cell of the four-cell stage of *Dentalium* segments partially, but yet gives rise to a complete trochophore because it has received the all-essential polar lobe while its companion AB, or A, B and C cells do not. Conversely in *Ilyanassa* the cleavage of separated blastomeres may be made to resemble that of whole ova, by lowering the temperature, yet their developmental capacities are strictly limited. In *Nereis* again the pattern of segmentation may be readily modified by pressure; but the trochophore is still normal; and most recently of all Lillie has shown that in *Chaetopterus* certain endoplasmic granules—associated with gut formation—may be driven by the centrifuge to any point of the egg, at will, but to whatever point they are driven the cleavage is always related to the

axis determined by the polar bodies in the same way as in the unaltered egg. Lillie has further succeeded, to some extent, in obtaining differentiation—the formation of the prototroch—though all division was entirely suppressed. We may add that cells which occupy an identical position in similar patterns of cleavage, may have, in different animals, a very diverse fate.

In "typical" development, as Roux would call it, the symmetry of the egg, the symmetry of segmentation, the symmetry of the embryo may all be coincident. This is well seen in Annelids and Molluscs—where the D cell is posterior; in Ascidians—where the First Furrow is in the Plane of Symmetry of the egg and also in the Sagittal Plane of the embryo, the second transverse, the third horizontal; again in the Cephalopoda—where the first is in the Symmetry Plane of the egg and in the Sagittal Plane, the second transverse; in Ctenophora—where the first and second divisions are respectively sagittal and transverse; in the Sea-urchins, and in the undisturbed egg of the Frog. But that coincidence of segmentation with differentiation is not necessary for normal development is made abundantly clear by the experiments we have just considered, experiments which bring out the latent distinction between the two processes.

We can only, it seems to me, conclude that there are present in the egg two sets of factors—those which cause a certain pattern of segmentation and those on which differentiation depends. The former must be sought for in the quantity of yolk in the egg (Balfour's rule), in the relation between the mitotic spindle and the cytoplasm (the rules of Hertwig), in the relation between the dividing centrosomes and the previous spindle axis (Sach's law) and in the capillary properties of the surfaces of the blastomeres. The latter on the other hand will be found—have, indeed, already been found—in certain necessary specific organ-forming substances of the cytoplasm.

A limited part might indeed be assigned to mitosis in the determination of embryonic axes; the asters at the poles of the fertilization spindle might conceivably impress a bilateral symmetry upon the cytoplasm,—and it is as well to bear in mind that there is in the Frog a slight correlation between First Furrow and Sagittal Plane; but speaking generally the two processes are very largely independent of one another. To the developing embryo the order in which its specific materials are cut up in segmentation is a matter of indifference—what is essential apparently is that the material should be subdivided until a definite quantitative relation is established between the dimensions of the nucleus and the dimensions of the cell (Boveri).

Postscript.

It is a very great pleasure to me to find that Professor Brachet, though working by different methods, has arrived at conclusions almost identical with mine.

In his first paper (*Arch. de Biol.* XXI. 1905) he states that the angle between Plane of Symmetry and First Furrow may have any value. One hundred eggs

were examined by means of *sections*. Further, according to him, the Plane of Symmetry *always* includes the "penetration" path, and *always* becomes the Sagittal Plane (here I cannot help thinking he goes too far). The relation between the Planes was further investigated by puncturing the egg in the two-celled stage. When the First Furrow coincided with the Plane of Symmetry the result was a hemiembryo lateralis, when at 90° to the Symmetry Plane a hemiembryo anterior (the blastomere not containing the grey crescent having been injured) and when oblique an embryo with one posterior side wanting.

In a second communication (*Arch. Ent. Mech.* XXII. 1906) the effect is described of removing part of the vegetative hemisphere by puncture. When this operation is performed prior to the formation of the grey crescent subsequent development is normal. Up to this time therefore the egg is isotropic about its axis. When the grey crescent has appeared, however, the loss of this substance may entail abnormality or death.

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MISCELLANEA.

I. The Distribution of the Means of Samples which are not drawn at Random.

By STUDENT.

It is one of the advantages of the normal curve that if samples are drawn at random from any population, no matter how distributed, the distributions of the statistical constants of the samples rapidly approach the Gaussian as the samples grow large.

This being so, the result of grouping 2000 in samples of 25 given in Drs Greenwood and White's very interesting paper in *Biometrika* is surprising.

For it is easy to show that if B_1, B_2 be the constants of the distribution of the means of samples of n drawn at random, corresponding to β_1, β_2 in the original frequency distribution, then*

$$B_1=\frac{\beta_1}{n} \quad \text{and} \quad B_2-3=\frac{\beta_2-3}{n}.$$

But in this case $\beta_1=1{\cdot}7977$ and $B_1={\cdot}4756 : \dfrac{\beta_1}{n}={\cdot}0719,$

$$\beta_2-3=2{\cdot}5790, \quad B_2-3={\cdot}3185 : \frac{\beta_2-3}{n}={\cdot}1032.$$

Now neither of these can be considered significant with a sample of 80 means but at the same time they are both sufficiently different to suggest that the conditions which led to the theoretical result have not been fulfilled.

The first thing which occurred to me was that as Sheppard's corrections had been used for the means but not for the original distribution it might be well to try applying them to both.

This however makes but little difference, for we get

$$\beta_1=1{\cdot}9898 : \frac{\beta_1}{25}={\cdot}0792,$$

$$\beta_2-3=2{\cdot}7725 : \frac{\beta_2-3}{25}={\cdot}1109.$$

I next considered the possibility that the samples were not strictly random but that there was some slight correlation between successive observations.

I therefore assumed that the individuals composing the sample were more like each other than to the rest of the population, that in fact there was homotyposis, and working from this

* Henderson, R.: *Journal Inst. of Actuaries*, Vol. XLI. pp. 429—442.

hypothesis I found that the slightest correlation produces a very marked retardation in the approach to normality with increase in the size of the sample.

It will be observed that this is essentially a 'small sample' problem, for with increase in the size of the sample the correlation due to likeness between successive individuals diminishes except in exceptional cases when it becomes manifest as a well-marked heterogeneity.

My results emphasize the necessity of avoiding anything which tends to produce secular variation and as far as possible to neutralise it by repeating observations only after some time has elapsed.

Thus repetitions of analyses in a technical laboratory should never follow one another but an interval of at least a day should occur between them. Otherwise a spurious accuracy will be obtained which greatly reduces the value of the analyses.

In the present case there is not sufficient evidence to show whether correlation was really present, but as in the course of a fairly extended practice I have not yet met with observations in which this tendency was altogether absent, I incline to the belief that it was.

In any case, being ignorant of the technique, I can only suggest as possibilities slight variations from point to point on the slide, differences in light or in the observer as the day went on.

The general problem is as follows:

Let samples of n be drawn from a population with constants $\mu_2, \mu_3, \mu_4, \beta_1, \beta_2$, and let the samples be drawn in such a manner that the individuals composing each sample are correlated with correlation coefficient r, then, assuming linear regression and homoscedastic arrays, the constants of the distribution of their means (M_2, M_3, M_4, B_1, B_2) are as follows:

$$M_2=\frac{\mu_2}{n}\{1+(n-1)r\},$$

$$M_3=\frac{\mu_3}{n^2}\{1+(n-1)r\}\{1+(2n-1)r\},$$

$$M_4=\frac{\{1+(n-1)r\}}{n^3(1+2r)}[\mu_4\{1+(3n-1)r+3n(n-1)r^2\}+3(n-1)(1-r)(1+nr)\mu_2^2],$$

$$B_1=\frac{\beta_1}{n}\frac{\{1+(2n-1)r\}^2}{(1+r)^2\{1+(n-1)r\}},$$

$$B_2=\frac{\beta_2}{n}\frac{\{1+(3n-1)r+3n(n-1)r^2\}}{(1+2r)\{1+(n-1)r\}}+\frac{3(n-1)(1-r)(1+nr)}{n(1+2r)\{1+(n-1)r\}}.$$

As the method of determining the three moment coefficients is the same in each case and it is merely a question of reduction to obtain B_1 and B_2, it will be sufficient for me to give the proof for M_4.

Let $x_1 x_2 \ldots x_n$ be the values, measured from the mean of the population, of the individuals composing the typical sample, and let there be N such samples.

Then
$$M_4=\frac{1}{N}\Sigma\left\{\frac{x_1+x_2+\ldots+x_n}{n}\right\}^4$$
$$=\frac{1}{N}\Sigma\frac{S(x_1^4)+4S(x_1^3x_2)+6S(x_1^2x_2^2)+12S(x_1^2x_2x_3)+24S(x_1x_2x_3x_4)}{n^4} \quad \ldots\ldots\ldots\text{(i)}.$$

Taking each of these six terms in turn we have

$$\frac{\Sigma\{S(x_1^4)\}}{Nn^4}=\frac{n\Sigma(n_{x_1}x_1^4)}{N.n^4}=\frac{\mu_4}{n^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}.$$

For $S(x_1^4)$ has n terms and when they are taken over all the N samples which compose the population there will be $n.n_{x_1}$ of x_1^4, n_{x_1} being the number of x_1's in the population and $n_{x_1x_2}$ the number of x_1's associated with x_2's and so on.

Again there are $n(n-1)$ terms in $S(x_1^3x_2)$,

$$\therefore \frac{\Sigma\{4S(x_1^3x_2)\}}{N.n^4} = \frac{4(n-1)\Sigma(n_{x_1x_2}x_1^3x_2)}{N.n^3}$$

$$= \frac{4(n-1)\Sigma(n_{x_1}.x_1^3.\text{mean value of } x_2)}{N.n^3}.$$

But the mean value of x_2 associated in the sample with x_1 will be $\frac{r\sigma_{x_2}}{\sigma_{x_1}}x_1$ or since $\sigma_{x_1}=\sigma_{x_2}$ it is rx_1,

$$\therefore \frac{\Sigma\{4S(x_1^3x_2)\}}{N.n^4} = \frac{4(n-1)\Sigma(n_{x_1}.x_1^4.r)}{N.n^3}$$

$$= \frac{4(n-1)r}{n^3}\mu_4 \quad \text{.....(iii).}$$

Next $$\frac{\Sigma\{6S(x_1^2x_2^2)\}}{N.n^4} = \frac{3(n-1)}{n^3}.\frac{\Sigma(n_{x_1x_2}.x_1^2x_2^2)}{N}$$

$$= \frac{3(n-1)}{n^3}\frac{\Sigma(n_{x_1}.x_1^2.\text{mean value of } x_2^2)}{N}$$

[Now the mean value of x_2^2 is equal to the square of the S.D. of the x_1 array of x_2's, $\{\mu_2(1-r^2)\}$, added to the square of the mean value of x_2, $(r^2x_1^2)$]

$$= \frac{3(n-1)}{n^3}\frac{\Sigma n_{x_1}\{r^2x_1^4+x_1^2\mu_2(1-r^2)\}}{N}$$

$$= \frac{3(n-1)}{n^3}\{r^2\mu_4+(1-r^2)\mu_2^2\} \quad \text{.....(iv).}$$

Again: $$\frac{\Sigma\{12S(x_1^2x_2x_3)\}}{Nn^4} = \frac{6(n-1)(n-2)}{n^3}\frac{\Sigma(n_{x_1x_2x_3}.x_1^2x_2x_3)}{N}$$

$$= \frac{6(n-1)(n-2)}{n^3}\frac{\Sigma(n_{x_1x_2}.x_1^2x_2.\text{mean value of } x_3)}{N}.$$

The mean value of x_3 for values x_1 and x_2 of the other two variables is given by the equation

$$m_{x_3} = -\frac{\sigma_{x_3}}{R_{33}}\left\{\frac{R_{31}x_1}{\sigma_{x_1}}+\frac{R_{32}x_2}{\sigma_{x_2}}\right\}$$

where the R's are the minors of the determinant

$$\begin{vmatrix} 1, & r, & r \\ r, & 1, & r \\ r, & r, & 1 \end{vmatrix} \quad \text{or} \quad m_{x_3}=(x_1+x_2).\frac{(r-r^2)}{1-r^2}=(x_1+x_2).\frac{r}{1+r}.$$

Substituting we get

$$\frac{\Sigma\{12S(x_1^2x_2x_3)\}}{N.n^4} = \frac{6(n-1)(n-2)}{n^3}.\frac{r}{1+r}.\frac{\Sigma(n_{x_1x_2}(x_1^3x_2+x_1^2x_2^2))}{N}.$$

By (iii) and (iv) $$= \frac{6(n-1)(n-2)}{n^3}.\frac{r}{1+r}\{r\mu_4+r^2\mu_4+(1-r^2)\mu_2^2\}$$

$$= \frac{6(n-1)(n-2)}{n^3}.r\{r\mu_4+(1-r)\mu_2^2\} \quad \text{.....(v).}$$

Lastly: $$\frac{\Sigma\{24S(x_1x_2x_3x_4)\}}{Nn^4} = \frac{(n-1)(n-2)(n-3)}{n^3}\frac{\Sigma(n_{x_1x_2x_3x_4}x_1x_2x_3x_4)}{N}$$

$$= \frac{(n-1)(n-2)(n-3)}{n^3}\frac{\Sigma(n_{x_1x_2x_3}.x_1x_2x_3.\text{mean value of } x_4)}{N}.$$

As before the mean value of x_4 comes from the multiple regression equation

$$m_{x_4} = -\frac{\sigma_{x_4}}{R_{44}}\left\{x_1\frac{R_{41}}{\sigma_{x_1}} + x_2\frac{R_{42}}{\sigma_{x_2}} + x_3\frac{R_{43}}{\sigma_{x_3}}\right\},$$

where the R's are minors of $\begin{vmatrix} 1, & r, & r, & r \\ r, & 1, & r, & r \\ r, & r, & 1, & r \\ r, & r, & r, & 1 \end{vmatrix}$.

$$\therefore\ m_{x_4} = (x_1+x_2+x_3)\frac{(r(1-r)^2)}{1-3r^2+2r^3} = (x_1+x_2+x_3)\cdot\frac{r}{1+2r}.$$

Substituting we get

$$\frac{\Sigma\{24S(x_1x_2x_3x_4)\}}{Nn^4} = \frac{(n-1)(n-2)(n-3)}{n^3}\cdot\frac{r}{1+2r}\cdot\frac{\Sigma\{n_{x_1x_2x_3}\cdot x_1x_2x_3(x_1+x_2+x_3)\}}{N}$$

$$= \frac{(n-1)(n-2)(n-3)}{n^3}\cdot\frac{r}{1+2r}\cdot\frac{3\Sigma(n_{x_1x_2x_3}\cdot x_1^2x_2x_3)}{N}.$$

Applying (v) $$= \frac{3(n-1)(n-2)(n-3)}{n^3}\cdot\frac{r^2}{1+2r}\{r\mu_4+(1-r)\mu_2^2\} \quad \text{.....(vi).}$$

Substituting (ii) ... (vi) in (i) we get

$$M_4 = \frac{1}{n^3}\Big\{\mu_4 + 4(n-1)r\mu_4 + 3(n-1)\{r^2\mu_4+(1-r^2)\mu_2^2\}$$

$$+6(n-1)(n-2).r.\{r\mu_4+(1-r)\mu_2^2\} + 3(n-1)(n-2)(n-3)\cdot\frac{r^2}{1+2r}\{r\mu_4+(1-r)\mu_2^2\}\Big\},$$

which reduces to the result given above, viz.

$$M_4 = \frac{\{1+(n-1)r\}}{n^3(1+2r)}[\{1+(3n-1)r+3n(n-1)r^2\}\mu_4 + 3(n-1)(1-r)(1+nr)\mu_2^2].$$

Using these equations it is possible to find values of r which would satisfy the conditions for the various constants.

Thus (using Sheppard's corrections for both sets of constants) I find that with the given values

of μ_2 and M_2 $r = \cdot003$,

of β_1 and B_1 $r = \cdot063$,

of β_2 and B_2 $r = \cdot033$.

Now clearly if r were fitted by least squares or in any other way from these three values it must clearly come closest to the μ_2 value owing to the lower prob. error of μ_2. As to fit it properly is clearly very complicated owing to the intercorrelations of the constants I have assumed a value $r = \cdot01$ as a nice round number which gives a value of M_2 higher than that found in the sample before us but not at all impossibly so.

This gives

$M_2 =$	·1101,	actual ·1074,
$B_1 =$	·1397,	„ ·4756,
$B_2 =$	3·2012,	„ 3·3185.

These constants give a type I. curve

$$y = 97{\cdot}57\left(1+\frac{x}{1{\cdot}65}\right)^{24{\cdot}64}\left(1-\frac{x}{47{\cdot}82}\right)^{714{\cdot}2}.$$

If we assume no correlation I get a curve

$$y = 109{\cdot}0\left(1+\frac{x}{1{\cdot}92}\right)^{41{\cdot}7}\left(1-\frac{x}{58{\cdot}31}\right)^{1266{\cdot}8},$$

whence I get the following 'fits'*.

* The figures given are really mid-ordinates, but for such small numbers the difference between the mid-ordinate and the area on the base unit is negligible.

		Below 1·10	1·10 1·22	1·22 1·34	1·34 1·46	1·46 1·58	1·58 1·70	1·70 1·82	1·82 1·94	1·94 2·06	2·06 2·18	2·18 2·30	2·30 2·42	2·42 2·54	2·54 2·66	2·66 2·78	Above 2·78
Actual ...		—	4	8	7	14	12	12	5	7	5	2	1	2	0	1	—
Calculated	No Correlation	1·01	2·42	5·28	8·86	11·69	13·07	12·18	9·97	6·90	4·27	2·36	1·18	—	—	·92	—
	Correlation ·01	1·85	3·27	6·02	8·92	11·01	11·71	10·84	8·95	6·64	4·52	2·82	1·64	·90	—	·85	—

These give $P = ·46$ and $P = ·86$ respectively, the first being a good deal helped by the convention that the tail should not be carried beyond the point at which a single unit may be expected, and the second much less so.

As the empirical curve fitted from the actual moments has a P of ·92, the second curve may be considered fairly good depending as it does on a guess following on calculation. On the other hand a P of ·46 with so few cases as 80 is not particularly good, and as Prof. Pearson has pointed out to me the graph distinctly gives an idea of greater skewness than is represented by the no correlation curve. I do not however wish to contend that the circumstances attending the production of the sample actually conformed to the arbitrary conditions which I found it necessary to assume in order to simplify the analysis. But seeing that the fit is good and that with such a small sample even the divergent B_1 is not altogether impossible, I think it likely that there was some sort of correlation, though probably not that particular kind which has been assumed in this note.

Conclusions:

(1) That the approach to normality of the distribution of means of samples drawn from a non-Gaussian population is delayed by the existence of correlation between the individuals composing the samples.

(2) That on certain arbitrary assumptions the constants of the new distribution can be found given the constants of the old one and r according to formulae given above.

(3) That using the above formulae and choosing a likely looking value of r, a curve can be drawn to represent the sample in Drs Greenwood and White's paper with fair likelihood.

II. A Short Method of Calculating the Coefficient of Correlation in the Case of Integral Variates.

By J. ARTHUR HARRIS, Ph.D., Cold Spring Harbor, Long Island, U.S.A.

For symmetrical correlation tables in which both variates have the same mean and standard deviation, Professor Pearson has suggested* that his difference method "may possibly be of good service," but warns the reader:—

"At the same time too much reliance must not be placed upon the difference method, not only because it assumes normality of distribution but because it involves a somewhat rough method of approximation in the case of the diagonal cell."

* *Drapers' Company Research Memoirs, Biometric Series*, IV. pp. 4—9, 1907.

In applying this difference method to series of material already worked out by the product moment method I have found considerable variation in the weight to be given to the diagonal cell, but many more series should be examined before suggesting any other value than the one-sixth proposed by Professor Pearson. While engaged in this work some modifications of method which, I think, will lighten considerably the calculation of r in some cases came to my attention.

It is well known that the formula

$$r_{xy}=(\sigma_x^2+\sigma_y^2-\sigma_v^2)/(2\sigma_x\sigma_y) \quad \text{(i)},$$

where σ_x and σ_y are the standard deviations of the two characters and σ_v is the standard deviation of their difference, gives r with the same accuracy as the product moment method irrespective of the nature of the distributions.

In symmetrical tables, in which each individual is used once as a first and once as a second member of a pair, $\sigma_x=\sigma_y$ and the above formula may be written

$$r_{xx}=\frac{2\sigma_x^2-\sigma_v^2}{2\sigma_x^2}=1-\frac{1}{2}\frac{\sigma_v^2}{\sigma_x^2} \quad \text{(ii)}.$$

From this formula the correlation coefficient may be calculated with great ease and rapidity as follows:

First, determine the positive differences between the first and second member of the pairs from the table, as suggested by Professor Pearson* and as illustrated in the example given below.

After the totals of the columns have been found they are multiplied by the squares of the differences as given at the heads of the several columns. Twice the sum of these products divided by N gives σ_v^2, for the origin is at 0, the plus and minus deviations are equally great and the first moment necessarily 0.

The standard deviation of the character will usually have been obtained for other purposes but if it is not wanted no roots need be extracted to obtain r by this method. The multiplication of the totals of the several columns by the squares of the differences which they represent requires only a little more work than their multiplication by the numbers themselves, and if a Brunsviga or Comptometer be used all of the arithmetical work can be done in a few minutes, even when the tables are rather large. Thus the whole work can be completed almost if not quite as quickly as that for Pearson's formula (vii)† and with the same accuracy as the product moment method.

Since all may not have access to Pearson's memoir referred to, I give the following illustration of the arithmetic of the method.

Illustration I. Calculation of correlation from symmetrical table. Dr Fernando De Helguero's symmetrical table for number of flowers per inflorescence in *Cicorium Intybus*‡ serves as an example. Beginning at the head of each column in the correlation table we copy down the frequencies to and including the diagonal cell—where the difference between the two variates is 0—under appropriate headings in a series of difference columns. The work can be done systematically and rapidly by copying the first number of the first column in the 0 column. In beginning the copying of each succeeding column from the correlation table a new column of differences, one higher than the last, will be begun and be represented by a single entry for that column while the remaining entries will be placed successively one place to the left. The whole process will be clear at once from a comparison of the work of the illustration with the table of data. The numbers in clarendon type show the order in which the entries from the 12th column were written down.

* *loc. cit.* p. 7.
† *loc. cit.* p. 6.
‡ *Biometrika*, Vol. v. p. 188.

The same value is obtained by the product moment method, the difference between this and the $r = \cdot 613$ obtained by De Helguero being due to the slip in his calculation of the standard deviation. It appears from the above that the calculation of r is a very simple process.

TABLE I.

Inflorescences of Cicorium.

	8	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
8	—	1	1	1	2	—	—	—	—	—	5
9	1	2	2	15	10	1	2	—	—	—	33
10	1	2	8	19	11	4	1	—	—	—	46
11	1	15	19	134	90	18	1	—	—	—	278
12	2	10	11	90	114	97	9	4	—	—	337
13	—	1	4	18	97	122	53	2	—	—	297
14	—	2	1	1	9	53	96	10	—	—	172
15	—	—	—	—	4	2	10	10	3	—	29
16	—	—	—	—	—	—	—	3	2	1	6
17	—	—	—	—	—	—	—	—	1	—	1
Totals	5	33	46	278	337	297	172	29	6	1	1204

v = *Difference of* x *and* y.

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	
0	1	1	1	**2**	0	0	0	0	0	
2	2	15	**10**	1	2	0	0	0	—	
8	19	**11**	4	1	0	0	0	—	0	
134	**90**	18	1	0	0	0	—	0		
114	97	9	4	0	0	—	0			$S(v^2)$
122	53	2	0	0	—	0				276
96	10	0	0	—	2					224
10	3	0	—	4						180
2	1	—	20							64
0	——	56								50
——	276									——
488										794

$N = 1204$,

$\sigma_x = 1 \cdot 3352$*, $\sigma_x^2 = 1 \cdot 7828$,

$$\sigma_v^2 = \frac{2 \times 794}{N} = 1 \cdot 318937,$$

$$r = \left(1 - \frac{1}{2}\frac{\sigma_v^2}{\sigma_x^2}\right) = \left(1 - \frac{1}{2}\frac{1 \cdot 318937}{1 \cdot 7828}\right) = \cdot 630.$$

In working with a symmetrical table we copy down only the diagonal cell and the positive differences. But it would be very easy to continue right down the columns: after passing the diagonal cell the differences would take the negative sign and we could calculate the standard deviation from all the differences instead of from a part of them. In the symmetrical table this would be merely a waste of time, but where the tables are not symmetrical (and the two variates have different means and standard deviations) this process can be used to advantage.

* The value 1·3536 given by De Helguero was apparently obtained by taking the root of the second rough moment, i.e. without first subtracting the square of the first rough moment.

In fact I believe that in the case of integral variates the calculation with formula (i) will be considerably more rapid than the conventional product moment method.

A further saving of time can be effected by doing away with the necessity for copying the entries from the correlation table into difference columns. This can be made clear by an illustration.

Illustration II. Calculation of correlation from non-symmetrical tables. Pearl's table for his Series V of Ceratophyllum*, showing correlation between the position on the branch and the number of leaves per whorl, is given below with the differences between the two variates indicated above each entry as small indices. These can be entered on a table—preferably in red

TABLE II.

Whorls of Leaves in Ceratophyllum.

Position of Whorl on Branch.

Leaves per Whorl.	1	2	3	4	5	6	7	8	9	10	Totals
4	2^{3}	2^{2}	$-^{1}$	$-^{0}$	$-^{-1}$	$-^{-2}$	—	—	—	—	4
5	17^{4}	5^{3}	1^{2}	2^{1}	$-^{0}$	1^{-1}	1^{-2}	—	—	—	27
6	16^{5}	6^{4}	10^{3}	1^{2}	1^{1}	1^{0}	$-^{-1}$	1^{-2}	—	—	36
7	40^{6}	29^{5}	18^{4}	14^{3}	9^{2}	1^{1}	2^{0}	4^{-1}	1^{-2}	—	118
8	19^{7}	28^{6}	25^{5}	17^{4}	8^{3}	7^{2}	3^{1}	$-^{0}$	3^{-1}	1^{-2}	111
9	5^{8}	20^{7}	19^{6}	26^{5}	22^{4}	17^{3}	13^{2}	6^{1}	4^{0}	5^{-1}	137
10	7^{9}	5^{8}	6^{7}	4^{6}	6^{5}	10^{4}	8^{3}	14^{2}	4^{1}	3^{0}	67
11	1^{10}	$-^{9}$	$-^{8}$	$-^{7}$	$-^{6}$	$-^{5}$	$-^{4}$	$-^{3}$	$-^{2}$	$-^{1}$	1
Totals	107	95	79	64	46	37	27	25	12	9	501

ink or pencil to avoid confusion—much more quickly than the products of the deviations of the two variables from their assumed origins, and with much less chance of error, especially in the case of large tables. The work can be most rapidly done by taking first the diagonal cell and then the diagonal rows lying on either side of it. All the entries of any diagonal row will have the same index, but it should be put down in every case to avoid confusion. All that now remains to be done is to sum these diagonal columns and calculate the second moment. For this r can be at once calculated from formula (i) above.

From our illustrative table we have for the sums of the diagonal rows :—

v	-2	-1	0	1	2	3	4	5	6	7	8	9	10	Total
Frequency ...	4	13	10	17	47	64	90	102	91	45	10	7	1	501

Note that a check for the accuracy of the work is furnished by the identity of the total with the N of the population. In the product moment method there is no convenient check for the accuracy of the multiplications leading to the $S(xy)$. In the present case we find

S. D. Position $= 2{\cdot}3733$,

S. D. Leaves $= 1{\cdot}4003$,

S. D.^{2} Difference $= 4{\cdot}4380$,

$r = {\cdot}475$,

agreeing with the value given by Pearl.

* Pub. 58, *Carnegie Institution of Washington*, p. 60, Table 31, 1907.

Obviously this method of obtaining the totals of the difference columns can be used for the symmetrical tables as well, where only the diagonal cell and half of the remaining diagonal rows need be considered, thus doing away with the work required by the method of copying the frequencies from the correlation table into difference columns as suggested by Pearson.

Unfortunately these difference methods are limited in their applicability. When integral variates are under consideration, they give the same result as the product moment method when Sheppard's Correction is not applied in the calculation of the standard deviations. But difficulties arise when other than integral variates are considered. The divergence of the coefficients of correlation calculated by different methods is shown by Wright, Lee and Pearson*. But where the limits of their applicability are borne in mind, I think, these difference methods deserve more consideration in practical work than they have hitherto received.

III. Note on Variation in Adoxa.

By J. ARTHUR HARRIS, Ph.D., Cold Spring Harbor, L.I., U.S.A.

In examining Whitehead's paper† on the Moschatel, certain points came to my attention which seem to merit more detailed consideration than he has given them. What is really needed is a more extensive quantitative investigation than is possible on the data hitherto published. Possibly the indication of certain points of interest not discussed by Whitehead may induce some one who has the opportunity of collecting fresh material to treat the problems in the detailed way which they deserve.

Two questions occur to one examining Whitehead's second table:

(*a*) Is there any relationship between the number of flowers which an inflorescence produces and the characteristics of these flowers?

(*b*) What is the similarity of the flowers of an inflorescence?

Both of these, I take it, are of considerable interest to morphologists.

Problem (*a*). The correlation between the number of flowers per inflorescence and the number of the divisions of the corolla.

In our almost complete ignorance of factors influencing the number of parts of the flower, it seems important to investigate every possible interrelationship between the characteristics of the flower and the characteristics of the individual which produces it. Such investigations may not be expected to yield more than a small part of the information which we desire, but before experimental methods are extensively applied, it seems logical to determine whether there is any dependence of the number of floral parts developed, or any dependence of any characteristics of these floral parts, upon the vegetative organs of the individuals which produce them.

In the present case only one characteristic other than those belonging to the flower itself is available for comparison, i.e. number of flowers per inflorescence. This character seems, however, a particularly interesting one for study. In the Moschatel there is one form of inflorescence which is conspicuously the modal one. This type is a very neat, compact, morphological structure and any departure from it appears at once to be "abnormal." The flowers, too, have a normal form, from which a considerable number of deviations are to be found. Are abnormalities in the number of lobes of the corolla associated with abnormalities in the number of flowers borne on the inflorescence? The data for determining the coefficients of correlation are extracted from Whitehead's Table II and presented in the correlation tables

* *Biometrika*, Vol. v. p. 410.

† *Biometrika*, Vol. II. pp. 108—113, 1902.

I—III. Here the inflorescences are necessarily weighted with the number of flowers which they produce. In all these cases the terminal flower seems to have been present. The number of lateral flowers, then, represents the real variable characteristic of the inflorescence, and has been taken as the first variable in preparing the correlation tables. The constants calculated are given in Table A. The weighting of the inflorescences must be borne in mind by anyone desiring to use these standard deviations or coefficients of variation for other purposes.

The means and variabilities of the three collections show some differences which may be significant, but until further collections from a wider range of habitats are available, little can be profitably said concerning them. Like other kinds of biological work, Biometry cannot make great progress until enough series of data have been reduced to permit of extensive comparisons.

The correlation coefficients are only about five to seven times their probable errors and so cannot be given too much significance*. Furthermore, two of the coefficients are negative while the other is positive. In such cases as these too great significance must not be attached to the probable errors. The actual number of inflorescences is really very small in two of the series, and the tables appear to contain adequate data merely because the inflorescences are necessarily weighted with the number of flowers which they bear. In cases in which deviations from the typical condition are very few the collection of a slightly larger series may materially change the constants for a race†. The Kent series is the only adequately large one of the three, and the correlations derived from it may be considered significantly negative. The other two collections would be sufficiently large to give fairly trustworthy results in characters with more normal variability. But where only 9 of the 163 inflorescences actually involved depart from the normal type—as is the case in the Essex series—it is dangerous to lay much stress upon the results from small samples.

Considering all these difficulties I think we can draw no final conclusion from these data. But it appears from the results of this first examination of the relationship between the number of flowers per inflorescence and the number of divisions of the floral envelopes that the problem is worthy of detailed study by someone living in a region where considerable quantities of *Adoxa* may be obtained‡.

Problem (*b*). The correlation between the number of corolla lobes of flowers of the same inflorescence.

Teratologists have long known that when one organ of an individual is abnormal there is some probability that a second homologous organ of the same individual will be abnormal also. How great is this probability?

For the small deviations from the type of the race which have generally been called variations —or fluctuations by many modern writers—Pearson and others have determined quantitatively the degree of resemblance between the undifferentiated like organs of the individual for a considerable series of species. The interindividual correlations for a form like *Adoxa* will be interesting to compare with the coefficients obtained from other homotypic relationships. *Adoxa* produces ordinarily only a single inflorescence, and the interest of the comparison lies in the fact that all the homotypes are so closely associated. *À priori*, I would have expected such organs to be more highly correlated than those more widely separated on the individual. Even

* All the probable errors are calculated from the formulae commonly used, but it must be noted that for the number of flowers per inflorescence the percentage of deviations from the normal approaches the limits beyond which these formulae cannot be used with perfect confidence.

† For another illustration see *Report Missouri Botanical Garden*, Vol. 20, 1909.

‡ In this note I have not considered the number of lobes of the terminal flower because the data are too few. This should be taken up when larger series of material are available. I note in passing that there is a slip in Whitehead's Table III, where the frequency of the 5-merous flowers should be given as 2479 instead of 2494.

if we suppose that proximity of organs does not indicate that they have any function to perform in common which would tend to render their correlation organic rather than homotypic*, it would seem quite reasonable to think that organs separated in their embryological development by only relatively few cell divisions would be more similar than those separated by an immensely large number. Concretely, the flowers of an inflorescence of *Adoxa* have been built up from a common vegetative point by only a few cell divisions as compared with the inconceivably large number which have intervened between the laying down of the leaves upon the outermost branches of a beech tree. In the laying down of the leaves of a forest tree there is also opportunity for considerable differences in the environments of the individual vegetative points. The actual correlation constants, calculated from the symmetrical homotypic tables IV—VI, are given in Table B.

The conspicuous thing about these constants is that they are not higher, but rather lower, than those usually found for vegetable homotypes. Comparing the results summarized in Pearson's Table XXXII†, we note, however, that Ash, Wild Ivy and one series of Holly give values of about the same general order of magnitude as these. In a study of the correlation between the two leaves of the flowering stalk of *Podophyllum* I have shown‡ that the relationship in two series of material is about ·45, although here too I should have expected higher values. Aside from any bearing which the problem of homotyposis may have on heredity, I think it may throw some light upon a number of morphogenetic problems. One of these is the question of a possible relationship between the homotypic constants for organs which are widely separated ontogenetically and those which are closely associated ontogenetically§.

One further point may be noted. In any critical study of interindividual correlation in the Moschatel the possibility of the individuals being related vegetatively must be taken into account. Pearson found that in *Malva rotundifolia* the homotypic coefficient was considerably lower than that found in most forms, and concluded that this was due to the plants having for the most part spread from one clump by stolons.

TABLE A.

Variation and Organic Correlation Constants for Adoxa.

Material	Means	Standard Deviation	Coefficient of Variation	Coefficient of Correlation
Kent, Flowers ...	4·042 ± ·007	·489 ± ·005	12·09	− ·135 ± ·014
„ Divisions ...	4·819 ± ·006	·440 ± ·005	9·14	
Surrey, Flowers ...	4·190 ± ·024	·604 ± ·017	14·423	− ·214 ± ·038
„ Divisions...	4·803 ± ·020	·500 ± ·014	10·41	
Essex, Flowers ...	3·994 ± ·007	·278 ± ·005	6·95	+ ·163 ± ·026
„ Divisions ...	4·862 ± ·009	·357 ± ·007	7·35	

Flowers = Number of lateral flowers per inflorescence; Divisions = Number of divisions of corolla. Sheppard's Correction not used.

* See in this connection Pearson's very clear statements of the differences between organic and homotypic correlation in *Biometrika*, Vol. I. p. 341, 1902.

† *Phil. Trans.* 1901, Vol. 197, A, p. 356.

‡ *Bot. Gaz.* Vol. 47, pp. 438—444, 1909.

§ Possibly the statistical difficulties introduced by the "small samples" necessarily used in studies of interindividual correlation in such forms as *Podophyllum* and *Adoxa* may obscure the real biological relationships. See Pearson, *Biometrika*, Vol. VI. p. 403, 1909.

TABLE B.

Interinflorescence Correlations for Number of Petals in Adoxa.

Series of Material	Correlation and Probable Error
Near Chislehurst, Kent ...	·346 ± ·013
Caterham, Surrey ...	·265 ± ·037
Theydon Garnon, Essex ...	·301 ± ·024

TABLE I.

Kent.

Petals.

Lateral Flowers.	3	4	5	6	7	8	Totals
2	—	—	9	—	—	1	10
3	1	15	70	6	1	—	93
4	12	319	1549	12	—	—	1892
5	—	14	36	—	—	—	50
6	8	16	42	—	—	—	66
7	—	1	6	—	—	—	7
Totals	21	365	1712	18	1	1	2118

TABLE II.

Surrey.

Petals.

Lateral Flowers.	3	4	5	6	Totals
3	—	1	11	—	12
4	—	41	173	10	224
5	2	15	12	1	30
6	—	6	12	—	18
Totals	2	63	208	11	284

TABLE III.

Essex.

Petals.

Lateral Flowers.	3	4	5	6	Totals
3	—	13	8	—	21
4	2	72	541	1	616
5	—	—	5	—	5
6	—	1	5	—	6
Totals	2	86	559	1	648

TABLE IV. *Kent.*

First Flower.

Second Flower.	3	4	5	6	7	8	Totals
3	12	44	22	—	—	—	78
4	44	474	599	12	—	—	1129
5	22	599	4532	30	2	1	5186
6	—	12	30	6	—	—	48
7	—	—	2	—	—	—	2
8	—	—	1	—	—	—	1
Totals	78	1129	5186	48	2	1	6444

TABLE V. *Surrey.*

First Flower.

Second Flower.	3	4	5	6	Totals
3	—	4	4	—	8
4	4	94	111	6	215
5	4	111	514	20	649
6	—	6	20	8	34
Totals	8	215	649	34	906

TABLE VI. *Essex.*

First Flower.

Second Flower.	3	4	5	6	Totals
3	2	2	2	—	6
4	2	92	152	1	247
5	2	152	1528	2	1684
6	—	1	2	—	3
Totals	6	247	1684	3	1940

IV. On the Association of Drawing with other Capacities in School Children.

By ETHEL M. ELDERTON, Galton Eugenics Laboratory.

An interesting paper* has recently been published by E. Ivanoff discussing the association of power of good drawing in boys and girls with other capacities. The relationships, however, are not expressed in terms of any modern statistical measure of correlation†, and it is accordingly not easy to determine whether drawing capacity is highly or slightly correlated with other

* Recherches expérimentales sur le Dessin des Écoliers de la Suisse Remande. *Archives de Psychologie*, III. VIII. 1908.

† From the standpoint of theory very grave objections indeed might be raised to M. Ivanoff's treatment of correlation, especially to his use of four correlations from a single table which are not independent.

characters, or what are the relative degrees of association between drawing and these characters. It seemed therefore worth while considering the data from another standpoint, and expressing it in a form more readily intelligible to the statistician. Each characteristic is divided into three categories, a 'mean' group, and a group above and a group below the mean. The classification of a group which contains 40 to 80% of the total frequency as 'moyens' is not to be commended. It corresponds, however, to a division into markedly good, markedly poor, and a middle group of 'indeterminates.' Thus for example Ivanoff uses for categories of attention 'Attentif,' 'Distrait' and 'Indéterminé,' and for temper 'Doux,' 'Violent' and 'Indéterminé,' while for most special studies including drawing he uses a 'moyens' group. He does not give, but from his data it is possible to deduce, contingency tables with 9-fold groupings. The coefficients of mean square contingency deduced from such tables will be comparable among themselves, but possibly 30 to 50 per cent. below the true value of the correlation coefficient; eight tables were worked out by the fourfold table method and showed on an average 40 per cent. increase on the contingency values. It is only needful to bear this in mind when we are considering the absolute importance of the contingencies investigated. I divide Ivanoff's characters into the following classes:

(A) Measures of General Ability, represented by 'Intelligence' and 'General Work.'

(B) Studies which we may *a priori* suppose to be associated with drawing, namely 'Writing,' 'Geography,' on the assumption that it covers map-making, and 'Manual Work.'

(C) Studies less obviously associated with Drawing, namely 'Composition,' 'Language,'* 'History' and 'Arithmetic.'

(D) Psycho-pedagogic Characters, i.e. such characters as are necessarily foremost in the estimate of the teachers. These are 'Attention,' 'Obedience,' 'Industry,' 'Cleanliness' and 'Temper.'

We reach the following values of the association of these characters with Drawing:

		Boys		Girls	
A.	Intelligence ...	·15	·17	·23	·23
	General work ...	·19		·23	
B.	Writing ...	·20	·16	·14	·20
	Geography ...	·14		·26	
	Manual work ...	·14		·21	
C.	Composition ...	·13	·11	·19	·18
	History ...	·12		·16	
	Language ...	·08		·14	
	Arithmetic ...	·09		·21	
D.	Attention ...	·16	·11	·11	·11
	Obedience ...	·13		·08	
	Energy ...	·08		·13	
	Industry ...	·10		·11	
	Cleanliness ...	·16		·10	
	Temper ...	·06		·12	
	Mean of 15 contingencies	·13		·16	

We see from this table several results flow at once:

(i) Ability in drawing is more closely associated with other characters in girls than in boys. This holds for every sub-group also, except the pedagogic characters, which in the girl are possibly slightly less associated with drawing capacity than in the case of the boy.

(ii) If we can trust the data, drawing ability has extraordinarily little relationship to any other of the characters observed. If we increase the contingency values by 40 per cent. to reach

* This is equivalent to 'reading' and 'grammar.'

the true correlation, the highest relationship will be ·36 for geography and drawing in girls; the highest in boys will be ·28 for writing and drawing; and the average value for the boys will be only ·18 and for girls ·22. These are distinctly small relationships, compared with what have been found for the interrelationship of other studies in previous investigations. This may arise possibly from capacity in drawing being a hereditary character having small association with other school measures of fitness.

(iii) Measures of general ability head the list for both boys and girls. Drawing is more closely related to general intelligence than even the subjects like writing, geography and manual work, more closely associated with the effective use of the hand.

(iv) In the case of both boys and girls the psycho-pedagogic characters have less relation to proficiency in drawing than the hand studies. For the girls all the correlations are the lowest on the list; efficiency in studies not obviously related to drawing like reading, grammar and arithmetic is far more important. The same result is screened in the boys by the comparatively high values of the attention and cleanliness correlations. Indeed nothing appears to be gained in drawing efficiency by a large capacity for industry or by energy.

(v) There do appear some slight sexual differences. Attention and cleanliness in boys take the place of the geography and manual work of the girls, whose arithmetic and composition are more vital than their handwriting.

The following table gives the exact order of intensity of the correlation of the characters considered with drawing for the two sexes. While Ivanoff's paper thus brings out a number of points of considerable interest and is suggestive of further work, we venture to put forward one or two points for consideration.

Table of contingency coefficients in order of magnitude:

Order	Boys	Girls
1	Writing (·20)	Geography (·26)
2	General work (·19)	Intelligence (·23)
3	Cleanliness (·16)	General work (·23)
4	Attention (·16)	Manual work (·21)
5	Intelligence (·15)	Arithmetic (·21)
6	Geography (·14)	Composition (·19)
7	Manual work (·14)	History (·16)
8	Composition (·13)	Writing (·14)
9	Obedience (·13)	Language (·14)
10	History (·12)	Energy (·13)
11	Industry (·10)	Temper (·12)
12	Arithmetic (·09)	Industry (·11)
13	Energy (·08)	Attention (·11)
14	Language (·08)	Cleanliness (·10)
15	Temper (·06)	Obedience (·08)

In the first place, in actual studies it would seem best to classify the children according to the decile position they take in their respective classes or standards. The ages also should be given, so that the correlations may be corrected for any age influence. In the next place, characters like intelligence, attention, obedience, etc., should, where it is feasible, be divided into five rather than three sub-classes. This would permit of a 25-fold table and the contingency could thus be calculated with considerably closer approach to the true correlation. In making from the statistical side these suggestions for future observations, we do not wish to underrate the value or interest of Ivanoff's data, but merely to press for additional information which will much strengthen the quantitative reasoning on such material.

BOYS.

Intelligence.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	69	207	18	294
Moy ...	117	749	79	945
Fa ...	13	132	21	166
Totals	199	1088	118	1405

General Work.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	135	117	42	294
Moy ...	301	372	272	945
Fa ...	34	66	66	166
Totals	470	555	380	1405

Writing.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	82	182	30	294
Moy ...	180	556	209	945
Fa ...	25	76	65	166
Totals	287	814	304	1405

Geography.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	74	199	21	294
Moy ...	136	706	103	945
Fa ...	17	124	25	166
Totals	227	1029	149	1405

Manual Work.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	44	237	13	294
Moy ...	67	808	70	945
Fa ...	8	139	19	166
Totals	119	1184	102	1405

Composition.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	54	215	25	294
Moy ...	87	745	113	945
Fa ...	13	135	18	166
Totals	154	1095	156	1405

History.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	72	201	21	294
Moy ...	147	698	100	945
Fa ...	17	130	19	166
Totals	236	1029	140	1405

Language.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	100	80	114	294
Moy ...	276	284	385	945
Fa ...	39	63	64	166
Totals	415	427	563	1405

Arithmetic.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	144	71	79	294
Moy ...	366	314	265	945
Fa ...	69	47	50	166
Totals	579	432	394	1405

Attention.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	41	201	52	294
Moy ...	85	649	211	945
Fa ...	20	82	64	166
Totals	146	932	327	1405

Obedience.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	70	214	10	294
Moy ...	172	701	72	945
Fa ...	29	112	25	166
Totals	271	1027	107	1405

Energy.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	76	177	41	294
Moy ...	189	591	165	945
Fa ...	24	112	30	166
Totals	289	880	236	1405

Industry.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	47	227	20	294
Moy ...	83	780	82	945
Fa ...	21	129	16	166
Totals	151	1136	118	1405

Cleanliness.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	41	239	14	294
Moy ...	64	808	73	945
Fa ...	16	121	29	166
Totals	121	1168	116	1405

Temper.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	61	209	24	294
Moy ...	164	685	96	945
Fa ...	24	128	14	166
Totals	249	1022	134	1405

Fo signifies fort; Moy, moyen; Fa, faible.

GIRLS.

Intelligence.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	26	37	5	68
Moy ...	94	578	65	737
Fa ...	17	174	40	231
Totals	137	789	110	1036

General Work.

	Fo	Moy	Fa	Totals
Fo ...	43	21	4	68
Moy ...	279	270	188	737
Fa ...	45	97	89	231
Totals	367	388	281	1036

Writing.

	Fo	Moy	Fa	Totals
Fo ...	19	37	12	68
Moy ...	113	513	111	737
Fa ...	31	141	59	231
Totals	163	691	182	1036

Geography.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	24	39	5	68
Moy ...	74	600	63	737
Fa ...	27	152	52	231
Totals	125	791	120	1036

Manual Work.

	Fo	Moy	Fa	Totals
Fo ...	26	33	9	68
Moy ...	90	575	72	737
Fa ...	31	158	42	231
Totals	147	766	123	1036

Composition.

	Fo	Moy	Fa	Totals
Fo ...	22	39	7	68
Moy ...	92	571	74	737
Fa ...	17	173	41	231
Totals	131	783	122	1036

History.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	21	43	4	68
Moy ...	102	558	77	737
Fa ...	20	172	39	231
Totals	143	773	120	1036

Language.

	Fo	Moy	Fa	Totals
Fo ...	43	13	12	68
Moy ...	285	236	216	737
Fa ...	91	57	83	231
Totals	419	306	311	1036

Arithmetic.

	Fo	Moy	Fa	Totals
Fo ...	37	15	16	68
Moy ...	182	300	255	737
Fa ...	43	75	113	231
Totals	262	390	384	1036

Attention.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	18	42	8	68
Moy ...	96	523	118	737
Fa ...	26	158	47	231
Totals	140	723	173	1036

Obedience.

	Fo	Moy	Fa	Totals
Fo ...	15	49	4	68
Moy ...	139	561	37	737
Fa ...	31	192	8	231
Totals	185	802	49	1036

Energy.

	Fo	Moy	Fa	Totals
Fo ...	19	39	10	68
Moy ...	155	457	125	737
Fa ...	25	152	54	231
Totals	199	648	189	1036

Industry.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	16	50	2	68
Moy ...	121	576	40	737
Fa ...	21	196	14	231
Totals	158	822	56	1036

Cleanliness.

	Fo	Moy	Fa	Totals
Fo ...	7	56	5	68
Moy ...	93	596	48	737
Fa ...	12	199	20	231
Totals	112	851	73	1036

Temper.

	Fo	Moy	Fa	Totals
Fo ...	22	44	2	68
Moy ...	123	583	31	737
Fa ...	30	194	7	231
Totals	175	821	40	1036

Fo signifies fort; Moy, moyen; Fa, faible.

V. Variability in Shirley Poppies from Pretoria.

Dr Maynard sends me the following data for the variability of the stigmatic bands on three Shirley Poppies, self-sown from a single poppy. The first poppy had no less than 468 capsules, the second 425, the third had probably hardly fewer, but when about one-third of the capsules (123) had been counted, the garden boy removed the plant.

The results obtained were as follows :

	Capsules	Mean	S. D.
First Poppy	468	11·079	1·233
Second Poppy	425	12·595	1·519
Third Poppy	(123+)	13·085	—
Together	1016	11·955	1·576

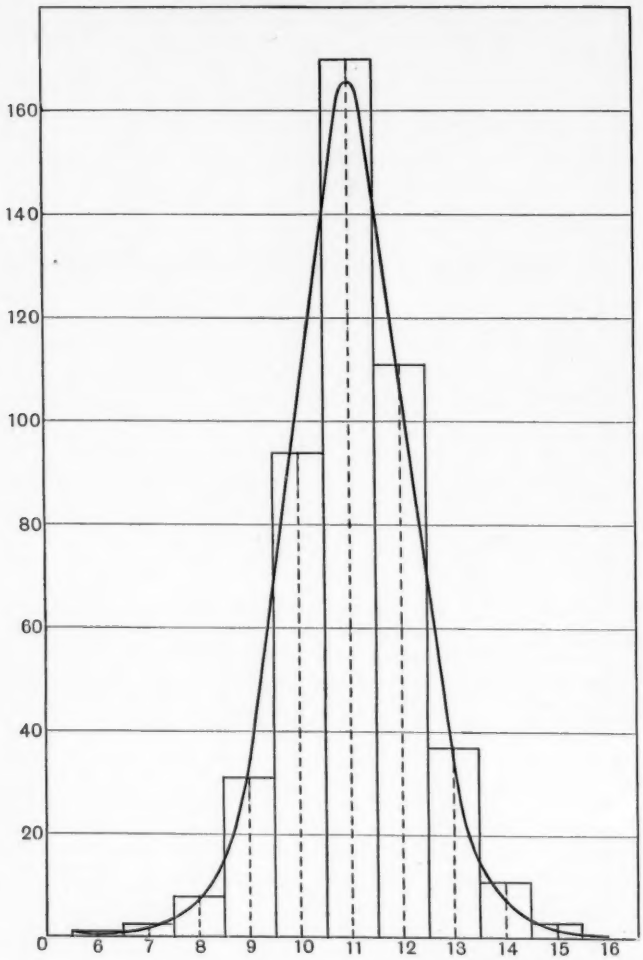
I found in 1899* that my two most prolific Shirley Poppies had 126 and 85 capsules respectively, giving S.D.'s of 1·329 and 1·589. There thus appears no striking increase in the variability of the individual plant after transfer to Pretoria. The mean values are possibly rather low, but plenty of individual poppies show as low a result. Only two out of 15 crops† gave a mean value less than 12 on English soil. On the whole the chief difference appears to be the prolific character of the Shirley Poppy in the Transvaal.

Dr Maynard has fitted his data to frequency curves, reproduced in the accompanying diagrams. It will be seen that the individual variation closely accords with Type II, or is symmetrical but hardly Gaussian. The total material exhibits a very slight skewness, caused probably by the heterogeneity resulting from three large groups. That this is not more marked is almost certainly due to the fact that the three poppies have all the same mother-plant, and were possibly the result of self-fertilisation.

K. P.

* *Phil. Trans.* Vol. 195, A, p. 316.
† *Biometrika*, Vol. IV. p. 401.

SHIRLEY POPPY. No. 1.



$N = 468$

$\sigma = 1{\cdot}233 \pm {\cdot}027$

$\mu_3 = -\ {\cdot}199 \pm {\cdot}143$

$\sqrt{\beta_1} = {\cdot}106 \pm {\cdot}076$

$\beta_2 = 4{\cdot}051 \pm {\cdot}153$

$\kappa = {\cdot}0042$

Mean $= 11{\cdot}079 \pm {\cdot}038$

TYPE II.

$m = -\ 5{\cdot}354$

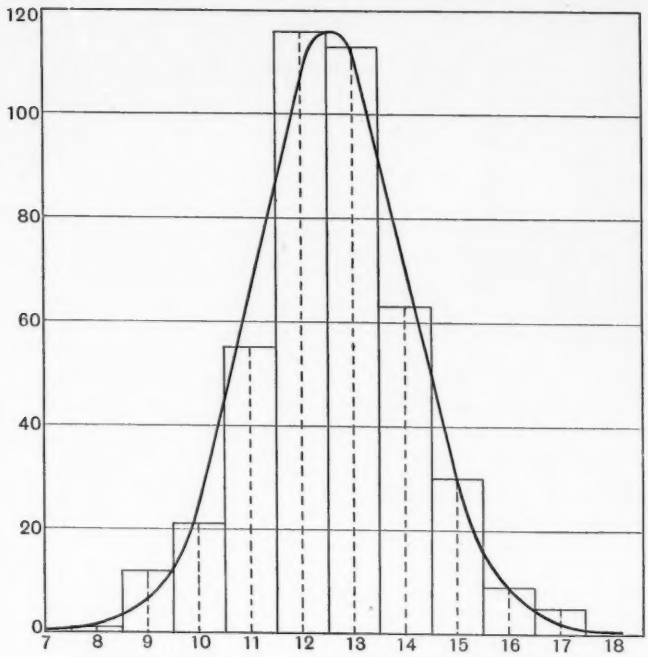
$a^2 = -11{\cdot}711$

BANDS ON CAPSULES

No.	Observed	Calculated
6	1	·37
7	2	1·65
8	8	7·83
9	31	34·36
10	94	106·84
11	170	165·70
12	111	106·84
13	37	34·36
14	11	7·83
15	3	1·65
16	0	·37
	468	467·8

$P = {\cdot}79$

SHIRLEY POPPY. No. 2.



$N = 425$

$\sigma = 1{\cdot}519 \pm {\cdot}035$

$\mu_3 = 0{\cdot}264 \pm {\cdot}286$

$\sqrt{\beta_1} = {\cdot}075 \pm {\cdot}081$

$\beta_2 = 3{\cdot}366 \pm {\cdot}160$

$\kappa = {\cdot}006$

Mean $= 12{\cdot}595 \pm {\cdot}050$

TYPE II.

$m = -10{\cdot}700$

$a^2 = -42{\cdot}470$

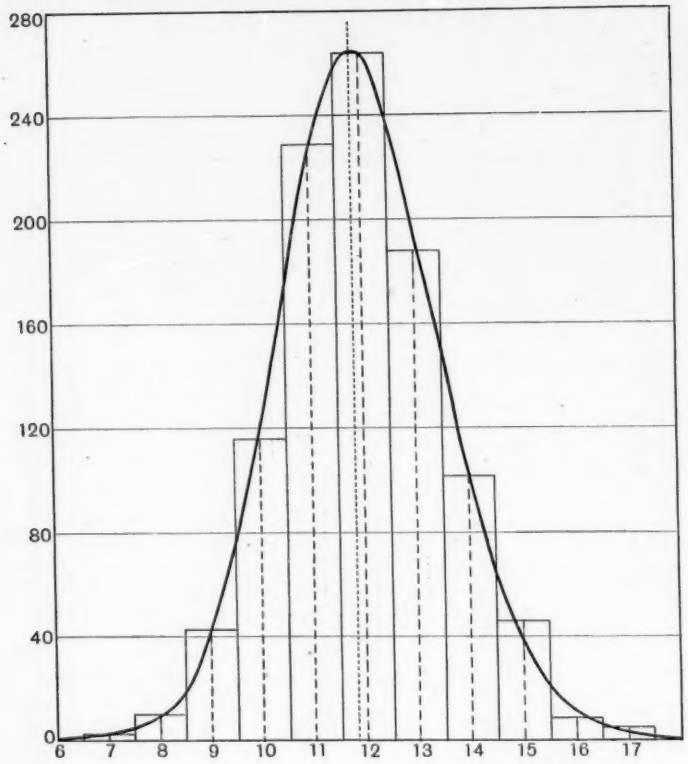
BANDS ON CAPSULES

No.	Observed	Calculated
7	0	·32
8	1	1·54
9	12	6·77
10	21	24·06
11	55	62·27
12	116	106·30
13	113	111·60
14	63	71·49
15	30	29·67
16	9	8·78
17	5	2·07
18	0	·43
	425	425·3

$P = {\cdot}34$

BANDS ON POPPY CAPSULES.

Three Plants. Nos. 1 and 2 + part of a third.



$N = 1016$
$\sigma = 1{\cdot}578 \pm {\cdot}024$
$\mu_3 = {\cdot}551 \pm {\cdot}203$
$\sqrt{\beta_1} = {\cdot}140 \pm {\cdot}052$
Mean $= 11{\cdot}956 \pm {\cdot}033$
Mode $= 11{\cdot}862$
$P = {\cdot}71$
$\kappa = {\cdot}027$
$\beta_2 = 3{\cdot}308 \pm {\cdot}104$
$r = 24{\cdot}694$
$m = 13{\cdot}347$
$v = -4{\cdot}098$
$a = 7{\cdot}575$

TYPE IV.

No.	Observed	Calculated
6	1	·25
7	2	1·75
8	9	9·66
9	43	40·74
10	116	118·70
11	229	223·5
12	264	264·2
13	189	200·0
14	102	102·7
15	46	38·71
16	9	11·61
17	6	3·0
	1016	1014·82

VI. Some Recent Criminological Works.

By CHARLES GORING, M.D.

In a bulky volume, entitled *Les femmes homicides**, Dr Pauline Tarnowsky records the results of an anthropometrical survey she has carried out upon 160 Russian women imprisoned for murder. The preface states, very modestly, the essential purpose of the author's undertaking. "I regard this book," she says, "as nothing more than a collection of material, and shall be happy if future workers may find some use for it in constructing the edifice of criminal anthropology which still awaits the great architect." Throughout the work, there is abundant evidence that Dr Tarnowsky has collected her material with a devotion and conscientious industry which insures its reliability; and these qualities, combined with the excellent arrangement and lucid exposition of her facts, cannot fail to make the book of value to all in need of criminal statistics.

The outline of the author's plan is, briefly, as follows. Each of the individuals studied was, firstly, submitted to physical measurement: all the principal measurements recognised by anthropologists were taken of head, face and body. Secondly, the majority of these criminals were examined with reference to the functional condition of their sense organs and, at the same time, the condition of their reflexes and muscular power and the presence of any deviation from the normal in physical structure, including the so-called stigmata of degeneration, were noted. Lastly, so far as they could be ascertained, the more important particulars of the family antecedents were noted for each individual:—such as the character, inclinations, habits, occupation and standard of living of the parents; special emphasis being laid upon the occurrence of heritable diseases and upon the more or less marked inclination towards alcoholic excess. Apart from some preliminary chapters dealing historically with general principles, the great bulk of the volume is made up of the records of these measurements and observations. The record of each individual is presented seriatim and upon a definite plan, and is supplemented in each case by a very readable and psychologically interesting narrative of the personal history of the murderess, the circumstances leading up to her crime, its probable motives, and the methods of its execution. Finally, the salient features contained in the records are brought together and classified in the form of tables. Accompanying tables of similar records relating to different classes of criminal and non-criminal Russian women are presented for purpose of comparison. The work concludes with an account of some general deductions which, in the opinion of the author, follow from the analytical and comparative study of her data. *Les femmes homicides* is a valuable contribution of conscientious spade-work, and as such will find general acceptance.

Having said this we are bound to add that, in the treatment of her evidence, the author has been animated too much by a spirit of partisanship for the old school of criminologists; we mean that school associated with the name of César Lombroso—to whom, by the way, the author dedicates her work. We think that the evidence from her data is altogether too slight to justify so emphatic and far-reaching a statement as the following, which the author affirms to be her fixed conviction—"that there exists an *enormous* physical and psychical difference between the female criminal and the normal woman of the same race and class and that this conclusion follows directly (1) from the measurements, (2) from the facts of heredity, and (3) from the evidence of the degeneration stigmata contained in the records." First, with regard to the evidence from the measurements. For comparison with the table of average measurements of "les femmes homicides," three corresponding tables are provided relating to (1) "femmes instruites,"

* Paris, Félix Alcan.

(2) "femmes villageoises illettrées," (3) "femmes voleuses et prostituées." As the author admits, between the murderesses and thieves the difference in the average measurements is insignificant and, we would add, is in every case within the limits of the probable error due to random sampling. But, says the author, "ce qu'il vous saute aux yeux" is the difference in length, breadth, and circumference of head between non-criminal peasant women and women of the criminal class. As a matter of fact what strikes the attention particularly is the precisely similar difference, in an opposite direction, between non-criminal peasant women and women of the educated class: and, the question arises, is not this difference due to the same cause in both cases? We note, from the records, that of the 160 individuals measured, five were of illegitimate parentage, and 13 were orphans from birth; and these, combined with the fact that 70°/₀ of the parents of the remainder were debauched with drink, suggest the provisional proposition that the descending scale of head measurements, from "les femmes instruites" at one extreme to "les femmes homicides" at the other, may be related to an increasing scale of poverty, and is more likely an expression of defective nutrition, during the early years of life and growth, than of inherited criminal diathesis.

The facts of heredity are interesting and suggestive, although we look in vain for the evidence in support of the alleged inheritance of a criminal type which the author contends is contained therein. We note from the records of these facts that 134 criminal children had 12 criminal parents, that seven criminal lunatics had two parents who were insane and two who were epileptic, and that 141 criminal children had 12 parents who died of phthisis. Now, while these figures suggest the existence of a positive correlation in the direct parental inheritance of criminality, insanity, and phthisis, their proportions are not in conformity with the plausible hypothesis in the mind of the author, that insanity and phthisis in one generation may influence the appearance of criminality in the next. In fact, from the evidence of the records, the only condition that might seem likely to give rise to physical degenerative changes associated with a criminal diathesis in the descendants, is parental alcoholism. The records give for the parents of "homicides," 71°/₀ alcoholic; for the parents of non-criminal peasant women, 16°/₀ alcoholic. However, in view of the fact, that the majority of the homicidal women were also themselves alcoholic, the significance of this alcoholism in the parents is somewhat modified. What we want to know is, not the relative incidence of alcoholism in the parents of criminal and non-criminal children, but its relative incidence amongst the parents of alcoholic children who are, or are not, criminal as well.

Of the value of the stigmata of degeneration, quoted as proof of the existence of a criminal type, we have only space here to say that the objection to this kind of observation, so popular with criminologists of the old school, is that no one can precisely define what it is that is being observed; and that the error due to any unconscious bias in the observer may be so large as to render the value of such evidence entirely nugatory. That the criminal women were found to have ten times as many stigmata as the women who were not criminal may seem to be conclusive evidence that ought to tell. Scientifically, however, such evidence is unfruitful and leaves the mind cold and unconvinced.

We wish we had more space in which to discuss some other points in this interesting book. But we have already strayed from our chief purpose which is to congratulate Dr Pauline Tarnowsky on her work and to recommend it to the attention of biometricians.

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*Britain's Blot** is the title of a book dealing with "Recidivism"—a general term including all kinds of Habitual Criminality and Habitual Petty Delinquency. The meaning of this figurative title is given in the introduction of the book. "The face of Society," writes the author, "is mottled all over by the actions of its law breakers and transgressors, and on this mottled area

* G. F. Sutherland, M.D., F.R.S.E.: W. Green and Sons, Edinburgh.

there stands out in bold relief the dark and discreditable blot of recidivism, perpetually hanging like a pall"..., &c. &c. The author has evidently had a wide experience of the evil he describes; but we are not convinced that he sees very clearly either the inherent nature or the intent or causation of this evil. For instance, in one part of the book the recidivist is described as "an irrepressible Frankenstein, rearing its hydra-head, and stalking through the land with limbs of brass." From this we take it that the recidivist looms in the imagination of the author as a monster, uncontrolled, and at large, a constant menace to Society, which is responsible for its creation. In another place, however, the writer states in plain, dogmatic English his confident belief that not only is every kind of social, economic, and environmental condition a principal cause of recidivism, but that another principal cause is to be found in the spiritual constitution of the delinquent. This second opinion, besides negativing the Frankenstein theory, leaves us in wonderment as to what other possible agency, apart from the supernatural, could be further quoted as a cause of recidivism. It is evident the book has been written to influence social reformers, and others of the general public interested in social questions, rather than to be a guide to serious students of sociology.

The subject of this book is treated in the main discursively, and with a general sprinkling of statistics that are more pretentious than helpful. In Chapter I, dealing with general statistics of crime, some figures and diagrams are presented of the number of the general population who are apprehended, imprisoned and convicted every year. It is stated that from these figures "Great Britain's recidivism can be seen at a glance"—a statement based upon the inference that the number of recidivists existing in prison corresponds to the total number of recidivists in existence; a most fallacious inference. To estimate the true dimensions of *Britain's Blot*, it is obviously necessary to know not only the number of imprisoned recidivists, relatively to the population at large, but also how many of the population at large are recidivists out of prison. That is to say, how many recidivists now free have already been in prison, and how many will find their way to prison before they die. No attempt has been made to answer this interesting, subtle and elusive statistical question. Chapters II and III deal with the age, education, occupation and geographical distribution of delinquents. The part played by illiteracy in the making of criminals, is asserted and deplored; as is also deplored the fact that a large majority of criminals are drawn from the labouring classes. In view of the fact that out of Britain's population of 38 millions, over 30 millions belong to the class of general labourers, and are presumably illiterate, it is quite idle, without the aid of a correlation calculus, to attempt to form any estimate of the causal association between these conditions and crime. In Chapter V, which consists of eight pages, a general survey is undertaken of no fewer matters than criminal anthropology, criminal anthropometry, criminal physiognomy, biometrics, and national eugenics. Some measurements are also given of the mean head length, head breadth, and stature of 370 criminals. The next chapter grants five pages to an exposition of criminal lunacy and eight pages to a discussion of legal and metaphysical conceptions of free will, criminal responsibility, and punishment. In Chapter VII, which deals with the causation of recidivism, degeneracy and heredity, answers to all of these problems are successfully begged in twelve pages. The conclusion that physical and mental characters are inherited in the same way, and at the same rate, seems to the author "a natural," as well as "a feasible" conclusion: but he asserts it to be "the climax of absurdity" to suppose that the criminal breeds the criminal. The remaining chapters deal with criminal jurisprudence and penology.

To those who prefer a wide approximate view to an accurate perception of detail, who prefer opinions to facts and rhetoric to figures the book should appeal sympathetically. To biometricians it will probably not be of very great interest.

VII. Note on Partial Leucosis in a Hen.

We owe the following account and photographs to the kindness of Mr W. Coles Finch of Luton, Chatham. Inquiry seems to show that the experience is uncommon among poultry-breeders, and its publication may lead to further observation of like phenomena, which would possibly throw light on whether plumage without moult can change by loss of pigment and thus otherwise than by abrasion.

The father of the hen in question was a pure Indian Game and the mother a true Houdan. Seven broods of this cross came under notice. In five the birds, watched to the third year, remained black. In the other two cases the broods of chickens were all true black. In one brood, however, one bird, a hen, turned from black to spotted in her first year, she has been spotted, not white, ever since, but the spots have diminished each year. Her present condition is given in Fig. 1. She has been crossed with an Indian Game and had two spotted black and white chickens. In the second brood one hen on the approach of her first winter gradually got paler and paler, turned into gray and finally pure white before the end of the year. No photograph unfortunately was taken. She remained white all next summer but after moult renewed her plumage to spotted black and white. She is shown in Fig. 2 with a normal sister for comparison. She remained spotted all next summer, but when her sisters moulted, she shed her tail-feathers only ; on wing and body feathers the black gradually changed to white. This commenced in beginning of October and by November 1 she was nearly quite white, as shown in Fig. 3*. By the middle of November she was practically pure white (the mark on wing occurring on one feather being the last vestige of black) as is shown in Fig. 4, a sister being given for comparison (the light colour is only reflection of sheen). On Fig. 5 black and white feathers plucked from time to time from October to November are given to indicate that the change appears to be one of pigment and scarcely of feather or due to abrasion. The bird has thus been once black, once white, then once spotted and again white. It was mated this year with an Indian Game but unfortunately has died without laying.

* The plumage is somewhat disarranged after a struggle in catching her.

Fig. 1.

FIG. 2.

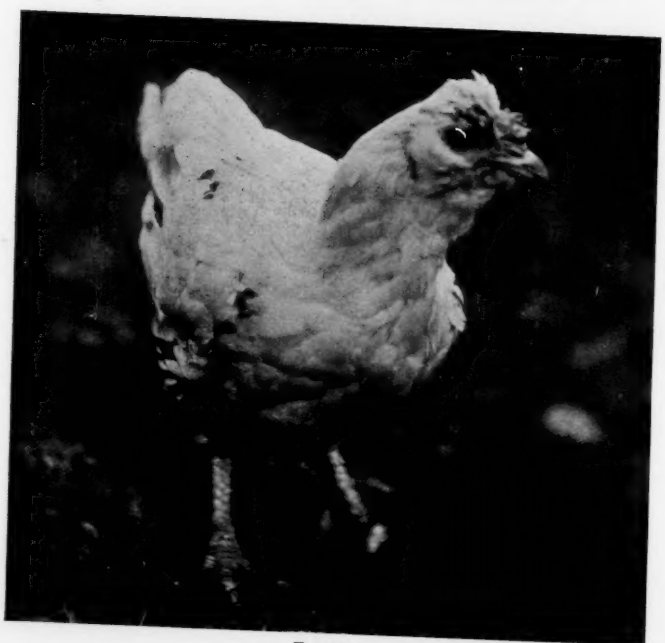

FIG. 3.

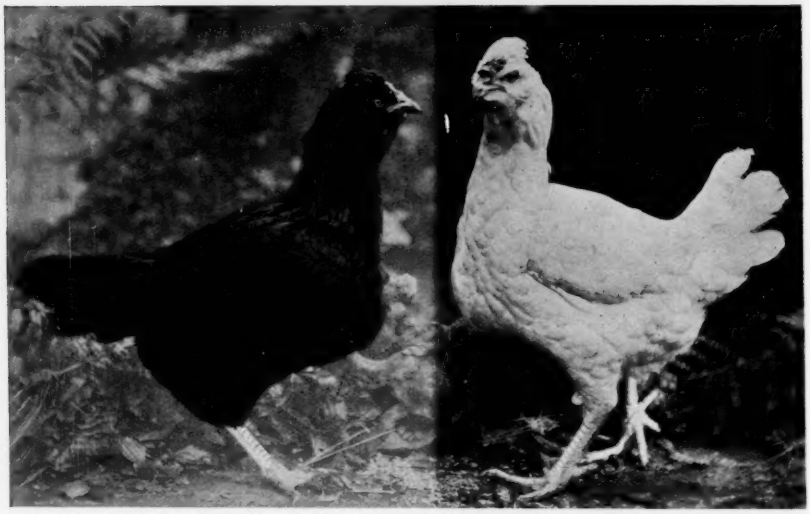

FIG. 4.

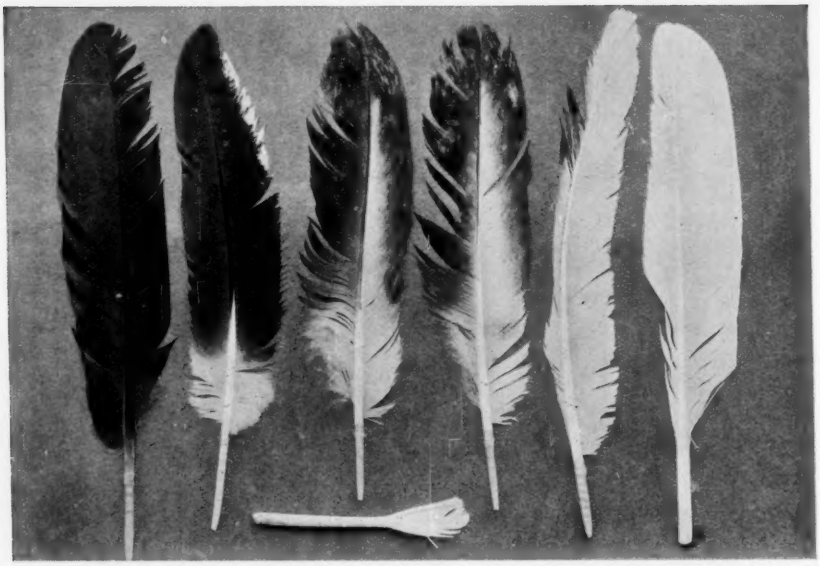

FIG. 5.